Asymptotic and Numerical Study of Resonant Tunneling in Quantum Waveguides with Several Equal Resonators

Oleg Sarafanov

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Statement of the problem

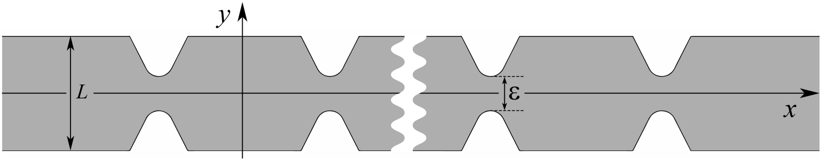


Figure: Waveguide $G(\varepsilon)$

The wave function u of a free electron of energy k^2 satisfies the boundary value problem

$$\left(-\Delta - k^2\right) u(x, y; \varepsilon) = 0, \text{ in } G(\varepsilon);$$
$$u(x, y; \varepsilon) = 0, \text{ on } \partial G(\varepsilon).$$

Auxiliary boundary value problem in the cross-section of the strip

$$\begin{aligned}&\Big(-\frac{\partial^2}{\partial y^2}-\lambda^2\Big)\Psi(y)=0,\\&\Psi(-L/2)=\Psi(L/2)=0.\end{aligned}\quad\Rightarrow\quad\begin{aligned}&\lambda_q^2=\Big(\frac{\pi q}{L}\Big)^2,\\&q=1,2,\dots.\end{aligned}$$

We assume that

$$\lambda_1^2<k^2<\lambda_2^2,$$

and set the radiation conditions

$$u(x,y;\varepsilon)=\begin{cases}e^{i\nu x}\Psi_1(y)+S_{11}(k;\varepsilon)e^{-i\nu x}\Psi_1(y)+O(e^{\delta x}), & x\to-\infty;\\ S_{12}(k;\varepsilon)e^{i\nu x}\Psi_1(y)+O(e^{-\delta x}), & x\to+\infty;\end{cases}$$

$$\delta>0\,,\qquad \nu=\sqrt{k^2-\lambda_1^2}\,,\qquad \Psi_1(y)=\sqrt{\frac{1}{L\nu}}\cos\frac{\pi y}{L}.$$

Reflection coefficient and transition coefficient

$$R(k;\varepsilon) = |S_{11}(k;\varepsilon)|^2, \qquad T(k;\varepsilon) = |S_{12}(k;\varepsilon)|^2.$$

We have

$$R(k;\varepsilon) + T(k;\varepsilon) = 1.$$

We are interested in

1. The asymptotics of resonant energies $k_r^2(\varepsilon)$ as $\varepsilon \to 0$;
2. The heights $T(k_r, \varepsilon)$ of resonant peaks;
3. The width $\Upsilon(\varepsilon)$ of resonant peaks at half-height.

Single resonator — reference

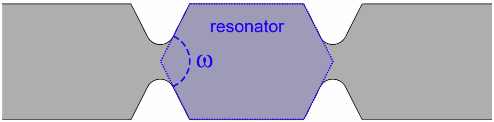


Figure: Waveguide $G(\varepsilon)$ with a single resonator

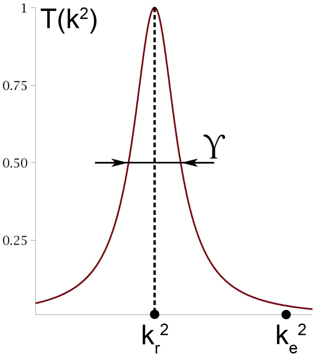
L. Baskin, P. Neittaanmäki, B. Plamenevskii, and O. Sarafanov. *Resonant Tunneling: Quantum Waveguides of Variable Cross-Section, Asymptotics, Numerics, and Applications* // Lecture Notes on Numerical Methods in Engineering and Sciences, Springer, 2015, 275 pp.

The results obtained:

$$k_r^2(\varepsilon) = k_e^2 - Q\varepsilon^{2\pi/\omega} + O(\varepsilon^{2\pi/\omega+2-\delta}),$$

$$T(k;\varepsilon) = \frac{1}{1 + P^2\left(\dfrac{k^2 - k_r^2}{\varepsilon^{4\pi/\omega}}\right)^2}\left(1 + O(\varepsilon^{2-\delta})\right),$$

$$\Upsilon(\varepsilon) = \frac{2}{P}\varepsilon^{4\pi/\omega}.$$



n equal resonators — resonant energies

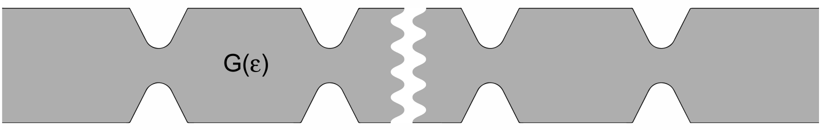
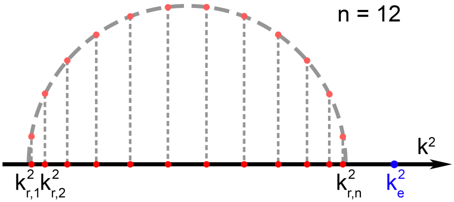


Figure: Waveguide $G(\varepsilon)$ with n equal resonators

The resonant energies

$$k^2_{r,j}(\varepsilon) = k_e^2 - Q_j \varepsilon^{2\pi/\omega} + O(\varepsilon^{4\pi/\omega}), \quad j = 1, \dots, n;$$

$$Q_j = A + 2B \cos \frac{\pi j}{n+1}.$$

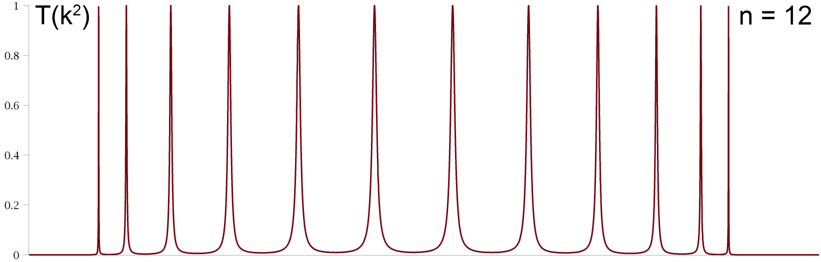


n equal resonators — peaks

The transition coefficient and the widths of peaks

$$T(k;\varepsilon) = \frac{1}{1 + P^2(n)\left(\frac{\prod_{j=1}^{n}(k^2 - k_{r,j}^2)}{\varepsilon^{2(n+1)\pi/\omega}}\right)^2}\left(1 + O(\varepsilon^{4\pi/3\omega})\right);$$

$$\Upsilon_j(\varepsilon) = \frac{4}{(n+1)P(n)B^{n-1}}\sin^2\left(\frac{\pi j}{n+1}\right)\varepsilon^{4\pi/\omega}.$$



Numerics for one resonator

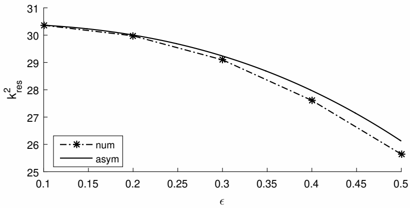


Figure: Resonant energy

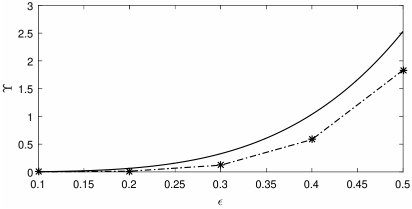


Figure: Width of a resonant peak

Numerics for 10 resonators

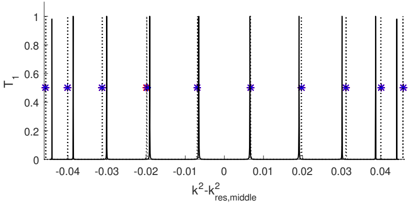


Figure: Resonant peaks for $\varepsilon = 0.1$

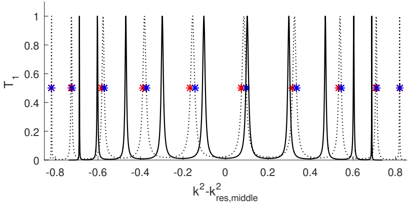


Figure: Resonant peaks for $\varepsilon = 0.3$

Thank you!