Differential Operators on Graphs and Waveguides

BOOK OF ABSTRACTS

February 25 – March 1, 2019

Graz University of Technology, Austria

Plenary talks

A NEW COMPLEX FREQUENCY SPECTRUM FOR THE ANALYSIS OF TRANSMISSION EFFICIENCY IN WAVEGUIDE-LIKE GEOMETRIES

Anne-Sophie Bonnet-Ben Dhia, *Palaiseau, France*

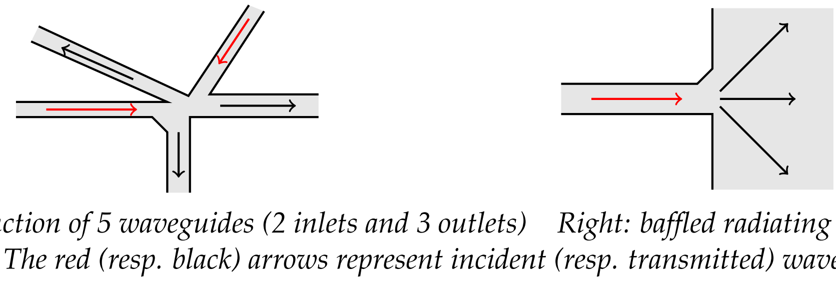
In several wave-based devices, one can define and quantify transmission and reflection phenomena. This is the case for instance in a junction of waveguides: typically, a tapered junction will be designed in order to maximize in a given frequency range the transmission from one waveguide into another one. Another example is given by diffraction gratings, which can reflect or transmit waves, depending on their frequency and incidence angle.

While spectral theory is widely used for the analysis of waveguides and gratings, the idea of introducing a spectral problem associated to these transmission-reflection phenomena is new.

A first attempt was done in [1] for a 1D toy problem. Then, in [2], we have found a way to generalize this idea to the case of a 2D infinite acoustic waveguide, with a local perturbation. In this work, we introduced an original non-self-adjoint spectral problem, using complex scalings (or perfectly matched layers) with imaginary parts of different signs on the two sides of the perturbation. This way, we select ingoing waves on one side of the perturbation and outgoing waves on the other side. Eventually, we obtain, in addition to the continuous spectrum, a discrete set of intrinsic complex eigenfrequencies. In particular, real eigenfrequencies correspond to one of the two following important situations: either there exists a trapped mode at this frequency, or there exists an incident wave (combination of propagating modes) which is completely transmitted through the perturbation (with only an evanescent reflected wave).

This approach can be generalized to several configurations of interest, that will be presented during the talk: (1) the junction of several waveguides, possibly different. A particular example with four branches, where a double PT-symmetry holds, will be presented as an illustration; (2) the radiation of a baffled waveguide, which can be seen as a junction between a half-space and a waveguide; (3) the diffractive grating where the angle of incidence enters in play.

This is a joint work with Lucas Chesnel (Palaiseau) and Vincent Pagneux (Le Mans).



Left: junction of 5 waveguides (2 inlets and 3 outlets)   Right: baffled radiating waveguide
The red (resp. black) arrows represent incident (resp. transmitted) waves.

[1] H. Hernandez-Coronado, D. Krejcirik, P. Siegl, Perfect transmission scattering as a PT-symmetric spectral problem, Phys. Lett. A 375 (2011), 2149–2152.

[2] A.S. Bonnet-Ben Dhia, L. Chesnel, V. Pagneux, Trapped modes and reflectionless modes as eigenfunctions of the same spectral problem, Proceedings of the Royal Society A 474 (2018), 20180050.

Spectra of periodic quantum graphs: more than one would expect

Pavel Exner, *Prague, Czech Republic*

Spectra of periodic quantum systems are usually expected to be absolutely continuous, consisting of bands and gaps, the number of the latter being determined by the dimensionality. Our aim is to show that if the systems in question are quantum graphs, many different sitautions may arise. Using simple examples, we show that the spectrum may then have a pure point or a fractal character, and also that it may have only a finite but nonzero number of open gaps. Furthermore, motivated by recent attempts to model the anomalous Hall effect, we investigate a class of vertex couplings that violate the time reversal invariance. We will find spectra of lattice graphs with the simplest coupling of this type, the one with 'maximum' non-invariance, and demonstrate that it depends substantially on the lattice topology, and discuss some consequences of this property.

Infinite quantum graphs

Aleksey Kostenko, *Ljubljana, Slovenia & Vienna, Austria*

We will review basic spectral properties of infinite quantum graphs (graphs having infinitely many vertices and edges). In particular, we will focus recently discovered fruitful connections between quantum graphs and discrete Laplacians on graphs.

Waveguides with asymptotically diverging twisting

David Krejčiřík, *Prague, Czech Republic*

We introduce new classes of tubular domains with geometric singularities at infinity. Among the peculiar spectral properties that we observe in respective models, there is the compactness of the resolvent despite the infinite volume of a twisted tube or sheared ribbon and the raise of dimension for the essential spectrum in a ruled strip.

Quantum graphs and almost periodic functions

Pavel Kurasov, *Stockholm, Sweden*

Spectra of Laplacians on metric graphs with scaling-invariant vertex conditions are best described as zeroes of certain trigonometric polynomials. We use this fact to prove describe the asymptotic structure of the spectra of Schrödinger operators with arbitrary vertex conditions. Obtained results are applied to inverse problems. In particular we prove the most general form of Ambartsumian theorem for quantum graphs as well as few additional results in the same spirit.

This is partially a joint work with Jan Boman and Rune Suhr.

SPECTRAL PARTITIONS OF QUANTUM GRAPHS

Delio Mugnolo, *Hagen, Germany*

There exist several possibilities of partitioning graphs or manifolds, including those based on Cheeger cuts or nodal domains. We present a different approach that elaborates on a theory developed in the last 15 years among others by Bonnaillie-Noël, Helffer, Hoffmann-Ostenhof, and Terracini. While these authors focus on domains, we are going to discuss the partitioning of graphs.

Unlike in the case of domains, several fundamental difficulties appear in our context: we are going to show how to solve them. We are going to introduce two well-defined classes of spectral partitions of graphs and show some of their features. While the complicated topology of metric graphs prevents us to recover all results that hold in the case of domains, new interesting features also arise.

This is joint work with James Kennedy, Pavel Kurasov, Corentin Léna.

ROBIN EIGENVALUES ON DOMAINS WITH PEAKS

Konstantin Pankrashkin, *Orsay, France*

Let $\Omega \subset \mathbf{R}^N$, $N \geq 2$, be a bounded domain with a suitably regular boundary. We consider the Laplacian T_α acting as $u \mapsto -\Delta u$ in Ω with the Robin boundary condition $\partial_n u = \alpha u$ on $\partial\Omega$ with ∂_n being the outward normal derivative and $\alpha > 0$ a large parameter. During the last decade, various authors obtained a number of results on the behavior of the eigenvalues of T_α for smooth domains and domains with corners. In the present talk we present first results for the case of domains Ω with outward pointing cusps. It appears that, under suitable assumptions on the cusp geometry, the jth eigenvalue $E_j(\alpha)$ of T_α behaves as

$$E_j(\alpha) \sim -\varepsilon_j\, \alpha^p \quad \text{as } \alpha \text{ tends to } +\infty,$$

with $p > 2$ and $\varepsilon_j > 0$ depending on geometric properties of the peak. In particular, the coefficients ε_j are obtained as the eigenvalues of an effective one-dimensional operator. This asymptotic behavior is in contrast with the well-known estimate $E_j(\alpha) = O(\alpha^2)$ for Lipschitz domains. We present the general scheme of the asymptotic analysis and mention some possible extensions to other classes of geometric differential operators involving domains with peaks.

Based on a joint work with Hynek Kovařík (Brescia).

TM-MODE SPECTRUM CREATED BY WAVEGUIDES IN PHOTONIC BANDGAP STRUCTURES

Michael Plum, *Karlsruhe, Germany*

We consider a Helmholtz-type spectral problem in 2D related to the propagation of TM-polarized electromagnetic waves in photonic crystal waveguides. The waveguide is created by introducing a linear defect into the two-dimensional periodic structure resulting from the polarization. In the talk we show that, under suitable assumptions, this perturbation generates guided mode spectrum inside the spectral gaps of the fully periodic unperturbed problem. We emphasize that this happens for arbitrarily "small" perturbations. For proving these results, we use Floquet-Bloch theory and the characterization of gap eigenvalues via a family of compact and symmetric operators resulting from a Birman-Schwinger approach. Furthermore, we characterize the number of created gap eigenvalues via the dispersion relation of the fully periodic problem.

The investigation of TE-modes will follow in a subsequent talk by Ian Wood.

Perturbations of manifolds and spectral convergence

Olaf Post, *Trier, Germany*

In this talk we present some recent results on wildly perturbed manifolds such as removing many little obstacles or adding small handles. We show that Laplacians on such perturbed spaces converge in a generalised norm resolvent sense to either the original space (when the perturbation has no effect) or a modified one (when the perturbation has an effect).

The talk is based on several papers, one part is joint work with Colette Anné (Nantes), another with Andrii Khrabustovskyi (Graz).

Spectral problems on star graphs

Christiane Tretter, *Bern, Switzerland*

In this talk various results for spectral problems on star graphs are presented, e.g. for Stieltjes strings or Dirac operators. Apart from direct and inverse spectral results, we establish an abstract reduction method collapsing the problem on a star graph to a path graph.

Joint with N. Rozhenko/V. Pivovarchik and B.M. Brown/H. Langer.

Contributed talks

SPECTRAL GEOMETRY IN A ROTATING FRAME: PROPERTIES OF THE GROUND STATE

Diana Barseghyan, *Ostrava, Czech Republic & Rež, Czech Republic*

We investigate spectral properties of the operator describing a quantum particle confined to a planar domain Ω rotating around a fixed point with an angular velocity ω and demonstrate several properties of its principal eigenvalue λ_1^ω.

The talk is based on a joint work with P. Exner [1].

[1] D. Barseghyan, P. Exner, Spectral estimates for Dirichlet Laplacian on tubes with exploding twisting velocity, arXiv:1902.03038 [math.SP], 2019.

PERIODIC QUANTUM GRAPHS IN HOMOGENEOUS MAGNETIC FIELDS

Simon Becker, *Cambridge, UK*

I will start by discussing the rich mathematical properties of periodic quantum graphs in homogeneous magnetic fields. After reviewing some results on the spectral theory of quasiperiodic operators and semiclassical analysis, I will explain how these techniques are used in the study of such quantum graphs to prove the existence of Cantor spectra and Landau levels. I will then discuss how randomness can be included in the model and how it affects the spectral properties. This will include a discussion of localization, delocalization, and the quantum hall effect.

[1] S. Becker, R. Han, S. Jitomirskaya, Cantor spectrum in graphene, preprint, 2018.

[2] S. Becker, M. Zworski, Cantor spectrum in graphene, preprint, 2018.

ABSENCE OF THE SINGULAR SPECTRUM IN A TWISTED DIRICHLET-NEUMANN WAVEGUIDE

Philippe Briet, *Marseille, France*

In this work we consider the Laplace operator defined in a straight strip $\Omega := \mathbb{R} \times (0, d)$ with $d > 0$ and satisfies Dirichlet boundary conditions on $\partial_D\Omega := [(-\infty, 0) \times \{0\}] \cup [(0, \infty) \times \{d\}]$ and Neumann boundary conditions on the other part of the boundary

$\partial_N\Omega := [(-\infty, 0) \times \{d\}] \cup [(0, \infty) \times \{0\}]$. Denote this operator as H, it is the self-adjoint operator in $L^2(\Omega)$ generated by the closed form

$$h[\psi] := \int_\Omega |\nabla\psi|^2 , \qquad Dom(h) := \{\psi \in W^{1,2}(\Omega) : \psi \restriction \partial_D\Omega = 0\} . \tag{1}$$

The model was introduced in [1]. Let $E_n := (n\pi)^2/(2d)^2$ with $n \in \mathbb{N}^*$ be the transverse modes i.e. the eigenvalues of the Laplacian in $L^2((0,d))$, subject to a Dirichlet boundary condition at 0 and a Neumann boundary condition at d. It is known that $\sigma(H) = \sigma_{\rm ess}(H) = [E_1, \infty)$ Then we prove that the operator H has only purely absolutely continuous spectrum namely,

Theorem.

$$\sigma_{\rm pp}(H) = \varnothing \qquad \textit{and} \qquad \sigma_{\rm sc}(H) = \varnothing \,.$$

This is a joint work with J. Dittrich (Řež, Czech Republic) and D. Krejčiřík (Prague, Czech Republic).

[1] J. Dittrich and J. Kříž, Bound states in straight quantum waveguides with combined boundary condition, J. Math. Phys. 43 (2002), 3892-3915.

NORM-RESOLVENT CONVERGENCE FOR ELLIPTIC OPERATORS IN A PLANAR WAVEGUIDE PERFORATED ALONG A CURVE

Giuseppe Cardone, *Benevento, Italy*

We consider a general second-order elliptic operator in an infinite planar straight strip perforated by small holes along a curve, subject to classical boundary conditions on the holes. Assuming that the perforation is non-periodic and satisfies rather weak assumptions, we describe all possible homogenized problems. Our main result is the norm-resolvent convergence of the perturbed operator to a homogenized one in various operator norms and the estimates for the rate of convergence. On the basis of the norm-resolvent convergence, we prove the convergence of the spectrum.

The results are obtained in a joint paper with D. Borisov (Russia) and T. Durante (Italy).

SELF-ADJOINTNESS AND SPECTRAL PROPERTIES FOR THE DIRAC OPERATOR WITH COULOMB-TYPE PERTURBATIONS

Biaggio Cassano, *Prague, Czech Republic*

We show a sharp Hardy-type inequality for the Dirac operator and we exploit this to obtain spectral properties of the *distinguished* extension of the Dirac operator perturbed with Hermitian matrix valued potentials V such that $\sup_x |x||V(x)| < 1$: we characterise its eigenvalues in terms of the Birman–Schwinger principle and we bound its discrete spectrum from below, showing that the *ground-state energy* is reached if and only if V verifies some rigidity conditions. In the particular case of an electrostatic potential, these imply that V is the Coulomb potential.

Moreover, we consider the case that $\sup_x |x||V(x)| = 1$: if V is spherically symmetric, a boundary triple is constructed and the notion of distinguished extension is discussed.

Joint work with Fabio Pizzichillo and Luis Vega.

Direct and inverse problems for one-dimensional Dirac operators with nonlocal potentials

Kamila Dębowska, *Krakow, Poland*

The main purpose of this talk is to give a brief outline of some aspects in the development of spectral theory and solvability of both direct and inverse problems of one-dimensional Dirac operators with nonlocal potentials. The problems will be discussed mainly for the usual case when the Dirac system is considered on a interval. However, results concerning Dirac operators on graphs will be also presented.

The talk is based on joint work with L.P. Nizhnik.

[1] S. Albeverio, L. Nizhnik, Schrödinger operators with nonlocal potentials, Methods Funct. Anal. Topology 19 (2013), 199–210.

[2] K. Debowska, L. Nizhnik, Direct and inverse spectral problems for Dirac systems with nonlocal potentials, (in preparation).

[3] B.M. Levitan, I.S. Sargsjan, Sturm-Liouville and Dirac Operators, Springer Science+Business Media, B.V., 1991.

[4] L. Nizhnik, Inverse spectral nonlocal problem for the first order ordinary differential equation, Tamkang Journal of Mathematics 42 (2011), 385–394.

[5] L. Nizhnik, Inverse eigenvalue problems for nonlocal Sturm-Liouville operators on a star graph, Methods Funct. Anal. Topology 18 (2012), 68–78.

Point interactions for 3D sub-Laplacians

Valentina Franceschi, *Orsay, France*

The aim of this seminar is to present some recent results on the essential self-adjointness of pointed sub-Laplacians in three dimensions. Given a n-dimensional manifold M, a sub-Laplacian is a hypoelliptic operator H, naturally associated to a (sub-Riemannian) geometric structure on it. We consider the case where $\mathrm{Dom}(H) = C_c^\infty(M \setminus \{p\})$, for $p \in M$. If $M = \mathbb{R}^n$ and the geometry is the Euclidean one, H acts as the standard Laplacian and it is well known that it is essentially self-adjoint with $\mathrm{Dom}(H) = C_c^\infty(\mathbb{R}^n \setminus \{p\})$ if and only if $n \geq 4$. Notice that, if $n \geq 4$, this follows, for instance, by the Hardy inequality in $\mathbb{R}^n$.

In this seminar we show that, unlike the Euclidean case, pointed sub-Laplacians (associated with smooth measures) are essentially self-adjoint in dimension 3. To this purpose, we focus on the case of the 3D Heisenberg sub-Laplacian, and we show its essential self-adjointness by exploiting non-commutative Fourier transform techniques. In connection with the latter result, we conclude with a discussion of Hardy inequalities in this setting.

Based on a joint work with R. Adami (Politecnico di Torino), U. Boscain (CNRS & UPMC, Sorbonne Université), and D. Prandi (CNRS & Centralesupélec)

CONSTRAINED EVOLUTION PROBLEMS ON A METRIC GRAPH

Luka Grubišić, *Zagreb, Croatia*

In this talk we present two 1D models of an endovascular stent. Endovascular stents are biomedical devices made of struts used for treating arterial stenosis. The state of the system in both models is described by a vector valued function on a metric graph which satisfies a system of ODEs and a set of algebraic constraints. Both models are obtained by Γ-convergence from 3D nonlinear elasticity. As a result of asymptotic analysis, solutions are contained in a set of functions which are constrained by a set of algebraic constraints in the nodes of the graph and by requiring that the middle line of a strut does not extend. The models differ in the way in which the constraints are handled, in one case the constraints are eliminated by an inclusion in the test space, whereas in the other all of the constraints are realized in a saddle point formulation. We will present convergence results for both models, but a much more puzzling question is which of the models will yield more efficient numerical methods. Namely, the second model yields a system matrix which is more than three times larger than in the first model. We present results of empirical comparison of the solution methods. We will further also study properties of the eigenvalue and the dynamical problem on a metric graph and discuss the solution methods and their efficiency. Finally, we will present validation experiments for the method by comparing it empirically to the 3D model solved by the standard legacy finite element code.

This is a joint work with M. Ljulj, V. Mehrmann and J. Tambaca.

QUANTUM GRAPH PROPERTIES VIA PSEUDO ORBITS AND LYNDON WORDS

Jon Harrison, *Baylor, US*

Spectral properties of quantum graphs can be expressed as sums over a finite set of *primitive pseudo orbits*, sets of primitive periodic orbits [1]. To investigate, for example, the characteristic polynomial of the graph scattering matrix, one wants to count primitive pseudo orbits of a given length.

A foundational result in the theory of *Lyndon words* (words that are strictly earlier in lexicographic order than their cyclic permutations) is the Chen-Fox-Lyndon theorem. This states that every word has a unique non-increasing decomposition into Lyndon words. Obtaining the proportion of these decompositions that are strictly decreasing can be used to count primitive pseudo orbits on q-nary graphs [2]. As an application we obtain the variance of the characteristic polynomial coefficients binary quantum graphs.

This is a joint work with Ram Band (Technion, Israel), Tori Hudgins and Mark Sepanski (Baylor, US).

[1] R. Band, J.M. Harrison, C.H. Joyner, Finite pseudo orbit expansions for spectral quantities of quantum graphs, J. Phys. A: Math. Theor. 45 (2012) 325204.

[2] R. Band, J.M. Harrison, M. Sepanski, Lyndon word decompositions and pseudo orbits on q-nary graphs, J. Math. Anal. Appl. 470 (2019), 135–144.

The Landau Hamiltonian coupled with an electric δ-potential

Markus Holzmann, *Graz, Austria*

In this talk the spectral properties of the Landau Hamiltonian, i.e. of the Schrödinger operator with a homogeneous magnetic field of strength B in $L^2(\mathbb{R}^2)$, coupled with an electric δ-potential supported on a curve Σ are discussed. Such operators are formally given by $A_\alpha := (i\nabla + A)^2 + \alpha\delta_\Sigma$, where $A = \frac{1}{2}B(-x_2, x_1)^\top$ and the interaction strength $\alpha : \Sigma \to \mathbb{R}$ is a bounded function, and they are used to describe the propagation of a particle on Σ taking quantum tunneling effects into account. It is known that the spectrum of the unperturbed Landau Hamiltonian consists of eigenvalues $\Lambda_q = B(2q+1)$, $q \in \mathbb{N}_0$, which have infinite multiplicity. It is shown under various conditions on α that the perturbation smears Λ_q into eigenvalue clusters and the accumulation rate of the eigenvalues in these clusters is found in terms of the capacity of the support of α. Furthermore, it is shown via an approximation procedure that A_α is a realistic model for Landau Hamiltonians with strongly localized potentials.

This talk is based on a joint work with J. Behrndt, P. Exner, and V. Lotoreichik.

Trace Hardy inequality for Euclidean space with cut

Michal Jex, *Karlsruhe, Germany*

In this talk we present trace Hardy inequalities for the Euclidean space with a bounded or an unbounded cut. In this novel geometric setting the respective trace Hardy inequality non-typically holds also in the two-dimensional case.

As an example of application, we show that for δ'-interaction supported by non-closed manifold the discrete spectrum disappears for weak coupling. This is surprising and previously unknown behaviour for δ'-interaction because the case of the attractive δ'-interaction supported by a closed manifold always has at least one negative eigenvalue.

This is a joint work with Vladimir Lotoreichik.

Spectral theory of an interacting two-particle system on the half-line

Joachim Kerner, *Hagen, Germany*

Motivated by Cooper's ground-breaking work in the area of superconductivity, we consider in this talk a system of two interacting particles moving on the positive half-line $\mathbb{R}_+$. Performing a spectral analysis of the underlying Hamiltonian, we determine the essential spectrum and prove, as a main result, the existence of a finite number of eigenvalues below the bottom of the essential spectrum. Moreover, for a specific interaction potential, we prove that there exists exactly one isolated eigenvalue. In general, we stress the importance of the geometry of the two-particle configuration space for the existence of a non-trivial discrete spectrum.

[1] Leon N. Cooper, Bound electron pairs in a degenerate Fermi gas, Phys. Rev. 104 (1956), 1189–1190.

[2] S. Egger, J. Kerner, K. Pankrashkin, Bound states of a pair of particles on the half-line with a general interaction potential, arXiv:1812.06500.

[3] J. Kerner, On the number of isolated eigenvalues of a pair of particles in a quantum wire, arXiv:1812.11804.

APPLICATION OF MATRIX-VALUED INTEGRAL CONTINUED FRACTIONS TO SPECTRAL PROBLEMS ON PERIODIC GRAPHS WITH DEFECTS

Anton Kutsenko, *Bremen, Germany*

We show that spectral problems for periodic operators on lattices with embedded defects of lower dimensions (planar and linear waveguides, local defects) can be solved with the help of matrix-valued integral continued fractions. While these continued fractions are usual in the approximation theory, they are less known in the context of spectral problems. We show that the spectral points can be expressed as zeros of determinants of the continued fractions. They are also useful in the analysis of inverse problems, e.g. one-to-one correspondence between spectral data and defects. Finally, the explicit formula for the resolvent in terms of the continued fractions is provided. We apply some of the results to the Schrödinger operator acting on the graphene with line and point defects.

[1] A.A. Kutsenko, Application of matrix-valued integral continued fractions to spectral problems on periodic graphs with defects, J. Math. Phys. 58 (2017), 063516.

[2] A.A. Kutsenko, Algebra of multidimensional periodic operators with defects, J. Math. Anal. Appl. 428(1) (2015), 217–226.

A GELFAND-LEVITAN TRACE FORMULA FOR GENERIC QUANTUM GRAPHS

Jiří Lipovský, *Hradec Králové, Czech Republic*

We generalize the result given by Gelfand and Levitan for the Schrödinger operator on a segment with Neumann coupling condition. We give a trace formula for the quantum graph with arbitrary edge lengths and generic coupling conditions. The only case of coupling conditions which is excluded is the condition with the unitary coupling matrix having eigenvalue -1 (hence it is a set of measure zero in the set of all self-adjoint couplings). However, the considered set does not include Dirichlet, standard or δ-conditions.

This is joint work with prof. Pedro Freitas.

SPECTRAL GAP FOR GRAPHENE QUANTUM DOTS

Vladimir Lotoreichik, *Prague, Czech Republic*

We will discuss the massless Dirac operator D_Ω on a bounded and sufficiently smooth domain $\Omega \subset \mathbb{R}^2$ with so-called *infinite mass boundary conditions*. This Dirac operator arises in an effective mathematical theory for graphene quantum dots. The operator D_Ω is self-adjoint in the Hilbert space $L^2(\Omega, \mathbb{C}^2)$ and non-semibounded. Its spectrum $\sigma(\mathsf{D}_\Omega)$ is discrete and symmetric with respect to the origin. The size $\mathcal{L}_\Omega := \mathrm{dist}\,(\sigma(\mathsf{D}_\Omega), 0) > 0$ of the spectral gap for D_Ω is known to be important in applications.

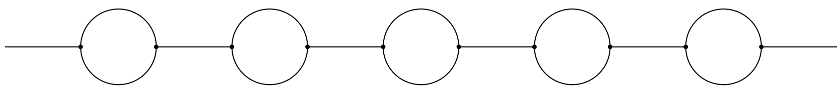
Our main result concerns the geometric control on $\mathcal{L}_\Omega$ for C^3-smooth simply connected domains. Namely, we obtain an upper bound on $\mathcal{L}_\Omega$ in terms of $\mathcal{L}_\mathbb{D}$ for the unit disk $\mathbb{D}$ with a pre-factor involving purely geometric quantities and a suitable Hardy norm of f' for a conformal map $f : \mathbb{D} \to \Omega$. This Hardy norm can be further estimated through geometric quantities for convex domains and for so-called nearly circular star-shaped domains. The obtained bounds on $\mathcal{L}_\Omega$ are attained for disks and are tight for domains which are close to a disk. These results can also be reformulated as reverse Faber-Krahn-type inequalities for D_Ω under suitable geometric constraints.

This talk is based on a joint work with Thomas Ourmières-Bonafos.

Breather solutions on periodic necklace graphs

Daniela Maier, *Stuttgart, Germany*

In this talk, I will explain the main steps of the construction of real-valued, time-periodic and spatially localized solutions of small amplitude of nonlinear Klein-Gordon equations on a periodic necklace graph.



Our results builds upon a spatial dynamics ansatz combined with center manifold reduction and bifurcation theory. Spectral gaps in the Floquet-Bloch spectrum of the linearized operator occur in a natural way for periodic necklace graphs and are essential for the construction. The major challenge arises from the irregularity of the solutions due to the Kirchhoff boundary conditions. Our approach is motivated by the existence result of Blank, Chirilus-Bruckner, Lescarret and Schneider, Breather solutions in periodic media, Comm. Math. Phys., 2011. The talk is based on arXiv:1812.02012 .

On the notion of effective impedance via ordered fields

Anna Muranova, *Bielefeld, Germany*

It is known that electrical networks with resistors are related to the Laplace operator and random walk on weighted graphs ([2], [4], [6]). More general electrical networks with coils, capacitors and resistors give rise to graphs with complex-valued weights. The central topic of the talk is the effective impedance of such networks ([3]). The corresponding Dirichlet problem with complex-valued coefficients does not necessary have a solution, and if it has, it may be not unique. This creates difficulties in definition of the effective impedance. In order to overcome these difficulties, we consider the impedances on each edge as rational functions of $\lambda = i\omega$, where ω is a frequency ([1]), and use the fact that rational functions form an ordered field ([5]). Hence, we develop a theory of weighted graphs with weights from an ordered field and prove that the effective impedance is always well defined in this case.

[1] O. Brune, Synthesis of a finite two-terminal network whose driving-point impedance is a prescribed function of frequency, Thesis (Sc. D.), Massachusetts Institute of Technology, Dept. of Electrical Engineering, 1931.

[2] P.G. Doyle, J.L. Snell, Random walks and electric networks, Carus Mathematical Monographs 22, Mathematical Association of America, Washington, DC, 1984.

[3] R.P. Feynman, The Feynman lectures on physics, Volume 2: Mainly Electromagnetism and Matter, 1964.

[4] A. GrigorâĂŹyan, Introduction to Analysis on Graphs, AMS University Lecture Series, Volume: 71, 2018.

[5] B. L. van der Waerden, Algebra. Volume I, 2003.

[6] David A. Levin, Yuval Peres, Elizabeth L. Wilmer Markov Chains and Mixing Times, 2009.

QUANTUM GRAPHS ON RADIALLY SYMMETRIC ANTITREES

Noema Nicolussi, *Vienna, Austria*

Whereas for finite metric graphs the Laplacian is (essentially) self-adjoint and has discrete spectrum, the spectral behavior of Laplacians on infinite graphs is much less understood. This talk is devoted to Laplacians on one particular class of infinite graphs, namely radially symmetric antitrees. This class of graphs played an important role in extending Grigoryan's stochastic completeness criterion from the manifold case to graphs. Employing symmetries in their structure, we perform a detailed spectral analysis including self-adjointness, spectral gap estimates, and spectral types (discrete, singular and absolutely continuous spectrum).

This is a joint work with Aleksey Kostenko.

AN EIGENVALUE BOUND FOR THE KIRCHHOFF-LAPLACIAN ON PLANAR QUANTUM GRAPHS

Marvin Plümer, *Hagen, Germany*

In [1] Spielman-Teng gave an upper bound for the first positive eigenvalue of the discrete Laplacian on planar graphs, that only depends on the number of vertices and the maximum degree of the graph. To achieve this bound they used a circle packing representation of the graph to obtain a sufficiently good test function in a Courant-Fischer principle for the first eigenvalue. In this talk we transfer these techniques to the metric setting, where we obtain a similiar bound for the first positive eigenvalue of the Kirchhoff-Laplacian on planar quantum graphs. Within this context we give a brief introduction to circle packings for planar graphs.

[1] D. A. Spielman, S. H. Teng, Spectral partitioning works: Planar graphs and finite element meshes, In: Linear Algebra and its applications 421 (2007), pp. 284-305.

Weyl asymptotics of resonances and resonance states completeness for quantum graphs

Igor Popov, *Saint-Petersburg, Russia*

A quantum graph Γ with the the second differentiation operator on edges and the Kirchhoff coupling condition at non-boundary vertices is considered. It is assumed that the graph consists of compact subgraph Γ_0 and a finite number of semi-infinite leads. Completeness of the system of resonance states in $L_2(\Gamma_0)$ on the compact subgraph is studied. It is proved that the completeness takes place if and only if each vertex having infinite leads attached is unbalanced, i.e. it has different numbers of finite and infinite edges attached. A relation between the completeness of the resonance states and the Weyl asymptotics [1, 2] of the resonances is established. The completeness proof is based on the factorization of the characteristic function in the Sz-Nagy functional model for the problem in question [3]. The main result is presented in the following theorem.

Theorem. *The system of resonance states for quantum graph Γ with the Kirchhoff coupling condition at vertices is complete in $L_2(\Gamma_0)$ if and only if the resonances has the Weyl asymptotics.*

[1] E. B. Davies, A. Pushnitski. Non-Weyl resonance asymptotics for quantum graphs. Analysis and PDE 4 (2011), 729Ű756.

[2] E. B. Davies, P. Exner, J. Lipovsky: Non-Weyl asymptotics for quantum graphs with general coupling conditions, J. Phys. A: Math. Theor. 43 (2010), 474013.

[3] N. Nikol'skii, Treatise on the shift operator: spectral function theory. Springer Science & Business Media, Berlin, 2012.

Estimate for the Weyl coefficient of a two-dimensional canonical system

Raphael Pruckner, *Vienna, Austria*

This is joint work with M.Langer and H.Woracek. We study two-dimensional canonical systems

$$y'(t) = zJH(t)y(t), \quad t \in [a,b), \tag{1}$$

where $-\infty < a < b \leq \infty$, $J = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$, $z \in \mathbb{C}$, and where the Hamiltonian $H : [a,b) \to \mathbb{R}^{2\times 2}$ is a.e. positive semidefinite, locally integrable and satisfies $\int_a^b \operatorname{tr} H(t)dt = \infty$.

The Weyl coefficient q_H of a Hamiltonian H is constructed from the fundamental solution of (1) and plays a crucial role in the spectral theory of canonical systems. Its construction goes back to H.Weyl and is based on a nested disks argument. The famous inverse spectral theorem of L.de Branges states that the correspondence between a Hamiltonian H and its Weyl coefficient q_H is essentially unique. It can be challenging to relate properties of the Hamiltonian to properties of its Weyl coefficient.

In our work we establish upper and lower bounds of the imaginary part of $q_H(ir)$ for $r > 0$ in terms of the Hamiltonian H. For large r, these bounds depend only on the behaviour of H locally at a. Our result can be seen as a generalisation of a theorem by I.S.Kac to the non-diagonal case.

For a large class of Hamiltonians our upper and lower bound coincide up to a factor, which is bounded and bounded away from zero. Hence, the behaviour of $\operatorname{Im} q_H(ir)$ for large r can be determined up to constant factors in many cases.

In the proof we follow an approach used by H.Winkler. Estimates for the power series coefficients of the fundamental solution of the system give rise to estimates of centres and radii of Weyl's nested discs.

Applying our results leads to conditions for $\mu_H(-r,r)$, i.e. the symmetrised distribution function of the measure in the Herglotz integral representation of q_H, to be integrable or bounded relative to a growth function.

MATRIX VALUED HERMITE-BIEHLER FUNCTIONS AND A GENERALIZED INTERLACING PROPERTY

Jakob Reiffenstein, *Vienna, Austria*

The Hermite-Biehler Theorem states - in essence - that an entire function of finite exponential type E satisfies the inequality $|E(\overline{z})| < |E(z)|$, $z \in \mathbb{C}_+$, if and only if its real and imaginary part $A(z) := \frac{1}{2}\big(E(z) + \overline{E(\overline{z})}\big)$ and $B(z) := \frac{1}{2i}\big(E(z) - \overline{E(\overline{z})}\big)$ have only real and interlacing zeros. In the field of Herglotz functions, this corresponds to the fact that the zeros and poles of a scalar-valued meromorphic Herglotz function are simple and interlacing. We investigate an analogue for matrix-valued functions.

For a meromorphic function f, real zeros and poles of f are simple and interlace if and only if for every finite interval (a,b) we have $\Big|\sum_{x\in(a,b)} \theta_f(x)\Big| \leq 1$, where θ_f is the divisor of f. In analogy to this, we can say that the zeros and poles of f are n-interlacing if

$$\Big|\sum_{x\in(a,b)} \theta_f(x)\Big| \leq n$$

for every finite interval (a,b). We characterize meromorphic matrix-valued Herglotz functions by means of an interlacing property of scalar-valued functions.

Theorem. *Let $Q = (Q_{jk})_{j,k=1}^n$ be a $n \times n$-matrix valued function which is meromorphic on $\mathbb{C}$, has no nonreal poles, and satisfies $Q(\overline{z}) = Q(z)^*$. Suppose that Q is of bounded type in the upper half plane, and that for each entry of Q we have $\limsup_{\tau\to+\infty} \frac{|Q_{jk}(i\tau)|}{\tau} < +\infty$. Then the following are equivalent:*

(i) Q is Herglotz;

(ii) For every $m \leq n$ and every $m \times m$-principal submatrix $\tilde{Q}$ of Q, $f(z) := \det \tilde{Q}(z)$ satisfies the following three properties:

a. The zeros and poles of f are all real and m-interlacing;

b. If the set of poles of multiplicity m of f is nonempty, then for at least one of those poles z_0 we have $\lim_{z\to z_0}\big[(z_0 - z)^m f(z)\big] > 0$.

c. $\lim_{\tau\to+\infty} \frac{f(i\tau)}{(i\tau)^m} \geq 0$.

HOT SPOTS OF QUANTUM GRAPHS

Jonathan Rohleder, *Stockholm, Sweden*

The Hot Spots Conjecture of J. Rauch asserts that the hottest and coldest points of an insulated body should move towards its boundary for large times, if the insulation is perfect. Via the semigroup associated with the Neumann Laplacian this reduces to proving that maximum and minimum of the eigenfunction(s) associated with the smallest positive eigenvalue are located on the boundary. This conjecture is not true in full generality but is currently open, for example, for convex domains.

In this talk we will examine the corresponding question on metric graphs: for the Laplacian on a finite metric graph with standard (continuity and Kirchhoff) vertex conditions we consider the possible distribution of maxima and minima of eigenfunctions associated with the smallest nonzero eigenvalue. Among other things, we give examples to show that the usual notion of "boundary" of a metric graph, namely the set of vertices of degree one, has limited relevance for determining the "hottest" and "coldest" parts of a graph.

This is joint work with James Kennedy (Lisbon).

LOCAL ENERGY DECAY FOR THE WAVE EQUATION IN A DISSIPATIVE WAVEGUIDE

Julien Royer, *Toulouse, France*

We discuss the problem of the local energy decay for the damped wave equation, focusing on the case of waveguides with dissipation at the boundary or at infinity. We will see in particular that for large time the wave has the behavior of the solution of some heat equation.

SCHRÖDINGER OPERATORS WITH GUIDED POTENTIALS ON PERIODIC GRAPHS

Natalia Saburova, *Arkhangelsk, Russia*

We consider discrete Schrödinger operators with periodic potentials on periodic graphs perturbed by guided positive potentials, which are periodic in some directions and finitely supported in other ones. The spectrum of the unperturbed operator is a union of a finite number of non-degenerate bands and eigenvalues of infinite multiplicity. We show that the spectrum of the perturbed operator consists of the ŤunperturbedŤ one plus the additional guided spectrum, which is a union of a finite number of bands. We estimate the number and the positions of the guided bands in gaps of the unperturbed operator in terms of eigenvalues of Schrödinger operators on some finite graphs. We also determine sufficient conditions for the guided potentials under which the guided bands do not appear in gaps of the unperturbed problem.

Asymptotic and numerical study of resonant tunneling in a 2D quantum waveguide with several resonators

Oleg Sarafanov, *Saint-Petersburg, Russia*

We consider a two-dimensional quantum waveguide, which occupies a strip with several narrows of small diameter ε. The wave function of a free electron satisfies the Dirichlet boundary value problem for the Helmholtz equation. The parts of the waveguide between two neighbouring narrows play the role of resonators. In such a waveguide, the resonant tunneling can occur, i.e. the transmission coefficient $T(k)$ has peaks at some "resonant" wave numbers k.

For applications, it is necessary to know the location and the shape of the resonant peaks. This information can be obtained by approximate numerical calculations if the diameter ε of the narrows is not too small. Otherwise numerical experiments meet difficulties and the asymptotic description of resonant tunneling (as $\varepsilon \to 0$) becomes actual. Moreover, the asymptotics provides a qualitative picture of the phenomenon.

The asymptotic theory of resonant tunneling in a quantum waveguide with several resonators was developed in [1]. After all the constant coefficients in the leading terms of the asymptotic formulas are calculated, the formulas can also be used to obtain numerical approximations for the resonant tunneling characteristics. We compare the results obtained by asymptotical methods and by direct calculation of the scattering matrix and show, that, for ε in some interval, they coincide with high accuracy. Analogous study for quantum waveguides with one resonator was carried out in [2].

[1] I. Gurianov, O. Sarafanov, Asymptotics of resonant tunneling in a two-dimensional quantum waveguide with several equal resonators, Applicable Analysis (2018), in press.

[2] L. Baskin, P. Neittaanmäki, B. Plamenevskii, O. Sarafanov, Resonant Tunneling: Quantum Waveguides of Variable Cross-Sections, Asymptotics, Numerics, and Applications, Lecture Notes on Numerical Methods in Engineering and Sciences, Springer, Cham, 2015.

Optimal potentials on quantum graphs with δ-couplings

Andrea Serio, *Stockholm, Sweden*

Schrödinger operators on finite compact metric graphs with delta couplings at the vertices are studied. We present which potential and which distribution of delta couplings on a given graph maximise the ground state energy, provided the integral of the potential and the sum of strengths of the delta couplings are fixed. It appears that the optimal potential, if it exists, is a constant function on its support formed by a set of intervals separated from the vertices. In the case where the optimal configuration does not exist explicit optimising sequences are presented.

This is a joint work with Pavel Kurasov from Stockholm University.

Approximation of the Laplacian on the Sierpinski Gasket: norm resolvent and spectral convergence

Jan Simmer, *Trier, Germany*

In this talk we briefly introduce the notion of *quasi-unitary equivalence* for energy forms (i.e., densely defined, closed quadratic forms) defined in a Hilbert space. The notion generalizes the two concepts of *norm-resolvent convergence* and *unitary equivalence* and provides a tool to compare energy forms (and their related operators, called Laplacians here) which are defined in different Hilbert spaces.

We apply this schema in the concrete example of the Sierpinski Gasket. Our first main result is that the standard energy form and the canonical sequence of (discrete) graph energy forms are quasi-unitary equivalent. Moreover, we present an *outer approximation*, where we approximate the Sierpinski Gasket by a suitable sequence of *graph-like manifolds*.

As a consequence (suitable) functions of the Laplacian (e.g. heat operator, spectral projections etc.) converge as well in operator norm. One also deduces the convergence of the spectrum and eigenfunctions in energy norm.

The talk is based on several papers which are a joint work with Olaf Post.

Localization for Anderson models on tree graphs

Selim Sukhtaiev, *Houston, US*

In this talk I will discuss Anderson localization for Bernoulli–type random models on metric and discrete tree graphs. Dynamical localization is proved on compact intervals contained in the complement of a discrete set of exceptional energies. This is based on joint work with D. Damanik and J. Fillman.

Boundary representations of λ-harmonic and polyharmonic functions on trees

Wolfgang Woess, *Graz, Austria*

On a countable tree T, allowing vertices with infinite degree, we consider an arbitrary stochastic irreducible nearest neighbour transition operator P. We provide a boundary integral representation for general eigenfunctions of P with eigenvalue $\lambda \in \mathbb{C}$. This is possible whenever λ is in the resolvent set of P as a self-adjoint operator on a suitable ℓ^2-space and the on-diagonal elements of the resolvent ("Green function") do not vanish at λ. We show that when P is invariant under a transitive group action, the latter condition holds for all $\lambda \neq 0$ in the resolvent set. These results extend and complete previous results by Cartier, by Figà-Talamanca and Steger, and by Woess. Furthermore, for those eigenvalues, we provide an integral representation of λ-polyharmonic functions of any order n, that is, functions $f : T \to \mathbb{C}$ for which $(\lambda \cdot I - P)^n f = 0$. This is a far-reaching extension of work of Cohen et al.

We can also provide an analogous result for polyharmonic functions on the unit disk with respect to the hyperbolic Laplacian, based on old results of Helgason.

This is joint work with Massimo Picardello (Rome).

WEAK LOCALIZATION RESULTS FOR TE-MODES

Ian Wood, *Kent, UK*

We study a spectral problem related to the propagation of electromagnetic waves in planar photonic crystal waveguides. The waveguide is created by introducing a linear defect into a periodic medium. The defect is infinitely extended and aligned with one of the coordinate axes. This perturbation introduces guided mode spectrum inside the band gaps of the fully periodic, unperturbed spectral problem.

Mathematically, the problem is described by a divergence form elliptic equation

$$Lu = -\mathrm{div}\frac{1}{\varepsilon}\nabla u.$$

The perturbation corresponds to changing the coefficient ε. After performing a Floquet decomposition in the axial direction of the waveguide, we study the spectrum created by the perturbation for any fixed value of the quasi-momentum. We use variational arguments in negative Sobolev spaces to prove that guided mode spectrum can be created by arbitrarily small perturbations to the coefficient.

This talk is closely related to the talk of Michael Plum on TM-modes.

[1] B. M. Brown, V. Hoang, M. Plum, M. Radosz, I. Wood, Gap Localization of TE-Modes by arbitrarily weak defects - multiband case, https://arxiv.org/abs/1901.05102.

[2] B. M. Brown, V. Hoang, M. Plum, M. Radosz, I. Wood, Gap localization of TE-Modes by arbitrarily weak defects, Journal of the LMS, 95 (2017), 942–962.

ON A PROBLEM OF LOUIS DE BRANGES

Harald Woracek, *Vienna, Austria*

A de Branges space is a reproducing kernel Hilbert space of entire functions with certain additional properties. For each de Branges space $\mathcal{H}$ there exists a unique maximal chain of de Branges subspaces which are contained isometrically in $\mathcal{H}$. This chain is determined by a two-dimensional canonical system $y'(t) = zJH(t)y(t)$ on the interval $(-\infty, 0]$. The Hamiltonian H of this system is called the structure Hamiltonian of the space $\mathcal{H}$.

De Branges identified in 1961 a particular class of Hamiltonians, each of them being the structure Hamiltonian of some de Branges space. A further class of structure Hamiltonians was identified (in a different language) by I.S.Kac and M.G.Krein in 1958. These classes, however, do by far not exhaust the set of all structure Hamiltonians.

In 1968, after having finalised his theory of Hilbert spaces of entire functions, de Branges posed the following question as a fundamental problem:

(Q) Which Hamiltonians H are the structure Hamiltonian of a de Branges space $\mathcal{H}$?

In the following decades there was no significant progress towards a solution of (Q). The only further result was claimed by I.S.Kac in 1995; proofs have never been published. He states a sufficient condition and a (different) necessary condition for H to be a structure Hamiltonian. Unfortunately, his conditions are difficult to handle and quite far apart from each other.

We give a — surprising and astonishingly simple — characterisation for H to be a structure Hamiltonian. It is "surprising" because it shows that the the answer does not depend on the off-diagonal entry of H. It is "astonishingly simple" because its proof is short and elementary. Our theorem also provides an alternative proof for the afore mentioned theorems of Kac-Krein and de Branges.

We give a characterisation for a canoncial system to have discrete spectrum which is explicit in terms of the Hamiltonian. This solves – in theory – the long open problems to determine which Jacobi matrices and which 1-dimensional Schrödinger operators have discrete spectrum. Unfortunately, in practice, rewriting such equations to the form of a canoncial system is difficult to handle.

This is a joint work with Roman Romanov (St.Petersburg, Russia).

Acknowledgments

It is a pleasure to acknowledge the substantial financial support from the

- FWF Der Wissenschaftsfonds
- AVL List GmbH
- City of Graz
- Graz University of Technology

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