Graz University of Technology

Institut für Diskrete Mathematik

Vortrag im Seminar für Kombinatorik und Optimierung

Freitag 1. Dezember, 14:00
Kaffeepause 13:30 (Steyrergasse 30, 2. Stock, Mathematik)

Hörsaal BE01, Steyrergasse 30, Erdgeschoss

The Partition Adjacency Matrix realization problem

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On Facebook, people with high number of connections tend to be connected more likely than randomness would suggest, while in biological networks vertices with high number of connections tend to be connected less likely than randomness would suggest. In terms of network science, the first network is assortative, while the second is disassortative.

Degrees (number of connections) do not tell if a network is assortative or disassortative. The Joint Degree Matrix (JDM) of a network (graph) counts number of edges between the sets of degree i and degree j vertices, for any i, j. The JDM realization problem asks whether a graph exists with prescribed number of connections (degree) at the vertices, and with prescribed number of edges between the sets of degree i and degree j vertices, for any i, j. The JDM realization problem is well understood. The usual measure for assortativity, the assortativity coefficient, the Pearson correlation coefficient of degree between pairs of linked nodes, is computable from the JDM.

A further generalization of the JDM is the following. Given a set W and numbers $d(w)$ associated with $w \in W$, and a $W_i : i \in I$ partition of W, with numbers $c(W_i, W_j)$ associated with unordered pairs of partition classes, the Partition Adjacency Matrix (PAM) realization problem asks whether there is a simple graph G on the vertex set W with degrees $d_G(w) = d(w)$ for $w \in W$, with exactly $c(W_i, W_j)$ edges with endpoints in W_i and W_j; and the PAM construction problem asks for such a graph, if they exist. (These problems are conjectured to be NP-hard.) The bipartite version of these problems are more restricted: $I = I_1 \cup I_2$ and $c(W_i, W_j) = 0$ whenever $i, j \in I_1$ or $i, j \in I_2$.

We provide algebraic Monte-Carlo algorithms for the bipartite Partition Adjacency Matrix realization and construction problems, which run in polynomial time, say, when $|I|$ is fixed. When the algorithms provide a positive answer, they are always correct, and when the truth is positive, the algorithms fail to report it with small probability.

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