Graz University of Technology

Institut für Diskrete Mathematik

Combinatorics Seminar (changed day and time)

Thursday 24th October 14:30

AE06, Steyrergasse 30

Component sizes in percolation on finite regular graphs

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A classical result by Erdős and Rényi from 1960 shows that the binomial random graph $G(n,p)$ undergoes a fundamental phase transition in its component structure when the expected average degree is around 1 (i.e., around $p = 1/n$). Specifically, for $p = (1-\varepsilon)/n$, where $\varepsilon > 0$ is a constant, all connected components are typically logarithmic in n, whereas for $p = (1+\varepsilon)/n$ a unique giant component of linear order emerges, and all other components are of order at most logarithmic in n.

A similar phenomenon — the typical emergence of a unique giant component surrounded by components of at most logarithmic order — has been observed in random subgraphs G_p of specific host graphs G, such as the d-dimensional binary hypercube, random d-regular graphs, and pseudo-random (n, d, λ)-graphs, though with quite different proofs.

This naturally leads to the question: What assumptions on a d-regular n-vertex graph G suffice for its random subgraph to typically exhibit this phase transition around the critical probability $p = 1/(d-1)$? Furthermore, is there a unified approach that encompasses these classical cases? In this talk, we demonstrate that it suffices for G to have mild global edge expansion and (almost-optimal) expansion of sets of (poly-)logarithmic order in n. This result covers many previously considered concrete setups. We also discuss the tightness of our sufficient conditions.

Based on joint works with Joshua Erde, Mihyun Kang, and Michael Krivelevich.

Joshua Erde, Mihyun Kang, Ronen Wdowinski