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Improving Christofides' Algorithm for the s-t Path TSP

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Abstract:

We present a deterministic (1+sqrt(5))/2-approximation algorithm for the s-t path TSP for an arbitrary metric. Given a symmetric metric cost on n vertices including two prespecified endpoints, the problem is to find a shortest Hamiltonian path between the two endpoints; Hoogeveen showed that the natural variant of Christofides' algorithm is a 5/3-approximation algorithm for this problem, and this asymptotically tight bound in fact had been the best approximation ratio known until now. We modify this algorithm so that it chooses the initial spanning tree based on an optimal solution to the Held-Karp relaxation rather than a minimum spanning tree; we prove this simple but crucial modification leads to an improved approximation ratio, surpassing the 20-year-old barrier set by the natural Christofides' algorithm variant. Our algorithm also proves an upper bound of (1+sqrt(5))/2 on the integrality gap of the path-variant Held-Karp relaxation. The techniques devised in this paper can be applied to other optimization problems as well: these applications include improved approximation algorithms and improved LP integrality gap upper bounds for the prize-collecting s-t path problem and the unit-weight graphical metric s-t path TSP.

This is joint work with Bobby Kleinberg and David B. Shmoys.

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