

Institut für Diskrete Mathematik

## Combinatorics Seminar

Friday 20th January 12:15

Online meeting (Webex)

# A Random Hall-Paige Conjecture

ALEXEY POKROVSKIY

(University College London)

A rainbow subgraph in an edge-coloured graph is one in which all edges have different colours. This talk will be about finding rainbow subgraphs in colourings of graphs that come from groups. An old question of this type was asked by Hall and Paige. Their question was equivalent to the following “Let  $G$  be a group of order  $n$  and consider an edge-coloured  $K_{n,n}$ , whose parts are each a copy of  $G$  and with the edge  $\{x, y\}$  coloured by the group element  $xy$ . For which groups  $G$ , does this coloured  $K_{n,n}$  contain a perfect rainbow matching?” This question is equivalent to asking “which groups  $G$  contain a complete mapping” and also “which multiplication tables of groups contain transversals”. Hall and Paige conjectured that the answer is “all groups in which the product of all the elements is in the commutator subgroup of  $G$ ”. They proved that this is a necessary condition, so the main part of the conjecture is to prove that that rainbow matchings exist under their condition. The Hall-Paige Conjecture was confirmed in 2009 by Wilcox, Evans, and Bray with a proof using the classification of finite simple groups. Recently, Eberhard, Manners, and Mrazovic found an alternative proof of the conjecture for sufficiently large groups using ideas from analytic number theory. Their proof gives a very precise estimate on the number of complete mappings that each group has. In this talk, a third proof of the conjecture will be presented using a different set of techniques, this time coming from probabilistic combinatorics. This proof only works for sufficiently large groups, but generalizes the conjecture in a new direction. Specifically we not only characterize when the edge coloured  $K_{n,n}$  contains a perfect rainbow matching, but also when random subgraphs of it contain a perfect rainbow matching. This extension has a number of applications, such as to problems of Snevily, Cichacz, Tannenbaum, Evans. This is joint work with Alp Mueyesser.

Meeting link:

<https://tugraz.webex.com/tugraz/j.php?MTID=m44797227fd680cc7956ebb840b6f033a>

Meeting number: 2730 500 3129

Password: vQydpq372D4

Joshua Erde, Mihyun Kang