

# SYMPOSIUM DISCRETE MATHEMATICS

15<sup>th</sup>-16<sup>th</sup> June 2018

Graz University of Technology

Plenary Speakers:

László Babai  
Amin Coja-Oghlan  
Ioannis Emiris  
Penny Haxell  
Martin Skutella

Executive Committee:

Volker Kaibel  
Mihyun Kang  
Tibor Szabó  
Thorsten Theobald

On the occasion of the symposium, the

**Richard-Rado Prize 2018**

for a dissertation in Discrete Mathematics will be awarded.

More information:

<https://www.math.tugraz.at/comb/sdm2018/>



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## Local Information and Public Transport

On-site registration and all talks will take place in Steyrergasse 30 on the New Campus of Graz University of Technology (Neue Technik). A map of the campus and surrounding area is available on the last page of this booklet.

The closest bus/tram stop to the conference venue is **Neue Technik**.  
All trams and almost all buses pass through Jakominiplatz (Central Transfer Point).

### Tickets and Prices

The following standard tickets can be used for any number of journeys within Graz by bus, tram and train (including to/from the airport) within the stated time period.

Ticket	Price	Validity
1-hour ticket (Stundenkarte)	€2.30	one hour from purchase
24-hour ticket (24-Stundenkarte)	€5.10	24 hours from purchase (rolls over to next day)

Tickets can be bought: on the bus (from the driver); on the tram (from the ticket machine); at train stations *before travel* (from a ticket machine).

### Old Town Tram (Altstadtbim)

All tram rides are free along certain stretches around the old town (within one stop of either Hauptplatz or Jakominiplatz). These stretches are marked by yellow highlighting on maps of the tram network. The closest such stop to the conference venue is Dietrichsteinplatz.

### Airport

Graz airport is *small* – arriving one hour before your flight home is *plenty* of time.  
To get to/from the airport you can use

Bus 630	from/to Jakominiplatz
Train S5	from/to Main station (Hauptbahnhof)

### Main Train Station

Train station → conference venue: Tram 6 towards St Peter, alight at Neue Technik.  
Conference venue → train station: Tram 6 towards Laudongasse, alight at Hauptbahnhof.

### Taxis

There are taxi ranks at the airport and the main station, as well as at Dietrichsteinplatz and Moserhofgasse/Petersgasse near the conference venue.

You can also order a taxi from one of the following companies:

Taxi 878	+43 316 878
Taxi 2801	+43 316 2801
ETaxiGraz	+43 664 4831413

A taxi from the airport to the conference venue typically costs around €25-30, from the main station around €10-15.

**More information on public transport can be found at [www.holding-graz.at](http://www.holding-graz.at).**

# Programme and Abstracts

## Friday, 15<sup>th</sup> June

8:30 – 9:00	Registration
9:00 – 9:15	Opening
9:15 – 10:15	<b>Amin Coja-Oghlan</b> <i>The satisfiability threshold for random linear equations</i>
10:15 – 10:45	Coffee break
10:45 – 11:15	<b>Kevin Schewior</b> <i>Handling Critical Tasks Online: Deadline Scheduling and Convex-Body Chasing</i>
11:15 – 11:45	<b>Maximilian Merkert</b> <i>Staircase Compatibility – a Generalization of Structures from Scheduling and Piecewise Linearization</i>
11:45 – 13:30	Lunch break
13:30 – 14:30	<b>Ioannis Emiris</b> <i>Randomized projections for geometric search in high dimension</i>
14:30 – 15:00	Coffee break
15:00 – 15:30	<b>Benjamin Schröter</b> <i>Matroidal Subdivisions, Dressians and Tropical Grassmannians</i>
15:30 – 16:00	<b>Rainer Sinn</b> <i>Graph invariants from positive semidefinite matrix completion</i>
16:00 – 16:30	Coffee break
16:30 – 17:30	<b>Martin Skutella</b> <i>Flows over time and submodular function minimization</i>
18:30 – 21:00	Conference dinner

## Saturday, 16<sup>th</sup> June

9:00 – 10:00	<b>Penny Haxell</b> <i>Independent transversals vs. small dominating sets</i>
10:00 – 10:30	Coffee break
10:30 – 11:00	<b>Michael Wallner</b> <i>Asymptotic Enumeration of Compacted Binary Trees</i>
11:00 – 11:30	<b>Frank Mousset</b> <i>The probability of non-existence in binomial subsets</i>
11:30 – 12:00	<b>Christoph Koch</b> <i>Phase transition phenomena and the evolution of random hypergraphs</i>
12:00 – 12:30	Meeting of the Fachgruppe
12:30 – 14:30	Lunch break
14:30 – 15:00	<b>2018 Richard-Rado Prize Award Ceremony</b>
15:00 – 15:50	<b>Stefan Glock</b> <i>Decompositions of graphs and hypergraphs</i>
15:50 – 16:20	Coffee break
16:20 – 17:20	<b>László Babai</b> <i>Groups, Graphs, Algorithms: The Graph Isomorphism Problem</i>
17:20 – 17:30	Closing

# The satisfiability threshold for random linear equations

AMIN COJA-OGHLAN

Goethe University Frankfurt

Friday, 15<sup>th</sup> June, 9:15 – 10:15

Let  $A$  be a random  $m \times n$  matrix over a finite field. We identify the threshold  $m/n$  up to which the linear system  $Ax = y$  has a solution with high probability and analyse the geometry of the set of solutions. In the special case  $q = 2$ , known as the random  $k$ -XORSAT problem, the threshold was determined by [Dubois and Mandler 2002, Dietzfelbinger et al. 2010, Pittel and Sorkin 2016]. The proof technique was subsequently extended to the cases  $q = 3, 4$  [Falke and Goerdts 2012]. But the argument depends on technically demanding second moment calculations that do not generalise to  $q > 3$ . Here we approach the problem from the viewpoint of a coding theory task, which leads to a transparent combinatorial proof.

# Handling Critical Tasks Online: Deadline Scheduling and Convex-Body Chasing

KEVIN SCHEWIOR

École Normale Supérieure Paris

Friday, 15<sup>th</sup> June, 10:30 – 11:00

Completing possibly unforeseen tasks in a timely manner is vital for many real-world problems. Under the hard constraint of timely fulfilling these tasks, one usually still wishes to use resources conscientiously. In this thesis, we consider such a setting in the form of two different fundamental mathematical models. We develop various online algorithms for them, that is, algorithms that do not require knowledge about future tasks. Our results improve upon previously known bounds on the resource usage, and thereby we answer long-standing questions. In this talk, we focus on Online Machine Minimization, where jobs arrive over time at their release dates and need to be scheduled preemptively on parallel identical machines until their deadlines. Here, the performance of a schedule is measured in the number of machines it requires. It has been an open question whether algorithms requiring a constant number of machines exist when the offline optimal number  $m$  of machines is constant [Phillips et al., STOC 1997 and Algorithmica 2002]. We answer this question in the affirmative by presenting a new algorithm and an analysis showing that  $O(m \log m)$  machines are sufficient for the algorithm to finish all jobs by their deadlines. We then review recent improvements of this algorithm [Azar and Cohen, OR Letter 2018; Im et al., RTSS 2017], which also utilize our analysis. We finally consider variants and special cases of the same problem.

# Staircase Compatibility – a Generalization of Structures from Scheduling and Piecewise Linearization

MAXIMILIAN MERKERT  
University Erlangen-Nürnberg  
Friday, 15<sup>th</sup> June, 11:00 – 11:30

We consider multiple-choice feasibility problems in which the items chosen have to fulfill a given pairwise compatibility relation. This is motivated by two applications that contain such a combinatorial problem as a substructure. One of them is on energy-efficient railway timetabling while the other arises in the context of piecewise linearization of nonlinear problems on transportation networks. The new definition of staircase compatibility generalizes key properties of both special cases which allow for a totally unimodular linear programming formulation of polynomial size. Furthermore, its constraint matrix is cographic, leading to a representation as a dual network flow problem. Computational results show that for the two applications mentioned above, using the resulting reformulations for representing the compatibility structure can make a huge difference with respect to solution times. Finally, we discuss issues on the recognizability of staircase compatibility in general multiple-choice problems.

## Randomized projections for geometric search in high dimension

IOANNIS EMIRIS  
NK University of Athens  
Friday, 15<sup>th</sup> June, 13:30 – 14:30

Typical algorithms for geometric search, even in the approximate domain, exhibit complexity exponential in the ambient dimension which, for a number of important applications, leads to the so-called curse of dimensionality. These algorithms usually rely on tree-based data structures. There is a significant body of work on methods addressing the curse by means of randomized data structures based on hashing, known as Locality Sensitive Hashing. We follow a completely different tack employing randomized projections, which are simpler to define and implement, but have long been overlooked since they did not seem powerful enough. We manage to show that they achieve competitive construction time and query time complexities as well as smaller space consumption for the fundamental question of approximate nearest-neighbor search in Euclidean spaces. The main idea is to establish that an approximate nearest neighbor lies among the preimages of a certain number of approximate nearest neighbors in the projection. Our algorithms yield relevant open-source implementations, which are compared to state of the art software. We extend our approach to more general metric spaces, including products of Euclidean metrics.

# Matroidal Subdivisions, Dressians and Tropical Grassmannians

BENJAMIN SCHRÖTER

TU Berlin

Friday, 15<sup>th</sup> June, 15:00 – 15:30

Classically the Grassmannian over some field is the moduli space of  $d$ -dimensional linear subspaces in a  $n$ -dimensional space. Its tropicalization yields a pure fan which only depends on the characteristic of the underlying field. As a set the tropical Grassmannian is contained in the Dressian, i.e., the moduli space of all tropical linear spaces. The Dressian is the non-pure subfan of matroidal subdivisions in the secondary fan of the hypersimplex. The relation between these fans is a field of active research that involves combinatorics, polyhedral geometry and algebraic geometry. In this talk we present constructions for rays in the Dressian that highlight the differences of Grassmannians and the Dressian. This construction relies on two ingredients. The first is a new class of matroids, that we call split matroids. A split is a subdivision into two maximal cells and corresponds to a ray in the secondary fan and in our situation also of the Dressian. The other ingredient is a special lifting vector that depends on a matroid, we call this vector the corank vector. We show that a corank vector lies in a tropical Grassmannian if and only if the matroid is representable by a point configuration in a vector space.

# Graph invariants from positive semidefinite matrix completion

RAINER SINN

FU Berlin

Friday, 15<sup>th</sup> June, 15:30 – 16:00

We will discuss the positive semidefinite matrix completion problem and how it leads to two graph invariants called generic completion ranks and maximum likelihood threshold. We discuss their relations to other invariants and some recent results, which are joint work with Greg Blekherman and Mauricio Velasco.

# Flows over time and submodular function minimization

MARTIN SKUTELLA

TU Berlin

Friday, 15<sup>th</sup> June, 16:30 – 17:30

Flow variation over time is an important feature in network flow problems arising in various real-world applications. Usually, flow does not travel instantaneously through a network but requires a certain amount of time to travel through each arc. Thus, when routing decisions are being made in one part of a network, the effects can be seen in other parts only after a certain time delay. Not only the amount of flow to be transmitted but also the time needed for the transmission plays an essential role. The lecture gives an introduction into the fascinating area of flows over time and, in particular, covers recent results on deep connections to submodular function minimization.

## Independent transversals vs. small dominating sets

PENNY HAXELL

University of Waterloo

Saturday, 16<sup>th</sup> June, 9:00 – 10:00

An *independent transversal* (IT) in a vertex-partitioned graph  $G$  is an independent set in  $G$  consisting of one vertex in each partition class. Many combinatorial problems can be formulated as existence problems for IT's in suitably chosen graphs. There are several known criteria that guarantee the existence of an IT, of the following general form: the graph  $G$  has an IT unless the subgraph  $G_S$  of  $G$ , induced by the union of some subset  $S$  of vertex classes, has a small dominating set. The known proofs of these criteria do not give efficient *algorithms* for actually finding an IT or a subset  $S$  of classes such that  $G_S$  has a small dominating set. Here we present appropriate weakenings of such results that do have effective proofs. We also discuss *stability* analogues for these criteria, that give detailed structural information on the near-extremal configurations.

## Asymptotic Enumeration of Compacted Binary Trees

MICHAEL WALLNER

University of Bordeaux

Saturday, 16<sup>th</sup> June, 10:30 – 11:00

Trees are a widely-used data structure in computer science. When storing such trees it is useful to consider compression techniques. A simple idea is to store isomorphic subtrees only once and mark repeated occurrences with a pointer. The resulting structure is called a compacted tree, being in fact no more a tree but a directed acyclic graph. Our goal is the enumeration of such structures. Since the enumeration turns out to be extremely difficult, we restrict it to a sub problem by imposing a bound on the so-called right height. We solve this enumeration problem with the help of generating functions. Due to the superexponential growth of the counting sequence we use exponential generating function despite the fact that these objects are unlabeled. We first derive a calculus on exponential generating function capturing their recursive nature. This leads to a sequence of differential equations for the generating functions (also implying their D-finiteness) for which a singularity analysis is carried out. At the end we comment on the unrestricted case and recent progress. This work is based on joint work with Antoine Genitrini, Bernhard Gittenberger, and Manuel Kauers.



# The probability of non-existence in binomial subsets

FRANK MOUSSET

Tel Aviv University

Saturday, 16<sup>th</sup> June, 11:00 – 11:30

Let  $F$  be a fixed graph. In this talk, we will consider the question of finding precise estimates for the probability that the binomial random graph  $G(n, p)$  avoids all copies of the graph  $F$ . I will focus specifically on the regime where the density  $p$  is small enough to ensure that the probability in question is still determined by essentially ‘local’ considerations (e.g., if  $F$  is 2-balanced, then  $p$  should be asymptotically smaller than the 2-density of  $F$ ). In this regime, a famous result of Janson (1987) states that the probability is given by  $\exp(-k_1 + O(k_2))$ , where  $k_1$  is the expected number of copies of  $F$  in  $G(n, p)$ , and  $k_2$  is a measure for the pairwise dependencies among the indicator variables for the different copies of  $F$  in  $G(n, p)$ . I will show that, more precisely, the probability is given by the formula  $\exp(-k_1 + k_2 - k_3 + \dots + O(k_{k+1}))$ , where each  $k_i$  is a natural measure for the  $k$ -wise dependencies among the indicator variables for the different copies of  $F$ . More precisely,  $k_i$  is the sum of all joint cumulants of the indicator variables for exactly  $i$  copies of  $F$ . This allows us to calculate the probability of being  $F$ -free to any desired degree of accuracy by evaluating sufficiently (but finitely) many values  $k_i$ . This method applies not just to random graphs but also to the more general problem of determining the probability that a  $p$ -random subset of the vertices of a given hypergraph forms an independent set in the hypergraph. Joint work with A. Noever, K. Panagiotou, and W. Samotij.

# Phase transition phenomena and the evolution of random hypergraphs

CHRISTOPH KOCH

University of Oxford

Saturday, 16<sup>th</sup> June, 11:30 – 12:00

A random (hyper-)graph exhibits a phase transition if there exists a critical value (called *threshold*) for some parameter (such as the edge density) that triggers a sudden drastic change in its structure whenever the parameter exceeds this threshold. The focus of this talk lies on the evolution of *high-order connected components* in  $k$ -uniform binomial random hypergraphs  $H^k(n, p)$ : for any integer  $1 \leq j \leq k - 1$ , two  $j$ -sets ( $j$ -tuples of distinct vertices) lie in the same  $j$ -component if there is a sequence of edges from one to the other such that consecutive edges intersect in at least  $j$  vertices. It is worth noting that  $j$ -components have a particularly rich and diverse structure: for instance, any given vertex may be covered by a large number of  $j$ -components (when  $j \geq 2$ ). To approach the challenges arising from this structural complexity, we develop the concept of smoothness for hypergraphs. In particular, we prove that reasonably large  $j$ -components in random hypergraphs are evenly distributed over all  $\ell$ -sets, where  $1 \leq \ell \leq j - 1$ . Based on this notion we analyse the emergence of the giant  $j$ -component and the thresholds for  $j$ -connectedness in  $H^k(n, p)$ , for all  $1 \leq j \leq k - 1$ . Moreover, we determine the threshold for jigsaw percolation, which aims to explain how a collective creative process of a group of individuals can achieve extraordinary results, such as a scientific breakthrough. The results in this talk are based on joint work with B. Bollobás, O. Cooley, and M. Kang.

# Decompositions of graphs and hypergraphs

STEFAN GLOCK

University of Birmingham

Saturday, 16<sup>th</sup> June, 15:00 – 15:50

In a recent breakthrough, Peter Keevash proved the existence conjecture for combinatorial designs, which has its roots in the 19<sup>th</sup> century. Using the iterative absorption method, we gave a new proof of this result and strengthened Keevash’s result in two directions: Firstly, our main result applies to decompositions into any uniform hypergraph (rather than just cliques), which generalises a fundamental theorem of Wilson to hypergraphs. Secondly, our proof framework applies beyond the quasirandom setting, enabling us e.g. to deduce a minimum degree version. (Similar results have subsequently also been proved by Keevash.) In this talk, we will present our new results within a brief outline of the history of the existence conjecture and sketch our proof method. We shall also discuss related results on so-called sparse designs and the decomposition threshold of any given graph. Based on joint work with Daniela Kühn, Allan Lo and Deryk Osthus.

# Groups, Graphs, Algorithms: The Graph Isomorphism Problem

LÁSZLÓ BABAI

University of Chicago

Saturday, 16<sup>th</sup> June, 16:20 – 17:20

Deciding whether two given finite graphs are isomorphic has long been known as one of a small number of natural computational problems with unsettled complexity status within the P/NP theory.

Building on a group theoretic “Divide and Conquer” framework introduced in a seminal 1980 paper by Eugene M. Luks, recent algorithmic progress on this problem has involved an interplay between finite permutation groups and highly regular partitions of the complete graph called “coherent configurations.”

The talk will attempt to illustrate some of the components of this work.

## Conference Dinner

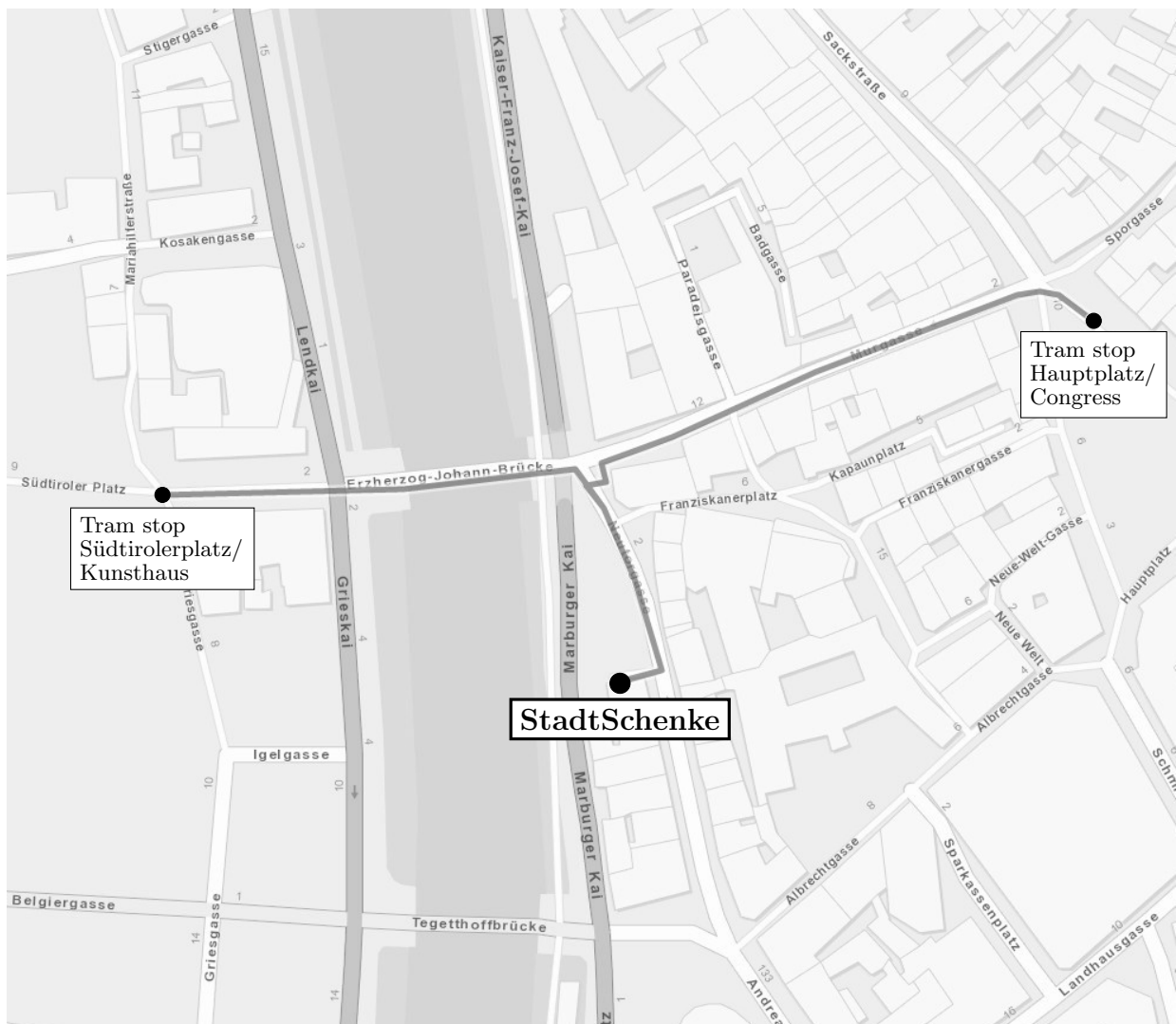
The conference dinner will take place on Friday evening, 6:30pm, at the restaurant

### StadtSchenke

Neutorgasse 22  
8010 Graz.

A group of local people will start walking to the restaurant from the conference venue at 6pm. If you want to join us, please make sure to gather in front of the lecture hall on time.

For those who plan to make their own way: the nearest tram stops are Hauptplatz/Congress and Südtirolerplatz/Kunsthhaus – both are within 300 metres walking distance.



## Conference Venue

On-site registration and all talks will take place on the ground floor of Steyrergasse 30 on the New Campus of Graz University of Technology (Neue Technik). The building is approximately 250 metres from the nearest tram stop Neue Technik.

The building has two entrances on its western edge. There is a step-free entrance on the eastern edge, from the parking lot at Steyrergasse. This entrance leads to the basement, from which the ground floor can be accessed using the elevator.

