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DISCRETE MATHEMATICS
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Institut für Optimierung und Diskrete Mathematik

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Finding Needles in Exponential Haystacks

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We discuss two recent methods in which an object with a certain property is sought. In both, using a straightforward random object would succeed with only exponentially small probability. The new randomized algorithms run efficiently and also give new proofs of the existence of the desired object. In both cases there is a potentially broad use of the methodology.

(i) Consider an instance of k-SAT in which each clause overlaps (has a variable in common, regardless of the negation symbol) with at most d others. Lovasz showed for certain d,k (regardless of the number of variables) the conjunction of the clauses was satisfiable. The new approach due to Moser is to start with a random true-false assignment. In a WHILE loop, if any clause is not satisfied we quot;fix itquot; by a random reassignment. The analysis (due, basically, to Don Knuth) of the algorithm is unusual, connecting the running of the algorithm with certain Tetris patterns, and leading to some algebraic combinatorics. [These results apply in a quite general setting with underlying independent "coin flips" and bad events (the clause not being satisfied) that depend on only a few of the coin flips.]

(ii) No Outliers. Given n vectors n-space with all coefficients between -1 and +1 one wants a vector x with all coordinates -1 or +1 so that its dot products with all the n vectors are at most K times the square root of n in absolute value, K an absolute constant. A random x would make the dot product Gaussian but there would be outliers. The existence of such an x was first shown by the speaker. The first algorithm was found by Nikhil Bansal. The approach here, due to Lovett and Meka, is to begin with x the all zero vector and let it float in a kind of restricted Brownian Motion until all the coordinates hit the boundary.

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