Polynomial bound on the expected value of the cardinality of the ILP constraint set for the traveling salesman problem in random Euclidean graphs

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The *traveling salesman problem* (*TSP*) is one of the most studied combinatorial optimization problems. Given a complete graph $G = (V, E)$ and distances $d\colon E(G) \to \mathbb{R}_0^+$, the *TSP* asks for a shortest tour with respect to the distances d.

Let $\delta(v) := \{e = (v, u) \in E \mid u \in V\}$ denote the set of all edges adjacent to $v \in V$. Introducing binary variables x_e for the possible inclusion of any edge $e \in E$ in the tour we get the following classical ILP formulation:

$$\begin{aligned}
\text{minimize} \quad & \sum_{e \in E} d_e x_e && && (1)\\
\text{s.t.} \quad & \sum_{e \in \delta(v)} x_e = 2 && \forall\, v \in V, && (2)\\
& \sum_{\substack{e=(u,v)\in E\\ u,v \in S}} x_e \leq |S| - 1 && \emptyset \neq S \subset V, && (3)\\
& x_e \in \{0,1\} && \forall\, e \in E && (4)
\end{aligned}$$

In general, this ILP formulation contains exponentially many constraints, but for an optimal solution of a concrete instance there exists a minimal set of subtours $\mathcal{S}^*$, such that the ILP model with only those subtour constraints implied by $\mathcal{S}^*$ yields an overall feasible, and thus optimal solution.

A key question is whether the expected value of $|\mathcal{S}^*|$ is polynomially bounded for random Euclidean graphs. We introduce an algorithm which yields an upper bound and present some ideas and observations which may lead to the full theoretical analysis of this algorithm.

This presentation does not provide a solution of the problem, it just states an open problem together with some research ideas and partial minor results.

Keywords: traveling salesman problem; polyhedra theory; random Euclidean graphs

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