25th LL-Seminar on Graph Theory

Montanuniversität Leoben

September 23–24, 2011

Program

All talks take place at Hörsaal für Allgemeinen Maschinenbau, main building, 1-st floor, above Institute of Mathematics

Friday, September 23, 2011

9:30	Opening and welcome addresses

9:35	Jernej Azarija (Universiy of Ljubljana)	*Euler's idoneal numbers, prime partitions and two problems related to the number of spanning trees*
10:00	Konstantin Klemm (University of Leipzig)	*Spanning tree encoding for the max-cut optimization problem*
10:00	Elmar Teufl (University of Tübingen)	*Counting spanning trees using electrical networks*

10:50 – 11:10	Coffee break

11:10	Vida Vukašinovic (Jožef Stefan Institute)	*On the mutually independent Hamiltonian cycles in faulty hypercubes*
11:45	Janez Žerovnik (University of LJubljana)	*On connectedness and hamiltonicity of direct graph bundles*

12:10 – 14:00	Lunch break

14:00	Clemens Brand (MU Leoben)	*Recognizing triangulated Cartesian products*
14:25	Wilfried Imrich (MU Leoben)	*The Cartesian skeleton of finite and infinite graphs*
14:50	Josef Leydold (WU Vienna)	*Convex cycle bases and Cartesian products*

15:15 – 15:35	Coffee break

Friday (continued)

15:35	Julia Wessely (University of Vienna)	*Consistent cycles in infinite graphs*
16:00	Bojan Mohar (University of Ljubljana)	*5-choosability of graphs with crossings*
16:25	Jing Qin (University of Leipzig)	*Cycle decomposition of small RNA configuration space*
16:50	Marc Hellmuth, Peter Stadler (Saarland University, University of Leipzig)	*Symbolic ultrametrics, ordered triples, orthology and co-graphs*

19:00	Seminar dinner, Hotel Kindler, 1-st floor

Saturday, September 24, 2011

9:00	Christiaan van de Woestijne (MU Leoben)	*Factorisation of graphs and polynomials with nonnegative integer coefficients*
9:25	Boštjan Kuzman (University of Ljubljana)	*Covers of doubled cycles and linear codes*
9:50	Aleksandar Jurišić (University of Ljubljana)	1-*codes in primitive distance-regular graphs of diameter* 3

10:15 – 10:35	Coffee break

10:35	Drago Bokal (University of Maribor)	*Crossing number is additive over edge cuts of size three*
11:00	Riste Škrekovski (University of Ljubljana)	*Some results on fullerene graphs*
11:25	Bernhard Krön (University of Vienna)	*Tutte's decomposition of graphs for arbitrary connectivity*

11:50 – 13:30	Lunch break

Saturday (continued)

13:30	Primož Potočnik (University of Ljubljana)	*How large can a group of automorphisms of a symmetric graph be?*
13:55	Aleksander Malnič (University of Ljubljana)	*On the split structure of lifted groups*
14:20	Jörg Lehnert (University of Frankfurt)	*Quasi-automorphisms of graphs*
14:45	Robert Gallant (University of Ljubljana	*Packings in grid graphs*

Abstracts

Abstracts are listed alphabetically with respect to Presenting Author.

Euler's idoneal numbers, prime partitions and two problems related to the number of spanning trees

Jernej Azarija
University of Ljubljana

In this talk we improve some bounds for two problems introduced by J. Sedláček. Let A_n denote the set of integers such that $x \in A_n$ whenever there exists a simple, connected graph on n vertices having precisely x spanning trees. One could think about $|A_n|$ as the maximal number of connected graphs on n vertices with mutually different numbers of spanning trees. Using a simple construction, Sedláček showed that $\lim_{n\to\infty} \frac{|A_n|}{n} = \infty$ and remarked that it is not clear how the fraction $\frac{|A_n|}{n^2}$ behaves as n tends to infinity. In this talk we make use of a number theoretical result in order to show that $|A_n|$ grows faster than $e^{\frac{2\pi}{3}\sqrt{n/\log n}}$, that is $|A_n| = \omega(e^{\frac{2\pi}{3}\sqrt{n/\log n}})$.

For the second problem, let $n \geq 3$ be an integer and let $\alpha(n)$ denote the least integer k such that there exists a graph on k vertices having precisely n spanning trees. Using a result related to the solvability of a specific *Diophantine equation* we show that:

$$\alpha(n) = \begin{cases} \frac{n+4}{3} & \text{if } n \equiv 2 \pmod 3, \\ \frac{n+9}{4} & \text{otherwise.} \end{cases}$$

if and only if $n \notin \{3, 4, 5, 6, 7, 9, 10, 13, 18, 22, 25\}$. This improves the currently known lower bound.

We finish our talk by presenting two open problems.

Crossing number is additive over edge cuts of size three

D. Bokal, M. Chimani, J. leanos
University of Maribor

We prove that if G is a graph with a minimal edge cut F of size three and G_1, G_2 are the two (augmented) components of $G - F$, then the crossing number of G is equal to the sum of crossing numbers of G_1 and G_2. Combining with known results, this implies that crossing number is additive over edge-cuts of size d for $d \in \{0, 1, 2, 3\}$, whereas there are counterexamples for every $d \geq 4$. The techniques generalize to show that minor crossing number is additive over edge cuts of arbitrary size, as well as to provide bounds for crossing number additivity in arbitrary surfaces.

Recognizing triangulated Cartesian products

Clemens Brand
Montanuniversität Leoben

Computational meshes for numerical simulation frequently show—at least locally—a structure resembling a triangulated 2D-grid, or a 3D-grid divided into tetrahedra. This motivates us to analyze the structural properties of *triangulated Cartesian products*: They are a special type of approximate graph product, somehow "in between" the Cartesian and the strong product. We show how to recognice and factorize graphs that are triangulated products, when the factors themselves are triangle-free graphs.

Packings in grid graphs

Robert Gallant
University of Ljubljana

A subset S of vertices in a graph such that no graph vertex is adjacent (or equal) to more than two such vertices in S is called a 2-limited packing in the graph. For arbitrary graphs the problem of finding a maximum 2-limited packing is NP-complete. We consider the problem of finding maximum 2-limited packings in grid graphs and show that in some cases the problem can be solved efficiently.

Symbolic ultrametrics, ordered triples, orthology, and co-graphs

Marc Hellmuth, Maribel Hernandez-Rosales, Katharina Huber, Vincent Moulton, Sonja Prohaska, Peter Stadler, Nicolas Wieseke
Saarland University, Leipzig University

The detailed reconstruction of gene phylogenies, and the determination whether two genes with a common ancestor are orthologs (derived by a speciation event) or paralogs (derived by a gene-duplication event), are two related tasks of fundamental importance in computational biology. Although much empirical work has been published in these areas, the problems, and in particular their connections, are not very well understood from a theoretical point of view. We show that the theory of symbolic ultrametrics developed by Böcker and Dress fills the gap. It helps to establish, for example, that orthology relations are co-graphs, and points a central role of ordered triples as a convient construct applicable both in proofs and in empirical calculations.

The Cartesian skeleton of finite and infinite graphs

Wilfried Imrich
Montanuniversität Leoben

The Cartesian skeleton is a subgraph of the square of a graph and plays an important role in the factorization of finite and infinite graphs with respect to the direct and the strong product.

In the finite case it helps in the construction of fast algorithms for the factorization, in the infinite case it helps to find new classes of graphs with unique factorization. In both cases it is essential in the description of the automorphism group of the product.

The talk outlines the main results and the ideas that are involved.

1-codes in primitive distance-regular graphs of diameter 3

Aleksandar Jurišić, Janoš Vidali
University of Ljubljana

We study distance-regular graphs with intersection arrays

$$\{(2r^2-1)(2r+1), 4r(r^2-1), 2r^2; 1, 2(r^2-1), r(4r^2-2)\}$$

and

$$\{2r^2(2r+1), (2r-1)(2r^2+r+1), 2r^2; 1, 2r^2, r(4r^2-1)\}$$

for $r > 1$. We use vanishing of certain Krein parameters to calculate some triple intersection numbers and show that they contain a maximal 1-code. Finally, we prove that no such graph exists. In particular, there is no distance-regular graph with intersection arrays $\{35, 24, 8; 1, 6, 28\}$, $\{119, 96, 18; 1, 16, 102\}$ or $\{40, 33, 8; 1, 8, 30\}$. The first one was the first member of the infinite family $\{7(m-1), 6(m-2), 4(m-4); 1, 6, 28\}$, $m \geq 6$ which is realized when m is a power of two by a bilinear forms graph.

Spanning tree encoding for the max-cut optimization problem

Konstantin Klemm
Leipzig University

Given a simple connected graph $G = (V, E)$, the max-cut optimization problem asks to find a cut of G with a maximum number of edges crossing. The cost function f maps a partition $\{X, Y\}$ of V to the the sum of the sizes (numbers of edges) of the subgraphs induced by X and Y. Here we approach the problem by encoding cuts with spanning trees. With an arbitrary fixed node x, a spanning tree T of G generates a cut $h(T) = \{Y(T), V \setminus Y(T)\}$ by taking $Y(T)$ as the set of nodes with even distance from x on T. For the landscape with the cost function $f \circ h$ and a suitably chosen move set for spanning trees, we find that all strict minima are global. Any adaptive walk eventually reaches a global minimum.

Tutte's decomposition of graphs for arbitrary connectivity

M. J. Dunwoody, Bernhard Krön
University of Vienna

As an application of an axiomatic theory of vertex cut systems we obtain a canonic tree decomposition of k-connected graphs. This answers a long-standing question of Tutte who did the same for 2- connected graphs. We also discuss in how far one can extend this to separators whose cardinality is larger than the connectivity of the graph, which is relevant for further applications.

Covers of doubled cycles and linear codes

Boštjan Kuzman, A. Malnič, P. Potočnik
University of Ljubljana

We apply the construction of minimal half-arc transitive and arc-transitive elementary abelian covers of doubled cycles in order to extend a result on symmetric tetravalent graphs by Gardiner and Praeger (1994). An interesting correspondence with a special class of linear codes appears along the way.

Quasi-automorphisms of graphs

Jörg Lehnert
University of Frankfurt

In the theory of decision problems in group theory groups of quasi-automorphisms of graphs turned out to be of some interest. Studying this groups one realizes, that some prominent examples of groups turn out to be quotients of groups of quasi-automorphisms of 'nice' graphs. We will focus on the question, which groups can be realized as such quotients.

Convex cycle bases and Cartesian products

Marc Hellmuth,Josef Leydold, Peter Stadler
WU Vienna

The set of all Eulerian subgraphs of some undirected graph G together with the geometric difference of edges forms a vector space over GF(2). Its bases have been intensively studied and various kinds like minimal length, fundamental or robust cycle bases have been described and investigated. In this talk we consider the convex cycle bases that entirely consist of (geodetically) convex elementary cycles, i.e., that contain all shortest path between any two vertices of these cycles. In particular we investigate convex cycle bases in isometric subgraphs of Cartesian products.

On the split structure of lifted groups

Aleksander Malnič, Rok Požar
University of Ljubljana

Some results about the split structure of lifted groups along regular covers of connected graphs will be presented.

5-choosability of graphs with crossings

Z. Dvorak, B. Lidicky, Bojan Mohar
University of Ljubljana

We give a new proof of the fact that every planar graph is 5-choosable, and use it to show that every graph drawn in the plane so that the distance between every pair of crossings is at least 19 is 5-choosable. At the same time we may allow some vertices to have lists of size four only, as long as they are far apart and far from the crossings.

How large can a group of automorphisms of a symmetric graph be?

Primož Potočnik
University of Ljubljana

An old conjecture of Richard Weiss states that for any valency $d \geq 2$ there exists a constant m such that whenever X is a d-valent graph and G an arc-transitive group of automorphisms of X such that the vertex-stabiliser G_v acts primitively on the neighbourhood of v, then the order of G_v is at most m. We will present the current status of the conjecture and present some possible generalizations and partial solutions.

Cycle decomposition of small RNA configuration space

Jing Qin
University of Leipzig

It is a well known fact that the behavior of simulated annealing algorithms is tightly related to the hierarchical decomposition of their configuration spaces in cycles. We here apply the iterative routine invented by Wentzell and Freidlin to construct the cycle decomposition of small RNA configuration spaces, for instance, hairpins. We furthermore explore the relationships of cycles and the barrier tree of the energy landscape.

Some results on fullerene graphs

Riste Škrekovski
University of Ljubljana

Fullerene graphs are 3-connected planar cubic graphs with all faces of size 5 or 6. Motivation to study this class of graphs came from chemistry. In my talk I will present few results on this topic.

Counting spanning trees using electrical networks

Elmar Teufl, Stephan Wagner
Tübingen University

Counting the number of spanning trees in graphs is an old combinatorial problem. Starting with the classical results of Cayley on the number of spanning trees in complete graphs many tools for this problem and many results on various graph classes have been obtained. One of these tools is the Matrix-Tree-Theorem, which is essentually contained in the work of Kirchhoff. In this talk another general approach is presented, which uses the connections indicated by Kirchhoff between spanning trees and electrical networks. An old determinant identity of Sylvester can be used for the proof of the presented method. As a by-product some statements on electrical networks and Laplace matrices are deduced.

On the mutually independent Hamiltonian cycles in faulty hypercubes

P. Gregor, R. Škrekovski, Vida Vukašinovic, Stephan Wagner
Jošef Stefan Institute, Ljubljana

Two ordered Hamiltonian paths in the n-dimensional hypercube Q_n are said to be independent if i-th vertices of the paths are distinct for every $1 \leq i \leq 2^n$. Similarly, two s-starting Hamiltonian cycles are independent if i-th vertices of the cycle are distinct for every $2 \leq i \leq 2^n$. A set S of Hamiltonian paths and s-starting Hamiltonian cycles are mutually independent if every two paths or cycles, respectively, from S are independent. We show that for every set F of f edges and $n-f$ pairs of adjacent vertices w_i and b_i, there are $n-f$ mutually independent Hamiltonian paths with endvertices w_i, b_i and avoiding edges of F in Q_n. We also show that Q_n contains $n-f$ fault-free mutually independent s-starting Hamiltonian cycles, for every set of $f \leq n-2$ faulty edges in Q_n and every vertex s. This improves previously known results on the numbers of mutually independent Hamiltonian paths and cycles in the hypercube with faulty edges.

Consistent cycles in infinite graphs

Julia Wessely
University of Vienna

Let Γ be a graph (finite or infinite) and let G be a subgroup of the automorphism group $\mathrm{Aut}(\Gamma)$ that is closed in $\mathrm{Aut}(\Gamma)$ and acts arc-transitively on Γ. A cycle C is called G-consistent if there exists an automorphism in G that rotates the cycle by one step. Consistent cycles in finite graphs were introduced by J. H. Conway. He observed that the number of orbits of G-consistent cycles of an arc-transitive group is precisely one less than the valency of the graph. We generalize this result of Conway to infinite graphs.

Factorisation of graphs and polynomials with nonnegative integer coefficients

Christiaan van de Woestijne
Montanuniversität Leoben

We show that factorisation of finite disconnected graphs, for any of the Cartesian, direct, or strong products, may be modelled almost isomorphically by factorisation of polynomials with nonnegative integer coefficients. We then give an algorithm to determine all non-unique factorisations of univariate such polynomials which take a given value at 1 (as opposed to having given degree) and give examples of the results for small values.

On connectedness and hamiltonicity of direct graph bundles

Irena Hrastnik, Janez Žerovnik
University of Ljubljana

A necessary and sufficient condition for connectedness of direct graph bundles where the fibers are cycles is given. We show that all connected direct graph bundles $X = C_s \times^{\alpha} C_t$ are Hamiltonian. A sufficient condition for hamiltonicity of direct graph bundles follows.

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