Institut für Diskrete Mathematik

Seminar für Kombinatorik und Optimierung

Friday 2nd October 14:15

Online meeting (Webex)

Non-concentration of the chromatic number and the Zig-zag conjecture

Annika Heckel

(Department Mathematik, LMU München)

The chromatic number of a graph is the minimum number of colours we need to colour all vertices so that adjacent vertices receive different colours.

What can we say about the chromatic number of a random graph $G(n,p)$? One main direction of past research has been the likely value of this random variable, i.e. proving that certain upper and lower bounds hold with high probability (whp). The other main direction of research has been the following question: how sharp is the concentration of the chromatic number? In other words, what is the length of the shortest interval (or rather sequence of intervals) which contains the chromatic number whp?

The starting point is a classic result of Shamir and Spencer who showed that the chromatic number of $G(n,p)$ is whp contained in some sequence of intervals of length at most about $n^{1/2}$. For sparse random graphs, this can be improved dramatically: Alon and Krivelevich proved that the chromatic number of $G(n,p)$ is two-point concentrated whenever $p < n^{-1/2-\epsilon}$.

In view of strong concentration results, Bollobás and Erdős asked the opposite question: can we find any examples where the chromatic number is not very narrowly concentrated? Specifically, can we show that the chromatic number of $G(n,1/2)$ is not whp concentrated on 100 integers?

In this talk, I will present a recent result showing that, at least for some values n, the chromatic number of $G(n,1/2)$ is not concentrated on fewer than $n^{1/2-o(1)}$ consecutive values, almost matching Shamir and Spencer's upper bound. I will also discuss and give evidence for a recent conjecture on the correct concentration interval length, which seems to depend on n.

Meeting link:

https://tugraz.webex.com/tugraz/j.php?MTID=m1cd0904285a119237aa9a7ce985ad803

Meeting number: 137 149 1265

Password: JYc3B3dunG2

Joshua Erde, Mihyun Kang