

**Mini-Workshop**  
**on "Probabilistic Combinatorics, Random Graphs, and Graph Theory"**

**Tuesday, 12 June 2012, TU Graz**

-----  
**Schedule and Place**  
-----

10:00 – 10:50, AE01, Anna Huber, Durham University  
"Randomized Rumor Spreading"

11:00 – 11:50, AE01, Kolja Knauer, TU Berlin  
"Three ways to cover a graph"

14:10 – 15:00, C208, Philipp Sprüssel, University of Haifa  
"Independent systems of representatives"

15:10 – 16:00, C208, Oliver Cooley, TU München  
"A hypergraph analogue of the Erdős-Gallai Theorem via a simplified  
hypergraph regularity lemma"

18:30 – 19:20, C209, Dieter Mitsche, Ryerson University, Toronto (Webkonferenz)  
"A new upper bound for 3-SAT"

-----  
The lecture rooms AE01, C208 and the office C209 are located at the Mathematics Building at Steyrergasse 30 at the Campus Neue Technik of TU Graz. These lecture rooms are equipped with large blackboards, one overhead projector and one digital projector.

The directions to the Mathematics Building can be found at  
<http://www.math.tugraz.at/mathb/index.php?link=contact>

For further information on the workshop please contact  
Univ.-Prof. Mihyun Kang, [kang@math.tugraz.at](mailto:kang@math.tugraz.at)

---

## Titles and Abstracts

---

Speaker: Anna Huber, Durham University

Titel: Randomized Rumor Spreading

Abstract: Randomized rumor spreading is a protocol for disseminating information on graphs.

The classical version of it has been introduced and first investigated by Frieze and Grimmett on the complete graph (1985). To start with, one vertex of a finite, undirected, connected graph has some piece of information ("rumor"). In each round, every vertex that knows the rumor informs a neighbor chosen uniformly at random. As a result, the neighbor vertex now also knows the rumor and begins to inform its neighbors in the next round.

Frieze and Grimmett showed that on the complete graph, the time needed to inform all  $n$  vertices is within  $(1 \pm o(1)) (\log_2 n + \ln n)$  with probability  $1 - o(1)$ .

I will talk about the performance and robustness of this protocol on random graphs  $G_{n,p}$ , and show that the same bounds as for the complete graph are achieved, as long as  $p \geq \frac{\alpha(n) \ln n}{n}$  for any function  $\alpha$  that tends to infinity as  $n$  grows.

I will also consider a quasirandom version of the rumor spreading protocol, proposed by Doerr, Friedrich, and Sauerwald (2008). The basic setup is the same as in the randomized rumor spreading model, where in each round every informed vertex contacts a neighbor. But in this model each vertex has a fixed, cyclic list of its neighbors which dictates the order in which the vertex contacts them. The first neighbor to be contacted is determined by choosing a starting position in this cyclic list at random, independently of the choices of the other vertices. From that point onwards, in each round the vertex contacts the next vertex on its list.

I will present the evolution of the quasirandom rumor spreading protocol and show that, on the complete graph, its performance and robustness match performance and robustness of the randomized rumor spreading protocol.

This is joint work with Benjamin Doerr, Spyros Angelopoulos, Nikolaos Fountoulakis, Ariel Levavi, and Konstantinos Panagiotou.

---

Speaker: Kolja Knauer, TU Berlin

Title: Three ways to cover a graph

Abstract: We consider the problem of covering a host graph  $G$  with several graphs from a fixed template class  $T$ . The classical covering number of  $G$  with respect to  $T$  is the minimum number of template graphs needed to cover the edges of  $G$ . Parameters that arise this way are for example thickness, track-number and all kinds of arboricities.

We introduce two new covering parameters: the local and the folded covering number. As in the global covering number each measures how far  $G$  is from the template class in a different way. The folded covering number has been investigated thoroughly for some template classes, e.g., interval graphs and planar graphs, yielding interval and splitting number, respectively. The local covering number was given only little attention.

The three covering numbers presented not only unify the notion in the literature, they as well seem interesting in their own right, e.g., provide new approaches to attack or support classical open

problems.

We provide new bounds on some covering numbers w.r.t. several template classes. The classical graph parameters turning up this way are interval-number, track-number, and linear-, star-, and caterpillar arboricity. As host graphs we consider graphs of bounded degeneracy, bounded degree, or bounded tree-width, as well as, outerplanar, planar bipartite and planar graphs. We also discuss some algorithmical questions.

This is joint-work with Torsten Ueckerdt.

-----

Speaker: Philipp Sprüssel, University of Haifa

Title: Independent systems of representatives

Abstract: Hall's theorem provides a necessary and sufficient condition for the existence of an injective choice function for a given collection of sets. In a more general setting, some structure is given on the union of these sets and the requirement is added that the range of the choice function belongs to this structure. For example, that the range is independent in some given matroid (this is the setting of Rado's theorem) or independent in some given graph. The elements chosen are then called an "independent system of representatives" (ISR). Many problems can be formulated in this setting for example coloring of graphs, or list coloring. ISR problems are typically NP-hard, so no necessary and sufficient condition is expected to be found for their existence, but some topological and algebraic tools have been developed that provide sufficient conditions. The talk provides an introduction to ISRs as well as presents some of the most recent results.

-----

Speaker: Oliver Cooley, TU München

Title: A hypergraph analogue of the Erdős-Gallai Theorem via a simplified hypergraph regularity lemma

Abstract: Erdős and Gallai determined the number of edges required in an  $n$ -vertex graph to guarantee the existence of a path or cycle of length  $l$ . We introduce a hypergraph analogue: We give an upper bound on the number of edges required in an  $n$ -vertex,  $k$ -uniform hypergraph to guarantee the existence of a tight path or cycle of length  $l$ . Our bound is asymptotically best possible up to lower order terms. The proof utilises a new, simplified form of the notoriously difficult strong hypergraph regularity lemma. In this talk I will briefly describe the strong hypergraph regularity lemma and sketch how we derive our simplified version from it using probabilistic methods.

Based on joint work with Peter Allen, Julia Böttcher and Richard Mycroft.

-----

Speaker: Dieter Mitsche, Ryerson University, Toronto

Title: A new upper bound for 3-SAT

Abstract: We present a new upper bound for randomly chosen 3-CNF-formulae. In particular we show that a random formula over  $n$  variables with clauses-to-variables ratio of at least 4.4898 is, as  $n$  grows large, asymptotically almost surely unsatisfiable. The previous best such bound, due to Dubois in 1999, was 4.506. The first such bound, independently discovered by many groups of researchers since 1983, was 5.19. Several decreasing values between 5.19 and 4.506 were published in the years between. Whereas the improvement is small, our focus is on the methods used. The probabilistic methods we use for the proof are, we believe, of independent interest.

Joint work with J. Diaz, L. Kirousis and X. Perez-Gimenez.