Spectral Theory and Differential Operators

Graz, September 15–19, 2025

Abstracts

Invited talks

1. **Anne-Sophie Bonnet-BenDhia**

   Title: *Trapped modes in electromagnetic waveguides*

   Abstract: This is a common work with Lucas Chesnel and Sonia Fliss. We consider the Maxwell's equations with perfect electric conductor boundary conditions in three-dimensional unbounded domains which are the union of a bounded resonator and one or several semi-infinite waveguides. We are interested in the existence of electromagnetic trapped modes, that is L^2 solutions of the problem without source term. These trapped modes are associated to eigenvalues of the Maxwell's operator, that can be either below the continuous spectrum or embedded in it. First for homogeneous waveguides, we present different families of geometries for which we can prove the existence of eigenvalues. Then we show that certain non homogeneous waveguides with local perturbations of the dielectric constants can support trapped modes. Let us mention that certain mechanisms we propose are very specific to Maxwell's equations and have no equivalent in the classical proofs of existence of trapped modes for the scalar Dirichlet or Neumann Laplacians.

2. **Pavel Exner**

   Title: *Spectral properties of time-reversal asymmetric quantum graphs*

   Abstract: We discuss quantum graphs with vertex couplings which violate the time-reversal invariance, the main attention being paid to the couplings described by circulant matrices. We show that such graph Laplacians may exhibit a nontrivial $\mathcal{PT}$-symmetry despite being self-adjoint [1]. They also have other interesting spectral properties which we illustrate through examples in which the matrix characterizing the coupling has a particularly simple structure. We show, for instance, that a sign change of this matrix can have a dramatic effect of the spectrum; we illustrate it in the situations when the underlying metric graphs are lattices of various types, square [2], Kagome [3], or Cairo [4]. We also analyze the magnetic Laplacian on a square lattice where two mechanisms of symmetry violation compete [5]. Finally, we address the eigenvalue optimization problem for finite graphs with the simplest vertex coupling of the indicated type [6].

   [1] P. Exner, M. Tater, Quantum graphs: self-adjoint, and yet exhibiting a nontrivial $\mathcal{PT}$-symmetry, Phys. Lett. A416 (2021), 127669

   [2] P. Exner, J. Pekař, Spectral properties of hexagonal lattices with the -R coupling, Rep. Math. Phys. 95 (2025), to appear

   [3] M. Baradaran, P. Exner, Kagome network with vertex coupling of a preferred orientation J. Math. Phys. 63 (2022), 083502

   [4] M. Baradaran, P. Exner, Cairo lattice with time-reversal non-invariant vertex couplings, J. Phys. A.: Math. Theor. 57 (2024), 265202

   [5] M. Baradaran, P. Exner, J. Lipovský, Magnetic square lattice with vertex coupling of a preferred orientation, Ann. Phys. 454 (2023), 169339

   [6] P. Exner, J. Rohleder, Optimization of quantum graph eigenvalues with preferred orientation vertex conditions, Ann H. Poincaré, to appear

3. **Fritz Gesztesy**

   Title: *Essential self-adjointness for a class of pseudo-differential operators perturbed by strongly singular potential coefficients*

   Abstract: We consider essential self-adjointness on $C_0^\infty(\mathbb{R}^n\backslash\{0\})$, $n \in \mathbb{N}$, $n \geq 2$, of pseudo-differential operators of the form $(-\Delta+m^2)^s$, $s > 0$, $m \geq 0$, perturbed by strongly singular homogeneous potential coefficients such as $|x|^{-2s}$. The results are obtained via a combination of Wüst's theorem on essential self-adjointness under the borderline case of relatively bounded perturbations with relative bound equal to one and fractional Birman–Hardy–Rellich-type inequalities. Subsequently, we extend some of these results to $C_0^\infty(\mathbb{R}^n\backslash X)$, where $X \subset \mathbb{R}^n$ is an appropriate discrete set.

   This is based on joint work with Markus Hunziker and Dorina Mitrea.

4. **Marjeta Kramar Fijavž**

   Title: *On delayed systems with boundary delays*

   Abstract: We consider abstract boundary delay systems, in which the delay term occurs in both the equations and the boundary conditions. We apply a perturbation approach to study the well-posedness of such systems. We also establish spectral theory for the associated delay operators and provide conditions for the positivity of solutions to these systems. Our results are illustrated using the example of a delayed transport equation on a metric graph, where the delays occur along the edges as well as at the vertices. This talk is based on the joint work with András Bátkai and Abdelaziz Rhandi, see arXiv:2503.08809.

5. **David Krejčiřík**

   Title: *The Laplacian with complex magnetic fields*

   Abstract: Motivated by stability of black holes in general relativity, we introduce a new concept of complex magnetic fields in a quantum-mechanical setting of Schroedinger operators. Spectral and pseudospectral properties are discussed. This is a joint birthday present with Nicolas Raymond and Tho Nguyen Duc.

6. **Marco Marletta**

   Title: *Essential spectra of Maxwell systems*

   Abstract: In this talk I shall review recent work with collaborators on essential spectra for non-selfadjoint Maxwell systems and Drude-Lorentz models. The emphasis will be on obtaining results that are valid in unbounded domains under minimal hypotheses on boundary regularity (just Lipschitz) and without assuming isotropy or homogeneity of the permeability, permittivity or conductivity. The essential spectra will be shown to arise partly from infinity, partly (possibly) from the boundary, and partly from interior phenomena such as black hole modes around regions of non-trivial conductivity - these provide the non-selfadjointness in the system.

   I shall also speak on limiting essential spectra, numerical ranges, and how spectral pollution for non-selfadjoint Maxwell systems turns out to be surprisingly less problematic, in some sense, than for their selfadjoint counterparts.

7. **Grigori Rozenblioum**

   Title: *Spectral properties of the resolvent difference under singular perturbations*

   Abstract: A common approach is proposed to three problems concerning the difference of resolvents: Robin problem with different weights, delta-potentials on a Lipschitz surface, and fractal potentials. We establish order sharp eigenvalue estimates, and for two first problems, eigenvalue asymptotics.

8. **Christiane Tretter**

Title: *Challenges for non-selfadjoint spectral problems in analysis and computation*

Abstract: Non-selfadjoint spectral problems appear frequently in a wide range of applications. Reliable information about their spectra is therefore crucial, yet extremely difficult to obtain. This talk focuses on tools to master these challenges such as spectral pollution or spectral invisibility. In particular, the concept of essential numerical range for unbounded linear operators is introduced and studied, including possible equivalent characterizations and perturbation results. Compared to the bounded case, new interesting phenomena arise which are illustrated by some striking examples. A key feature of the essential numerical range is that it captures, in a unified and minimal way, spectral pollution which may affect e.g. spectral approximations of PDEs by projection methods or domain truncation methods. As an application, Maxwell's equations with conductivity will be considered.

(Joint work with S. Boegli, M. Marletta, and also F. Ferraresso)

Contributed talks

1. **Olena Atlasiuk**

Title: *Convergence and approximation of solutions of inhomogeneous boundary-value problems in Sobolev spaces*

Abstract: We consider the most general (generic) class of linear inhomogeneous boundary-value problems for systems of ordinary differential equations of arbitrary order, the solutions and right-hand sides of which belong to the corresponding Sobolev spaces.

Let $(a, b) \subset \mathbb{R}$ be a finite interval and the parameters

$$\{n, k\} \subset \mathbb{N} \cup \{0\}, \quad \{m, r\} \subset \mathbb{N}, \quad 1 \le p \le \infty,$$

are arbitrarily chosen. $W_p^{n+r}\big([a, b]; \mathbb{C}\big)$ be a complex Sobolev space. We consider a sequence of linear boundary-value problems

$$(L_k y_k)(t) := y_k^{(r)}(t) + \sum_{j=1}^{r} A_{r-j}(t, k) y_k^{(r-j)}(t) = f(t), \quad t \in (a, b), \tag{1}$$

$$B_k y_k = c, \tag{2}$$

where the matrix-valued functions $A_{r-j}(\cdot, k) \in (W_p^n)^{m\times m}$, the vector-valued function $f(\cdot) \in (W_p^n)^m$, the vector $c \in \mathbb{C}^{rm}$, and the linear continuous operators

$$B_k \colon (W_p^{n+r})^m \to \mathbb{C}^{rm}.$$

As proven in the articles [1, 2], each of the operators $\big(L_k, B_k\big)$ is a bounded Fredholm operator with index zero.

We find constructive sufficient conditions on the left-hand sides of boundary-value problems under which these problems are well-posed for arbitrary right-hand sides of the problems (1), (2)

$$y_k \to y_0 \quad \text{in} \quad (W_p^{n+r})^m \quad \text{as} \quad k \to \infty. \tag{3}$$

Moreover, we prove that for an arbitrary well-posed problem $\big(L_0, B_0\big)y_0 = (f, c)$, there exists a sequence of boundary-value problems (1), (2) with polynomial coefficients and multipoint boundary conditions of the form (1), which satisfy boundary condition (3) for arbitrary right-hand sides of the problems.

[1] V. Mikhailets, O. Atlasiuk, The solvability of inhomogeneous boundary-value problems in Sobolev spaces, Banach J. Math. Anal. 18(2):12 (2024).

[2] V. Mikhailets, O. Atlasiuk, Differential systems in Sobolev spaces with generic inhomogeneous boundary conditions, Carpathian Math. Publ. 16:2 (2024), 523–538.

2. **Matthias Baur**

Title: *Optimizing the ground state energy of the three-dimensional magnetic Dirichlet Laplacian with constant magnetic field*

Abstract: We consider the shape optimization problem of minimizing the ground state energy of the magnetic Dirichlet Laplacian with constant magnetic field on three-dimensional geometries of fixed volume. In contrast to the two-dimensional case, a generalized "magnetic" Faber-Krahn inequality does not hold and the minimizers are not expected to be balls when the magnetic field is turned on. An analysis of the problem among cylindrical domains reveals geometric constraints for general minimizers. In particular, minimizers must elongate with a certain rate along the direction of the magnetic field as the field strength increases. In addition to theoretical analysis, we present numerical minimizers which confirm this prediction and give rise to further conjectures.

3. **Cristian Cazazu**

Title: *New regularity results for elliptic operators with unbounded coefficients*

Abstract: In this talk we present some recent regularity results for solutions of elliptic equations with singular coefficients of the form $-\Delta u + \lambda \frac{x\cdot\nabla u}{|x|^2} = f$, which turn out to be new in the literature. We show that the influence of the unbounded perturbation term $\lambda \frac{x\cdot\nabla u}{|x|^2}$ in the regularity behavior of the solutions depends in a deep way on the value of the spectral parameter λ. This talk is based on a joint work with Adelina Călina (PhD Student, University of Bucharest).

4. **Fabrizio Colombo**

Title: *Evolution of superoscillations*

Abstract: Superoscillations manifest across various domains in science and technology, particularly arising from weak measurements in quantum mechanics. Therefore, it is crucial to explore the evolution of superoscillatory functions as initial data for the Schrödinger equation when the Hamiltonian operator involves different potentials. Given that the prototypical functions appearing in weak measurements lack squared integrability, it becomes important to investigate the evolution problem using methods employing infinite order differential operators that act on spaces of entire functions. This presentation provides an overview of the current advancements in the function theory and the Schrödinger evolution of superoscillations.

5. **Tomáš Dohnal**

Title: *Spectrum of the Maxwell Equations for a Flat Interface between Dispersive Media*

Abstract: The study of time harmonic electromagnetic waves at the interface of dispersive (i.e. frequency dependent) media leads to the non-self-adjoint operator pencil problem

$$\nabla \times \nabla \times E - \omega^2 \mu_0 \epsilon(x_1, \omega) E = 0, \quad \nabla \cdot (\epsilon E) = 0,$$

where $\epsilon(x_1, \omega) \in \mathbb{C}$ is the dielectric function. A classical application is to surface plasmon polaritons at the interface of a metal and a dielectric. We assume that the interface is located at $x_1 = 0$, i.e. $\epsilon(\cdot, \omega)$ jumps at $x_1 = 0$ and the media in the two half spaces are either homogeneous or depend only on x_1. The dependence of ϵ on the spectral parameter ω (frequency) is arbitrary and generally nonlinear.

The whole spectrum of the pencil L, with $L(\omega)E := \nabla \times \nabla \times E - \omega^2 \mu_0 \epsilon(\cdot, \omega) E$, consists of eigenvalues and the essential spectrum, but the various standard types of essential spectra do not coincide in all cases. The main tool for determining the essential spectra are Weyl sequences. The functional setting is such that the operator domain is not a subset of the range which brings about a difficulty in defining the discrete spectrum. We study the reductions to 1D, 2D, as well as the full 3D setting.

This work is in collaboration with M. Brown (Cardiff), M. Plum (Karlsruhe), K. M. Schmidt (Cardiff) and I. Wood (Kent).

[1] M. Brown, T. Dohnal, M. Plum, and I. Wood, Spectrum of the Maxwell Equations for a Flat Interface between Homogeneous Dispersive Media, *Comm. Math. Phys.*, 406, 3 (2025).

[2] T. Dohnal, M. Plum, K.M. Schmidt, and I. Wood, Spectrum of the Maxwell Equations for a Flat Interface between Dispersive Media in 3D, *in preparation*, 2025.

6. **Joaquim Duran**

Title: *Seeing quantum dot Dirac operators through $\overline{\partial}$-Robin Laplacians*

Abstract: This work addresses Faber-Krahn type inequalities for quantum dot Dirac operators with nonnegative mass on domains in $\mathbb{R}^2$. We show that this family of inequalities is equivalent to a family of Faber-Krahn type inequalities for $\overline{\partial}$-Robin Laplacians. Thanks to this, we prove them in the case of simply connected domains for quantum dot boundary conditions asymptotically close to zigzag boundary conditions.

7. **Tom ter Elst**

Title: *Commutator estimates and Poisson bounds for Dirichlet-to-Neumann operators with variable coefficients*

Abstract: We consider the Dirichlet-to-Neumann operator $\mathcal{N}$ associated with a general elliptic operator

$$\mathcal{A}u = -\sum_{k,l=1}^{d} \partial_k(c_{kl}\,\partial_l u) + \sum_{k=1}^{d}\Big(c_k\,\partial_k u - \partial_k(b_k\,u)\Big) + c_0\,u \in \mathcal{D}'(\Omega)$$

with possibly complex coefficients. We study three problems:
1) Boundedness on C^ν and on L_p of the commutator $[\mathcal{N}, M_g]$, where M_g denotes the multiplication operator by a smooth function g.
2) Hölder and L_p-bounds for the harmonic lifting associated with $\mathcal{A}$.
3) Poisson bounds for the heat kernel of $\mathcal{N}$.
We solve these problems in the case where the coefficients are Hölder continuous and the underlying domain is bounded and of class $C^{1+\kappa}$ for some $\kappa > 0$. For the Poisson bounds we assume in addition that the coefficients are real-valued. We also prove gradient estimates for the heat kernel and the Green function G of the elliptic operator with Dirichlet boundary conditions.

This is joint work with E.M. Ouhabaz.

8. **Francesco Ferraresso**

Title: *The dissipative Maxwell system in Drude-Lorentz materials*

Abstract: I will discuss spectral enclosures and spectral approximation results for the inhomogeneous, dissipative Drude-Lorentz system with purely imaginary poles, in a possibly unbounded Lipschitz domain of $\mathbb{R}^3$. Under the assumption that the coefficients θ_e, θ_m of the material are asymptotically constant at infinity, I will show that spectral pollution due to domain truncation is confined in the essential numerical range of a $\operatorname{curl}\operatorname{curl} - f(\omega)$ pencil. Based on joint work with M. Marletta (Cardiff).

9. **Borbala Gerhat**

Title: *Eigenvalue bounds in the linear stability of the Ekman spiral*

Abstract: Originating in fluid dynamics, the stability analysis of Ekman boundary layers leads to a spectral problem for a family of non-selfadjoint linear operator matrices. We present new enclosures for the point spectrum (leading to the solution of an open problem posed by L. Greenberg and M. Marletta in 2004) and investigate the number of eigenvalues. Our analysis is based on a Birman–Schwinger type argument which exploits underlying similarities to Schrödinger operators.

Based on joint work with O. Ibrogimov and P. Siegl.

10. **Denis Grebenkov**

Title: *The exterior Steklov problem: formulations, asymptotic behavior, and applications*

Abstract: In this talk, we present recent advances on the Steklov spectral problem in the complement $\Omega = R^d\backslash\Omega_0$ of a compact set Ω_0 that consists in finding eigenpairs $\{\mu, V\}$ satisfying $\Delta V = 0$ in Ω, with the Robin-type boundary condition $\partial_n V = \mu V$ on a Lipschitz boundary $\partial\Omega$. We first overview several distinct formulations of this exterior problem and discuss their equivalence. We then switch to the generalized Steklov problem, in which $\Delta V = 0$ is replaced by $(\Delta - p)V = 0$, with a parameter $p > 0$. We obtain the asymptotic behavior of the Steklov eigenvalues as $p \to 0$ and reveal their non-analytical dependence on p that depends on the space dimension. Several applications of these asymptotic results to reflected diffusion in chemical physics and statistics physics are presented. Some open problems and further perspectives are discussed.

[1] D. S. Grebenkov and A. Chaigneau, *The Steklov problem for exterior domains: asymptotic behavior and applications* (accepted to J. Math. Phys.; preprint 2407.09864v2).

[2] L. Bundrock, A. Girouard, D. S. Grebenkov, M. Levitin and I. Polterovich, *Exterior Steklov problems* (in preparation).

11. **Seppo Hassi**

Title: *An extension of the theorems of Douglas and Sebestyén for unbounded operators*

Abstract: Let T and B be closed densely defined operators from a Hilbert space $\mathfrak{H}$ to a Hilbert space $\mathfrak{K}$. Necessary and sufficient conditions are established for the factorization of T as a product of B and a bounded

nonnegative operator X on $\mathfrak{K}$. This result yields a new extension and a refinement of a well-known theorem of R.G. Douglas (1966) who showed that for bounded operators the operator inequality $A^*A \leq \lambda^2 B^*B, \lambda \geq 0$, is equivalent to the factorization $A = CB$ with $\|C\| \leq \lambda$. The main results give necessary and sufficient conditions for the existence of an intermediate selfadjoint operator $H \geq 0$, such that $A^*A \leq \lambda H \leq \lambda^2 B^*B$. The connection of the operator H to the operators T and B is made explicit. The key results are proved by first extending a theorem of Z. Sebestyén (1983) to the setting of unbounded operators.

The talk is based on joint work with Yosra Barkaoui (Vaasa).

12. **Andrii Khrabustovskyi**

Title: *Geometric approximation of non-local interface and boundary conditions*

Abstract: In this talk, we discuss an approximation of the Laplacian with non-local interface conditions (resembling surface δ' interactions) by Neumann Laplacians defined on a family of Riemannian manifolds. These manifolds consist of two domains whose boundaries are connected via an array of intertwined passages. We establish a kind of strong resolvent convergence for the associated operators, along with the convergence of spectra, eigenspaces, and corresponding semigroups. Furthermore, we construct an explicit example of a manifold that realizes any prescribed integral kernel appearing in the interface conditions. Finally, we extend our analysis to similar approximations for Laplacians subject to non-local Robin-type boundary conditions.

This is a joint work with Pavel Exner (Prague).

13. **Alexander Komech**

Title: *On well posedness for autonomous damped driven Jaynes–Cummings equation*

Abstract: We consider damped driven version of the Jaynes–Cummings equations describing the laser action [1, 2, 3]. The equation describes quantised one-mode Maxwell field coupled to a two-level molecule.

Our main result is construction of the contraction semigroup in the Hilbert space of Hermitian Hilbert–Schmidt operators in the case of time-independent pumping. The main novelty is a non-positivity of the Jaynes–Cummings generator with the damping.

[1] E.T. Jaynes, F.W. Cummings, Comparison of quantum and semiclassical radiation theories with application to the beam maser, *Proc. IEEE* **51** (1963), no. 1, 89–109.

[2] J. Larson, T. Mavrogordatos, The Jaynes–Cummings Model and Its Descendants. Modern research directions, IOP Publishing, Bristol, UK, 2021.

[3] G. Lindblad, On the Generators of Quantum Dynamical Semigroups, *Comm. Math. Physics* **48** (1976), 119–130.

14. **Elena Kopylova**

Title: *Asymptotic stability of solitons for 2D Maxwell-Lorentz equations with rotaiting particle*

Abstract: We consider 2D Maxwell-Lorentz equations with extended charged rotating particle. The system admits solitons which are solutions corresponding to a particle moving with a constant velocity and rotating with a constant angular velocity. Our main result is asymptotic stability of the solitons.

15. **Yuri Latushkin**

Title: *The ubiquitous Duistermaat index and eigenvalues counting*

Abstract: Eigenvalue interlacing is a useful tool in linear algebra and spectral analysis. We discuss a sharp version of the interlacing inequalities expressed as bounds on the spectral shift between two self-adjoint extensions of a fixed symmetric operator with finite and equal defect numbers. The bounds are given in terms of the Duistermaat index, a topological invariant describing the relative position of three Lagrangian planes in a symplectic space. We give an axiomatic description of the Duistermaat index, study its connections to the Maslov index and give applications to a formula for the Morse indices of difference of Hermitian matrices.

This is a joint work with G. Berkolaiko, G. Cox and S. Sukhtaiev.

16. **Michael Levitin**

Title: *From asymptotics to bounds: Pólya's conjecture, ODEs, and number theory*

Abstract:

There are many situations when asymptotic behaviour of some multi-parameter-dependent quantity (when one of the parameters gets large) provides an upper or lower bound for this quantity for all values of parameters. A classical example of this phenomenon is Pólya's conjecture in spectral geometry [1] stating that the eigenvalue counting functions of the Dirichlet or Neumann Laplacians in a bounded Euclidean domain are bounded above (in the Dirichlet case) or below (in the Neumann case) by the leading term of their Weyl's asymptotics (which does not depend on the boundary conditions) for *all* values of the parameter. This conjecture is still open in full generality: I will discuss its recent resolution for disks, balls, sectors and annuli, as well as other similar or related phenomena arising in other subjects, from ODEs to number theory [2, 3, 4, 5].

[1] G. Pólya, Mathematics and plausible reasoning. Princeton University Press, Princeton, N. J., 1954.

[2] N. Filonov, M. Levitin, I. Polterovich, and D. A. Sher, Pólya's conjecture for Euclidean balls, Invent. Math. 234 (2023), 129–169.

[3] N. Filonov, M. Levitin, I. Polterovich, and D. A. Sher, Inequalities à la Pólya for the Aharonov–Bohm eigenvalues of the disk, J. Spectr. Theory 14:2 (2024), 597–618.

[4] N. Filonov, M. Levitin, I. Polterovich, and D. A. Sher, Uniform enclosures for the phase and zeros of Bessel functions and their derivatives, SIAM J. Math. Anal. 56:6 (2024), 7644–7682.

[5] N. Filonov, M. Levitin, I. Polterovich, and D. A. Sher, Pólya's conjecture for Dirichlet eigenvalues of annuli, arXiv, 2025 (to appear shortly).

17. **Vladimir Lotoreichik**

Title: *Convergence of Schrödinger operators on domains with scaled resonant potentials*

Abstract: In this talk, we will consider Schrödinger operators on a bounded, smooth domain of dimension $d \geq 2$ with Dirichlet boundary conditions and a properly scaled potential, which depends only on the distance to the boundary of the domain. We will discuss the convergence of these operators as the scaling parameter tends to zero. It turns out that if the scaled potential is resonant, the limit in strong resolvent sense is a Robin Laplacian with boundary coefficient expressed in terms of the mean curvature of the boundary. Remarkably, a counterexample shows that norm resolvent convergence cannot hold in general in this setting. On the contrary, if the scaled potential is non-negative (hence non-resonant), the limit in strong resolvent sense is the Dirichlet Laplacian. We conjecture that we can drop the non-negativity assumption in the non-resonant case. Our results extend the one-dimensional case analysed by P. Šeba to higher dimensions, where now the curvature of the boundary starts to play a role. This talk is based on a joint work with Olaf Post.

18. **Annemarie Luger**

Title: *Rational Herglotz-Nevanlinna functions*

Abstract: In this talk we first review a characterization of rational Herglotz-Nevanlinna functions in one variable and discuss then the situation for functions in several variables. In particular, we focus on the situation of low degree in the denominator.

19. **Anton Lunyov**

Title: *On the formula for characteristic determinants of boundary value problems for $n\times n$ Dirac type systems and its applications*

Abstract: In this talk we investigate the spectral properties of the boundary value problems (BVP) associated with the following $n \times n$ Dirac type equation:

$$-iy' - iQ(x)y = \lambda B(x)y, \qquad y = \operatorname{col}(y_1, \dots, y_n), \qquad x \in [0, \ell],$$

on a finite interval $[0, \ell]$ subject to the general two-point boundary conditions $Cy(0) + Dy(\ell) = 0$ with $C, D \in \mathbb{C}^{n\times n}$. Here $Q = (Q_{jk})_{j,k=1}^n$ is an integrable potential matrix and $B = \operatorname{diag}(\beta_1, \dots, \beta_n) = B^*$ is a diagonal integrable matrix "weight". If $n = 2m$ and $B(\cdot) = \operatorname{diag}(-I_m, I_m)$, this equation turns into $n \times n$ Dirac equation.

First, assuming that $\operatorname{supp}(Q_{jk}) \subset \operatorname{supp}(\beta_k - \beta_j)$, we show that the deviation $\Phi_Q(\cdot, \lambda) - \Phi_0(\cdot, \lambda)$ of the fundamental matrix solutions to the above perturbed and unperturbed ($Q = 0$) equation is represented as a Fourier transform of a certain matrix kernel $K_Q(\cdot, \cdot)$ from a special Banach space. This result is used to

prove our main result, the following formula for the deviation of the characteristic determinants $\Delta_Q(\cdot)$ and $\Delta_0(\cdot)$ of two (perturbed and unperturbed) BVPs as a Fourier transform,

$$\Delta_Q(\lambda) = \Delta_0(\lambda) + \int_{b_-}^{b_+} g(u)e^{i\lambda u}\,du, \qquad g \in L^1[b_-, b_+],$$

where $b_\pm$ are explicitly expressed via entries of the matrix function $B(\cdot)$.

In turn, assuming that each function $\beta_k(\cdot)$ is of fixed sign, this representation yields asymptotic behavior of the spectrum in the case of regular boundary conditions. Namely, we show that $\lambda_m = \lambda_m^0 + o(1)$ as $m \to \infty$, where $\{\lambda_m\}_{m\in\mathbb{Z}}$ and $\{\lambda_m^0\}_{m\in\mathbb{Z}}$ are sequences of eigenvalues of perturbed and unperturbed BVP, respectively. It is also shown that for $Q \in L^p$, $p \in (1, 2]$, the following estimate holds under additional condition on $B(\cdot)$:

$$\sum_{m\in\mathbb{Z}} |\lambda_m - \lambda_m^0|^{p'} + \sum_{m\in\mathbb{Z}} (1 + |m|)^{p-2}|\lambda_m - \lambda_m^0|^p < \infty, \qquad p' := p/(p-1).$$

In the case of Dirac operator, we show that the sequence of the eigenvalues splits into the union of n branches asymptotically close to arithmetic progressions $\{2\pi k - i\mathrm{Log}(-\mu_s)\}_{k\in\mathbb{Z}}$, $s \in \{1, \dots, n\}$, where $\mu_1, \dots, \mu_n$ are the eigenvalues of a certain $n \times n$ matrix explicitly constructed via the matrices C and D.

The talk is based on a joint work [1] with Mark Malamud.

[1] A.A. Lunyov and M.M. Malamud, On the formula for characteristic determinants of boundary value problems for $n \times n$ Dirac type systems and its applications, *Advances in Math.*, **478** (2025), 110389.

20. **Mark Malamud**

Title: *To the Birman-Krein-Visic Theory. Solution to the Birman problem.*

Abstract: Let A be a closed non-negative symmetric densely defined operator in a Hilbert space $\mathfrak{H}$ and let $\mathfrak{H}_1 := \mathrm{ran}(A + I)$. By the Stone – Friedrichs theorem the set $\mathrm{Ext}_A(0, \infty)$ of all nonnegative selfadjoint extensions $\widetilde{A} = \widetilde{A}^*$ of A is nonempty. Complete theory of extensions of $A \geq 0$ was built by M. Krein. In particular, he proved that $\mathrm{Ext}_A(0, \infty)$ forms an operator segment with two endpoints: the maximal (the Friedrichs) and the minimal (the Krein) extensions $\widehat{A}_F$ and $\widehat{A}_K$. They are uniquely characterized by means of the inequalities: $\widehat{A}_K \leq \widetilde{A} \leq \widehat{A}_F$ for each $\widetilde{A} \in \mathrm{Ext}_A(0, \infty)$ which are understood in quadratic forms sense.

Krein's theory has substantially been completed by M. Vicik and M. Birman.

If A is positive definite, then $\widehat{A}_K$ admits a representation $\widehat{A}_K = \widehat{A}'_K \oplus (\mathbb{O} \restriction \mathfrak{N}_0)$ where $\mathfrak{N}_0 := \ker A^*$. The operator $\widehat{A}'_K$ is called the reduced Krein extension.

Let P_1 be the orthoprojection in $\mathfrak{H}$ onto $\mathfrak{H}_0$ and let $\mathfrak{M}_0 = \mathfrak{N}_0^\perp$. We will discuss the equivalence

$$P_1(I_\mathfrak{H} + A)^{-1} \in \mathfrak{S}_p(\mathfrak{H}_1) \quad \Longleftrightarrow \quad (I_{\mathfrak{M}_0} + \widehat{A}'_K)^{-1} \in \mathfrak{S}_p(\mathfrak{M}_0),$$

which improves Krein's result: $(I_\mathfrak{H} + \widehat{A}_F)^{-1} \in \mathfrak{S}_p \implies (I_{\mathfrak{M}_0} + \widehat{A}'_K)^{-1} \in \mathfrak{S}_p$.

In early 2000s M.S. Birman posed the following problem.

Problem.

Assume that the operator $(I + A)^{-1} : \mathfrak{H}_1 \to \mathfrak{H}$ is compact. Is it true that the resolvent of the Friedrichs' extension $\widehat{A}_F$ of A is also compact?

We will also discuss an explicit solution to this problem for elliptic operators. The results of the talk were announced in [1], [2], and partially published in [3].

[1] M. M.Malamud, To Birman–Krein–Vishik Theory, Doklady Mathematics, Vol. 107, No. 1 (2023), pp. 44–48.

[2] M.M. Malamud, On the Birman problem on positive symmetric operators with compact inverse, Func. Anal. Appl., V.57, No 2 (2023), p. 111–116.

[3] M. Malamud, Explicit Solution to the Birman Problem for the $2D$-Laplace operator, Russian Jour. of Math. Phys., 2024, Vol. 31, No.3, p. 495-504.

21. **Andrea Mantile**

Title: *Acoustic operators with piecewise constant coefficients: scattering theory and resonant asymptotic regimes*

Abstract: We revise the mixed perturbation theory elaborated in [1] to build Kreĭn-like resolvent formulae for acoustic operators with piecewise constant coefficients. The key of this approach is the introduction of an acoustic Q-function describing the interaction of free waves with sharply discontinuous inhomogeneities.Such formulae are new and allow us to build the acoustic scattering theory in such regimes. In these frameworks, resolvent's limiting absorption estimates, explicit formulae for the generalised eigenfunctions and a modified Fourier transform, leading to an explicit representation of the scattering matrix, are obtained. This provides the framework for a rigorous approach to acoustic resonances excited by sharp inhomogeneities. Such problems are considered in the asymptotic settings of small inhomogeneities that are highly contrasted with respect to an homogeneous background: it is known that these regimes can produce localisation phenomena where the scattered wave concentrates at the inhomogeneities sites at specific incoming frequencies, referred to as scattering-resonances. In the acoustic case, micro-resonator settings have been investigated in connection with volume or surface stationary-modes. By now, a proper spectral characterisation of these scattering-frequencies is an open question. Our construction allows us to model different micro-resonator designs, providing an explicit representation of the scattering amplitude in terms of the acoustic Q-function. For those configurations decoupling volume and surface contributions, implicit-function arguments adapted to such functional frameworks allows to derive complete asymptotic expansions of resonances close to their limit values: in these cases, our results confirm the expected links of the physical resonances with known scattering frequencies. We also consider configurations, not yet explored in mathematical application studies, where the contributions from the relevant volume and surface operators are of the same size. In such cases, using a simplified geometry, we explicitly compute the dominant part of the scattering matrix and implement the Gohberg-Sigal approach to localize the relevant resonances at a small scale, proving the survival of volume modes. This work is jointly elaborated with A. Posilicano.

[1] A. Mantile, A. Posilicano. Scattering theory with both regular and singular perturbations. *J. Spectr. Theory* **13** no. 3, 1057-1109, 2023.

22. **Francesco Mantovani**

Title: *The H^∞-functional calculus for Clifford bisectorial operators*

Abstract: In this talk we introduce the H^∞-functional calculus for unbounded bisectorial operators defined on Banach modules over the Clifford algebra $\mathbb{R}_n$. This calculus is defined in a three-step procedure. Firstly, we generate the ω-functional calculus for decaying functions at zero and at infinity, then we extend this calculus to functions with a constant value at zero and at infinity. Finally, using a subsequent regularization procedure, we can define the H^∞-functional calculus for the class of regularizable functions, which in particular includes functions with polynomial growth at infinity and, if T is injective, also functions with polynomial growth at zero.

We will also present a relevant example of bisectorial operator, which is the gradient with nonconstant coefficients defined on $L^2(\mathbb{R}^n, \mathbb{R}_n)$.

This talk is based on a joint work with F. Colombo and P. Schlosser.

[1] F. M., P. Schlosser: *The H^∞-functional calculus for bisectorial Clifford operators.* J. Spectr. Theory. 15 , no. 2, pp. 751-818. (2025).

[2] F. Colombo, F. M., P. Schlosser: *Spectral properties of the gradient operator with nonconstant coefficients.* Anal. Math. Phys. 14 , no. 5, Paper No. 108, 31 p. (2024).

23. **Anna Muranova**

Title: *Recurrence and transience in non-Archimedean weighted graphs*

Abstract: We introduce notions of recurrence and transience for graphs over a non-Archimedean ordered field. To achieve this, we establish a connection between these graphs and random walks. We give a characterization of the Markov chains, which can arise in such a way. Moreover, we define a quantity related to the capacity to a graph which allows to establish a type (recurrent or transient) of the vertex in the corresponding Markov chain. It is a joint work with Matthias Keller.

24. **Paolo Musolino**

Title: *The functional analytic approach for domain perturbation problems in spectral theory*

Abstract: The functional analytic approach (FAA) is a method whose goal is to represent the solutions of perturbed boundary value problems in terms of real analytic maps and known functions of the perturbation parameters. Typical examples are regular domain perturbation problems, where a reference domain Ω is perturbed through a suitable diffeomorphism ϕ (and one considers a problem in the perturbed set $\phi(\Omega)$), or singular domain perturbations, where, for example, one perturbs Ω by making a small hole of size ϵ and then studies a certain boundary value problem in the perforated set Ω_ϵ. By exploiting the FAA, one would try to represent the solution of the boundary value problem in $\phi(\Omega)$ as a real analytic map of the diffeomorphism ϕ (which we think as a point in a certain Banach space) or of a problem in Ω_ϵ as a real analytic map of the size ϵ of the small perforation, for ϵ close to 0.

In this talk, we present some recent applications of the FAA to the study of the behavior of eigenvalues, both in the case of regular perturbations and of singular perturbations.

Acknowledgment: P.M. acknowledges the support from EU through the H2020-MSCA-RISE-2020 project EffectFact, Grant agreement ID: 101008140, and by NextGeneration EU under the NRRP, Call PRIN 2022 No. 104 of February 2, 2022 of MUR; Project code 2022SENJZ3.

25. **Konstantin Pankrashkin**

Title: *Recent results on Laplacian eigenvalues with large negative Robin parameters*

Abstract: The eigenvalues of Laplacians with large negative Robin parameter in the boundary condition have attracted a lot of attention in the last 20 years due to interesting links between the eigenvalue behavior and the geometry of the domain or of its boundary. We give an overview of recent results for smooth and non-smooth domains and mention some ongoing work.

26. **Irina Pettersson**

Title: *Homogenization of some indefinite spectral problems arising in population genetics*

Abstract: The work concerns the homogenization of a spectral problem for a second order self-adjoint elliptic operator in an asymptotically thin cylinder. We assume that the operator coefficients and the spectral density function are locally periodic in the axial direction of the cylinder, and that the spectral density function changes sign. Because of the latter condition, there are infinitely many positive and negative eigenvalues. The asymptotic behavior of the spectrum, as the thickness of the rod tends to zero, depends essentially on the sign of the average of the density function. We study the asymptotics of the positive eigenvalues under the assumption that the local average of the spectral weight is negative. We derive a one-dimensional effective spectral problem that is a harmonic oscillator on the real line and prove the convergence of the spectrum. A key auxiliary result is the existence of a positive principal eigenvalue of an indefinite spectral problem with the Neumann boundary condition on a periodicity cell. Under the structural assumption that this principal eigenvalue attains a global minimum in the domain, we show that the eigenfunctions of the original problem are oscillating and localized near the minimum point of this principal eigenvalue. The study is motivated by applications in population genetics where the least positive eigenvalue determines the bifurcation point near a stationary solution.

27. **Yehuda Pinchover**

Title: *Weak perturbation of critical quasilinear operators*

Abstract: We consider a critical quasilinear operator $-\Delta_p+V$ in $\mathbb{R}^N$ perturbed by a weakly coupled potential. For $1 < p < N$ we find the leading asymptotic of the lowest eigenvalue of such an operator in the weak coupling limit separately for $p < N \leq p^2$ and $p^2 < N$.

This is a joint work with Ujjal Das and Hynek Kovařík.

28. **Mateusz Piorkowski**

Title: *Regularity properties of Sturm–Liouville operators with trace class resolvents*

Abstract: Consider the general three-term Sturm–Liouville differential expression

$$\tau = \frac{1}{r(x)}\Big[-\frac{d}{dx}p(x)\frac{d}{dx} + q(x)\Big], \qquad x \in (a,b) \subset \mathbb{R}$$

with locally integrable coefficients. In a recent collaboration with J. Stanfill (arXiv:2407.04847) we have shown that the property of τ having self-adjoint realizations with trace class resolvents can be fully characterized in terms of two arbitrary classical solutions f_1, f_2 of the ODE $\tau f_i = \lambda_i f_i$ with different eigenparameters $\lambda_1 \neq \lambda_2$; in the special case $\lambda_1 = \lambda_2$, a known characterization can be obtained from Mercer's Theorem. In case the resolvent trace class property holds, our results allow us to define what we call the "regularization index" at each of the endpoints $\{a, b\}$. I will show how this index refines the limit circle/limit point classification due to Weyl.

If time permits, I will present an application of the regularization index to the commutation method for Schrödinger operators.

29. **David Ploß**

Title: *Spectral Properties of Elliptic Operators with Non-local Dynamic Boundary Conditions*

Abstract: The interchange of energy between a (Lipschitz) domain Ω and its boundary Γ can be modeled by dynamic or Wentzell–Robin boundary conditions, which also allow for concentration of energy on Γ. Our main focus lies in adding bounded non-local operators to an elliptic operator, e.g. $-\Delta$, which may be used to include integral kernels arising from stochastic processes into the dynamic boundary condition. As usual, when dealing with those conditions, we work in the product space $\mathcal{H} = L^2(\Omega) \times L^2(\Gamma)$, which gives rise to operator families of type

$$\mathcal{A} = \mathcal{L} - \varepsilon\mathcal{B} = \begin{pmatrix} -\Delta & 0 \\ \partial_\nu & 0 \end{pmatrix} - \varepsilon \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}.$$

We investigate the spectral properties of $-\mathcal{A}$ and show that it generates an analytic semigroup in $\mathcal{H}$. Furthermore, we give a full characterization of positivity and Markovianity. Also, when positivity fails, we give criteria to establish eventual positivity for symmetric and non-symmetric perturbations. In the latter case, if the non-locality is of a special structure, we also consider $\mathcal{A}$ in Krein-spaces and self-adjoint perturbation techniques therein.

30. **Andrea Posilicano**

Title: *The Born-Oppenheimer approximation for a 1D 2+1 particle system with zero-range interactions*

Abstract: We study the spectrum of the self-adjoint Hamiltonian that models the quantum dynamics of a 1D three-body system consisting of a light particle interacting with two heavy ones through an attractive zero-range force. We are mainly concerned with the behavior of the eigenvalues below the essential spectrum in the regime $\varepsilon \ll 1$, where ε is proportional to the square root of the mass ratio. We show that the n-th eigenvalue behaves as

$$\lambda_n(\varepsilon) = -\alpha^2 + |\sigma_n|\,\alpha^2\varepsilon^{2/3} + O(\varepsilon)\,,$$

where α explicitly relates to the physical parameters and σ_n is either the n-th extremum or the n-th zero of the Airy function Ai, depending on the kind (respectively bosons or fermions) of the two heavy particles.

This is a joint work with Claudio Cacciapuoti and Hamidreza Saberbaghi.

31. **Olaf Post**

Title: *Convergence of the Dirichlet-to-Neumann map on thin branched manifolds — a never ending story — joint work with Jussi Behrndt*

Abstract: We show that the time to finish an article can become infinitely long: the convergence of the Dirichlet-to-Neumann maps associated with a thin branched manifold with cylindrical ends to the Dirichlet-to-Neumann map on the corresponding metric graph may take a lot of time. As boundary for the Dirichlet-to-Neumann map we choose the boundary of the cylindrical ends of the manifold and the free ends of the metric graph.

32. **Jakob Reiffenstein**

Title: *Eigenvalues of self-adjoint exit space extensions*

Abstract: This talk is about a natural question that occurs in the context of the Krein-Naimark formula. The setting is as follows: Let S be a closed symmetric operator in a Hilbert space $\mathcal{H}$ with equal defect numbers (n, n). We fix a canonical self-adjoint extension A of S and a defect family $\gamma(\lambda)$. Then there is a one-to-one correspondence between all minimal self-adjoint extensions $\widetilde{A}$ of S in Hilbert spaces containing $\mathcal{H}$ (up to unitary equivalence) and all Nevanlinna families $\tau(\lambda)$ by means of the Krein-Naimark formula:

$$\mathcal{P}_{\mathcal{H}}(\widetilde{A} - \lambda)^{-1}|_{\mathcal{H}} = (A - \lambda)^{-1} - \gamma(\lambda)(m(\lambda) + \tau(\lambda))^{-1}\gamma(\overline{\lambda})^{*}. \tag{4}$$

Here $m(\lambda)$ is the Q-function of $(S, A, \gamma(\lambda)$, which is unique up to a self-adjoint additive constant.

We give a complete characterization of the eigenvalues of $\widetilde{A}$ in terms of the interplay between m and τ close to the real line. I will explain the role of *pole cancellation functions* in the description of eigenvalues and why we need to extend this notion in order to make the characterization explicit in m and τ.

Based on joint work with A. Luger.

33. **Jonathan Rohleder**

Title: *Inequalities between Neumann and Dirichlet Laplacian eigenvalues*

Abstract: Non-trivial inequalities between Neumann and Dirichlet Laplacian eigenvalues have a long history going back at least to work of Pólya and Payne in the 1950s, followed by significant contributions by Levine and Weinberger, Friedlander, Filonov, and others. In this talk, some recent developments around the topic are presented. In particular, we show that an optimal inequality proven by Levine and Weinberger for convex domains remains true for simply connected planar domains.

34. **Christian Rose**

Title: *Off-diagonal Upper heat kernel bounds on graphs with unbounded geometry*

Abstract: Recent characterizations of localized Gaussian upper heat kernel bounds on graphs with unbounded geometry in terms of Faber-Krahn inequalities are presented. Due to the non-local character of the associated Laplacian, characterisations only hold on large scales. The involved localized functional inequalities defined in distance balls depend on variable dimensions which can be related to the doubling dimension and the vertex degree growth.

35. **Sofia Rumiantseva**

Title: *Geometric Algebra Approach to Exploring the Multidimensional Dirac Equation*

Abstract: In the talk, we will introduce the multidimensional Dirac–Hestenes equation, a real-valued analogue of the Dirac equation formulated in the geometric algebra $\mathrm{Cl}_{1,n}$, and discuss some of its fundamental properties. It is known that the classical four-dimensional Dirac equation is equivalent to the Dirac–Hestenes equation in geometric algebra $\mathrm{Cl}_{1,3}$. It means that one might obtain a solution to the Dirac–Hestenes equation using a solution to the Dirac equation and vice versa. The Dirac–Hestenes equation may provide a deeper understanding of geometry in various tasks, as the considering wave function is entirely real. We will show that the theory is extended to the multidimensional case. Since the matrix representation of complex geometric algebra depends on the parity of n, the cases of even and odd n are analyzed separately. In the even-dimensional case, there are two types of spinors which are solutions to the Dirac equation: semi-spinors and double spinors. Additionally, we will demonstrate that the multidimensional Dirac–Hestenes equation exhibits both gauge invariance and Lorentz invariance.

36. **Tomas Sanz-Perela**

Title: *Eigenvalue curves for Dirac operators: a strategy for shape optimization*

Abstract: In this talk I will describe two recent works (in collaboration with A. Arrizabalaga, J. Duran, A. Mas, and L. Vega) in which we consider families of Dirac operators with a boundary condition depending on a parameter, both in the 2D and 3D setting. I will explain how to describe the spectrum of the operators through eigenvalue curves, and how the properties of these curves can be used to obtain positive answers to some questions in shape optimization.

37. **Timotheus Schmatzler**

Title: *Eigenvalue inequalities for some classes of elliptic operators*

Abstract: For the Laplace operator on a bounded Lipschitz domain, several inequalities of the form

$$\mu_{k+r} \leq \lambda_k \qquad (\star)$$

(with $k, r \in \mathbb{N}$) between the Dirichlet eigenvalues $\{\lambda_k\}_{k\geq 1}$ and the Neumann ones $\{\mu_k\}_{k\geq 1}$ have been been proven. One example is Pólya's inequality $\mu_2 < \lambda_1$, which has interesting consequences on the geometry of eigenfunctions. However, for elliptic operators with non-constant coefficients, for example the inhomogeneous membrane operator $-\frac{1}{\rho}\Delta$ with density ρ, Pólya's inequality does not hold in general. In this talk we motivate and discuss some inequalities of the form $(\star)$ for certain classes of elliptic operators with non-constant coefficients.

38. **Konrad Schmüdgen**

Title: *C-Self-adjoint Extensions of C-Symmetric Operators*

Abstract: Let C be a contraction on a Hilbert space H. A linear operator T on H is called C-symmetric if $\langle Tx, Cy\rangle = \langle x, CTy\rangle$ for all $x, y \in \mathrm{dom}(T)$. If T is densely defined, then T is called C-self-adjoint if $CT = T^*C$.

In the first part of the talk, we characterize all contractive C-self-adjoint extensions of a non-densely defined C-symmetric contraction. Applying this to the Cayley transform we obtain a proof of an announced result of Glazman which states that each densely defined C-symmetric dissipative operator admits a C-self-adjoint maximal dissipative extension.

In the second part, we describe all C-self-adjoint extensions of a densely defined (unbounded) C-symmetric operator.

The talk is based on joint work with Yu. M. Arlinskii.

39. **Nathanael Skrepek**

Title: *A boundary triple for the wave equation in the classical first order (in time) form*

Abstract: We study the Lagrangian representation of the wave equation with generalized Laplacian $\operatorname{div} T\nabla$. We allow the coefficients—the Young modulus T and the density ρ—to be L^∞ or even nonlocal operators. Moreover, the Lipschitz boundary of the domain Ω can be split into several parts admitting Dirichlet, Neumann and/or Robin-boundary conditions of displacement, velocity and stress. We show well-posedness of this classical model of the wave equation utilizing boundary triple theory. In addition we show semi-uniform stability of solutions by means of a spectral result.

40. **Sandeep Kumar Soni**

Title: *Boundary systems for Friedrichs operators*

Abstract: The concept of boundary triples is well-known for dual-pairs of operators, symmetric and skew-symmetric operators. In addition to providing an extension theory, these allow us to explore the spectral properties of the operators.

In this talk, we shall see a suitable version of boundary triples for abstract Friedrichs operators (and naming as such). As a first step, we shall see an extension theory in terms of boundary triples. Some examples will be presented.

41. **Eduard Stefanescu**

Title: *Random Schrödinger operators on compact manifolds*

Abstract: We investigate the eigenvalues of a random Schrödinger operator of Anderson type, given by $-\Delta_g + V_\omega$, on compact manifolds with complex-valued potential. We show that, with high probability, the eigenvalues can be bounded in terms of the L^q-norm of the potential for all $q \geq 1$. Through the use of randomization techniques, we obtain improved bounds that surpass the sharp deterministic estimates established by Sogge. This advancement draws on ideas introduced by Bourgain in the context of almost-sure scattering for lattice Schrödinger operators. This is joint work with Jean-Claude Cuenin and Konstantin Merz.

42. **Grzegorz Swiderski**

Title: *On spectral analysis of some Sturm-Liouville operators*

Abstract: We introduce a new class of Sturm-Liouville operators with periodically modulated parameters. Their spectral properties depend on the monodromy matrix of the underlying periodic problem computed for the spectral parameter equal to 0. Under some assumptions, by studying the asymptotics of Christoffel functions and density of states, we are able to prove that the spectral density is a continuous function positive everywhere on the real line. This is a joint work with Bartosz Trojan (Wrocław University of Science and Technology).

43. **Jari Taskinen**

Title: *Spectral Laplace problem and heat equation in periodic domains*

Abstract: We treat the classical linear heat equation in unbounded, periodic waveguides $\Pi \subset \mathbb{R}^d$ by using Floquet (or Bloch) transform methods. As well known, the behavior of the solution of the heat equation is determined by the underlying spectral Laplace problem. The Floquet transform F turns the elliptic problem on the unbounded domain Π into a corresponding elliptic model problem on the bounded periodic cell ϖ of Π. The Floquet transform can also be directly applied to the original heat equation, which yields a heat equation with mixed boundary conditions on the periodic cell ϖ. We analyse the connection between the two approaches to the problems.

In the case Π is periodic in d directions, we observe that for a general, integrable initial data, the solution decays at the same rate $t^{-d/2}$ as in the case of the Cauchy problem in the entire Euclidean space. Initial data with vanishing x-integral leads to faster decay rates. In this respect, we also analyze initial data with more detailed conditions related to the lowest eigenvalue and eigenfunction of the Floquet model problem.

[RT1] M. Rosenberg, J. Taskinen, Some aspects of the Floquet theory for the heat equation in a periodic domain, J. Evolution Equations 24 (2024), 23.

[RT2] M. Rosenberg, J. Taskinen, Heat equation in a periodic domain with special initial data, submitted.

44. **Carsten Trunk**

Title: *From student to professor*

Abstract: I will report on some observations about Jussi with the tendency to leave out positive ones. In the same style we also discuss his mathematical oevre. This abstract also serves for the research about how many attendees read abstracts.

45. **Matěj Tušek**

Title: *Non-local delta-shell potentials*

Abstract: New self-adjoint realizations of Dirac and Schrödinger operators, that may be associated with non-local distributional potentials supported on hypersurfaces, will be presented. They are described by transmission conditions along the hypersurfaces. For proving their self-adjointness and studying their spectral properties we will use suitable generalized boundary triples. Whereas for the Dirac operators it was convenient to work with the triple introduced in [1], for the Schrödinger operators we constructed a new triple, whose boundary mappings contain Wirtinger derivatives, in [2].

[1] J. Behrndt, M. Holzmann, C. Stelzer-Landauer, G. Stenzel: Boundary triples and Weyl functions for Dirac operators with singular interaction. Rev. Math. Phys. 36, 2350036, 2024.

[2] L. Heriban, M. Holzmann, C. Stelzer-Landauer, G. Stenzel, M. Tušek: Two-dimensional Schrödinger operators with non-local singular potentials. J. Math. Anal. Appl. 549 (2), 2025.

46. **Oktay Veliev**

Title: *On Differential Operators with Periodic Coefficients*

Abstract: We study the spectral analysis of non-self-adjoint differential operators generated in the space of square-integrable functions on the entire real line by ordinary differential expressions of arbitrary order with periodic, complex-valued coefficients. We discuss methods for constructing the spectral expansion and describe the structure of the spectrum of such operators. Special attention is given to the case when the coefficients are PT-symmetric functions, and the corresponding spectral properties are analyzed in detail.

47. **Nicolas Weber**

Title: *Eigenvalues of operator functions: A commutativity result with an application to 1D Dirac operators*

Abstract: We introduce the notion of eigenvalues of finite type for analytic operator functions $W(\cdot) : \Omega \to \mathcal{L}(\mathcal{H}, \mathcal{G})$, where $\mathcal{H}$ and $\mathcal{G}$ are Hilbert spaces and $\Omega \subset \mathbb{C}$ is open and connected. Given two analytic operator functions $A(\cdot) : \Omega \to \mathcal{L}(\mathcal{H}, \mathcal{G})$ and $B(\cdot) : \Omega \to \mathcal{L}(\mathcal{G}, \mathcal{H})$, we then show that the operator families

$$I - A(\cdot)B(\cdot) : \Omega \to \mathcal{L}(\mathcal{G})$$
$$I - B(\cdot)A(\cdot) : \Omega \to \mathcal{L}(\mathcal{H})$$

have the same eigenvalues of finite type, with coinciding algebraic multiplicities. Finally, the usefulness of the above commutativity result is demonstrated, by applying it to the Birman-Schwinger family of a weakly coupled one-dimensional Dirac operator H_ε, that is given for $\varepsilon > 0$ and $m \geq 0$ in $L^2(\mathbb{R}; \mathbb{C}^2)$ by

$$H_\varepsilon = -i\partial_x \sigma_1 + m\sigma_3 + \varepsilon V, \quad \operatorname{dom} H_\varepsilon = H^1(\mathbb{R}, \mathbb{C}^2),$$

where $V : \mathbb{R} \to \mathbb{C}^{2\times 2}$ is a sufficiently regular and decaying potential, to show that the bound states of H_ε in the weak coupling limit, that were obtained in [1], must be simple. This talk is based on joint work with J. Behrndt and P. Siegl.

[1] J.-C. Cuenin, P. Siegl, *Eigenvalues of one-dimensional non-self-adjoint Dirac operators and applications*, Lett. Math. Phys., **108** (2018), 1757-1778.

48. **Timo Weidl**

Title: *Sharp Lieb-Thirring inequalities for the shifted Coulomb Hamiltonian*

Abstract: We prove sharp Lieb-Thirring (LT inequalities for the family of shifted Coulomb Hamiltonians. More precisely, we prove the classical LT inequalities with the semi- classical constant for this family of operators in any dimension $d \geq 3$ and any $\gamma \geq 1$. We also prove that the semi-classical constant is never optimal for the Cwikel-Lieb-Rozenblum (CLR) inequalities for this family of operators in any dimension. In this case, we characterize the optimal constant as the minimum of a finite set and provide an asymptotic expansion as the dimension grows. Using the same method to prove the CLR inequalities for Coulomb, we obtain more information about the conjectured optimal constant in the CLR inequality for arbitrary potentials.

Joint work with Thiago Carvalho Corso and Zhuoyao Zeng.

Posters

1. **Marcus Dykes**

Title: *Vibration spectrum of an elastic string resting on a Winkler foundation*

Abstract: The eigen-spectrum of the Sturm-Liouville boundary value problem for an elastic string resting on a Winkler foundation is studied. The most general boundary conditions that showcase the behaviour of both positive and negative stiffness springs are considered. Similar setups arise in modern meta-materials, where negative stiffness mechanisms exhibit novel wave phenomena. It is shown that within specific intervals of physical parameters, the analysed boundary conditions support notable spectral features, e.g. non-oscillatory eigenfunctions, including trapped modes, localized near the string's ends. Numerical examples demonstrating peculiarities of the problem are presented.