Doctoral Program (DK-plus) C75-N13

Discrete Mathematics

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Univ.-Prof. Dipl.-Ing. Dr. W. Woess
(Speaker of the DK-plus)

Antrag zur ersten Förderperiode

Contents

1. General information about the DK-plus

1.1. Overall scientific concept with short-term and long-term goals.

Discrete mathematics is an important part of modern mathematics that comprises the complete spectrum from pure to applied mathematics. Our program covers many mathematical disciplines including classical fields such as combinatorics, number theory, probability theory and group theory as well as important applications, for instance analysis of discrete geometric data, optimisation and certain aspects of cryptography. For this reason we are able to provide a broad variety of lectures, seminars and research topics for the post graduate education of our doctoral students.

Following a current trend, we can formulate our *mission and vision* as

Advanced mathematical education through advanced mathematical research
and
advanced mathematical research through advanced mathematical education.

In the mathematical landscape of Styria, multifaceted research in discrete mathematics is certainly one of the significant features that unites the mathematical competences present at Graz University (Karl-Franzens-Universität Graz, KFU), Graz University of Technology (TU Graz, TUG) and University of Leoben (Montanuniversität Leoben, MUL), with multiple and well-established interactions between these three institutions. These intensive activities are documented by numerous joint research projects (mostly funded by the Austrian Science Fund FWF), for example the national research network S96 "Analytic combinatorics and probabilistic number theory", the former special research area (SFB) "Optimisation and control", the former START project of Peter Grabner and by the research project P19115 "Random walks, random configurations, and horocyclic products" of Wolfgang Woess which is also linked to the EU training network "GROUPS: European Training Courses and conferences in Group Theory" and two further postdoc projects (funded by FWF and by Deutsche Forschungsgemeinschaft) in Woess' research group. On the algebraic-combinatorial side, we have Alfred Geroldinger's FWF project P18779 "factorisations of algebraic integers"

The core of the research themes of the proposed doctoral program is mainly focussed on pure mathematics with an outreach towards applied mathematics, whose strong presence is particularly visible in the proposed Projects 5, 7 and 10, see below. Furthermore we mention here ongoing FWF research projects with specific applications such as the project P18918 "Efficiently solvable variants of location problems" of Bettina Klinz and our senior associated scientist Rainer Burkard, and Johannes Wallner's project "Computational Differential Geometry", which is part of the FWF research network S92 on industrial geometry. All together, this forms an excellent environment for a high level doctoral program in the framework of the FWF funding system.

We see the main advantage of such a program in the synergy effects building upon the critical mass of ten research groups tied together by common scientific and educational goals as well as an appropriate organisational structure. Breaking up the classical Austrian structure of PhD education, each PhD student profits from the entire breadth of the proposed program and the complete pool of scientific expertise in Discrete Mathematics that is present in the three participating universities. This is granted by continuous interaction with the whole faculty and supported by our mentoring system.

Our long range goal for the 12 years of the DK is to create an internationally visible *School of Discrete Mathematics* whose graduates are recognized for that provenience and maintain a corporate identity on the basis of their common experience in that school. Our short range goal for the first four years is to quickly establish the necessary bases for attracting excellent PhD students from all over the world with whom we can create that school. Of course, this internationality requires that the official language of the DK, in every respect, is English.

Taking care of their status as Early Stage Researchers, the doctoral students shall be educated on a high scientific and ethic standard benefitting from the inner mathematical multidisciplinarity and from the different personalities that form the faculty of the DK. The permanent scientific exchange between the DK-students ("collegiates") and the faculty members is an essential ingredient. This is guaranteed by regular seminars and frequently organised workshops, where we can build upon year-long individual experiences of faculty members. These workshops will be held including international speakers and can be linked to other current projects mentioned above.

Already in the past, the faculty members acted as joint advisors of PhD theses. The approach to joint supervisions shall be intensified via a mentoring system that is described in more detail below and by involving foreign visitors, with a substantial role, in our educational program. This is going to constitute an enormous added value.

The cornerstone of the permanent interplay of the different groups within this educational program is the joint recruitment of the DK students. This interplay is continued by running joint internal seminars where scientific problems are discussed and ethic, pedagogical and personal skills are trained.

A further important aspect of the proposed doctoral program is its long term influence on the importance of Discrete Mathematics in Austria in general and at the participating universities in particular. Specifically, it will also influence the hiring policy for positions in mathematics.

We are aware of the fact that women are under-represented in mathematical research. Efforts in order to overcome this imbalance have begun well before the elaboration of this program. This can be seen from the several former and present female PhD students supervised by faculty members. While they are certainly encouraged to contribute to this program, they are still in a too early stage of their career to act as faculty members. It is one of the long-range goals that they as well as other female researchers will be able to join the faculty in the second,

resp. third funding period of the present program. On the level of the DK students, it is our goal to attract further female candidates in the recruitment phase. This is enhanced by the additional associated DK student position reserved for female applicants which is going to be funded by TU Graz.

We also point out that the speaker of this program is (since 2005) the head of the curriculum committee for doctoral studies of Graz University of Technology and thus very much involved in conceptual as well as organisational matters concerning PhD studies. Also, our faculty member Peter Grabner as the vice-dean (resp. dean starting with autumn, 2009) for mathematical studies at TUG is responsible for administrative-organisational questions regarding PhD studies in mathematics at TU Graz. This guarantees an optimal cooperation between this doctoral program and the local university structure at its seat, which is Graz University of Technology.

1.2. **List of participating researchers.**

At its current status, the DK has 10 faculty members, each of whom contributes to the supervision of the 10 DK students funded by this program, as well as the projected 11 associated DK students. Furthermore, we have an associated scientist (Franz Lehner) and a senior associated scientist (Rainer Burkard, who – in view of his imminent retirement – will contribute to the DK in this role). Compare with the order of the DK in 2.5. The associated scientists support the research activities with a formal status of external collaborators.

Table 1a: List of Faculty members participating in the DK-plus

Project leader	Sex	Research Institution	Coordinates of the research institution
Woess, Wolfgang, Univ.-Prof. Dr.	M	Institut für Mathematische Strukturtheorie Technische Universität Graz	Steyrergasse 30 A-8010 Graz
Berkes, István, Univ.-Prof. Dr.	M	Institut für Statistik Technische Universität Graz	Münzgrabenstraße 11 A-8010 Graz
Geroldinger, Alfred, Ao.Univ.-Prof. Dr.	M	Institut für Mathematik und Wissenschaftliches Rechnen Universität Graz	Heinrichstrasse 36 A-8010 Graz
Grabner, Peter, Univ.-Prof. Dr.	M	Institut für Analysis und Computational Number Theory Technische Universität Graz	Steyrergasse 30 A-8010 Graz
Heuberger, Clemens, Ao.Univ.-Prof. Dr.	M	Institut für Optimierung und Diskrete Mathematik Technische Universität Graz	Steyrergasse 30 A-8010 Graz
Kirschenhofer, Peter, O.Univ.-Prof. Dr.	M	Institut für Mathematik und Informationstechnologie Montanuniversität Leoben	Franz-Josef-Strasse 18 A-8700 Leoben

Table 1a (continued)

Project leader	Sex	Research Institution	Coordinates of the research institution
Klinz, Bettina, Ao.Univ.-Prof. Dr.	F	Institut für Optimierung und Diskrete Mathematik Technische Universität Graz	Steyrergasse 30 A-8010 Graz
Thuswaldner, Jörg, Ao.Univ.-Prof. Dr.	M	Institut für Mathematik und Informationstechnologie Montanuniversität Leoben	Franz-Josef-Strasse 18 A-8700 Leoben
Tichy, Robert, O.Univ.-Prof. Dr.	M	Institut für Analysis und Computational Number Theory Technische Universität Graz	Steyrergasse 30 A-8010 Graz
Wallner, Johannes, Univ.-Prof. Dr.	M	Institut für Geometrie Technische Universität Graz	Kopernikusgasse 24 A-8010 Graz

Table 1b: List of associated and senior associated scientists

Associated scientist	Sex	Research Institution	Coordinates of the research institution
Burkard, Rainer, O.Univ.-Prof. Dr. (Project 07: Klinz)	M	Institut für Optimierung und Diskrete Mathematik Technische Universität Graz	Steyrergasse 30 A-8010 Graz
Lehner, Franz, Univ.-Doz. Dr. (Project 01: Woess)	M	Institut für Mathematische Strukturtheorie Technische Universität Graz	Steyrergasse 30 A-8010 Graz

1.3. **Research areas, interdependencies and synergies.**

We understand Discrete Mathematics as a broad and modern discipline that is not closed into itself but interacts with, is open to and benefits from almost all mathematical areas as well as many other branches of science. The specific themes that will be pursued in the DK are of course influenced by the competences of the faculty members. They cover a very wide range of research directions that are well embedded in the scientific landscape and bear a considerable potential for future research. This guarantees a wide variety of interesting topics for PhD theses which are part of research directions that are pursued by many research groups all over the world. As the track records of many of our past PhD students show, they have been well prepared to continue their scientific career abroad. We are confident that the synergy effects of the tight collaboration within the DK will equip our students with still better scientific as well as personal abilities, and that after receiving their PhD they will be welcome as applicants for positions at distinguished research institutions all over the world.

The faculty members have produced many joint publications and have been involved in various common research projects. We also have shared teaching experiences in many courses and

seminars within and across the three universities. This provides the fruitful base for the choice of the themes that are the core of our educational concept.
Our fields of expertise can be subsumed in the following items which also form the core of the proposed doctoral program consisting of 10 projects.

1.3.1. **Combinatorial, additive and algorithmic number theory.** Several of the projects have a number theoretic flavour. In particular, various projects deal with specific problems from algebraic (Project 03), additive (Projects 03 and 09) and analytic (Projects 02, 04, 08, 09) number theory. The main focus of this item is devoted to additive, algorithmic and combinatorial aspects. Various problems are attacked by tools from Diophantine approximation (such as Wolfgang Schmidt's subspace theorem [7] or Alan Baker's method on linear forms in logarithms). In particular, Projects 04, 05, 06, 08 and 09 focus on number theoretic algorithms.

1.3.2. **Discrete dynamics and fractals.** Starting with Mandelbrot's [23] fundamental investigations, fractals became an important and extremely useful tool for the understanding of discrete dynamical systems. In particular, fractal structures appear in several projects as fundamental domains of digital expansions and dynamical systems and as tilings originating in self-affine structures (Projects 04, 05 and 08). Other aspects of dynamical systems (e.g. random number generators, percolation and ergodicity) can be found in Projects 01 and 02. In particular, we refer to Project 06 and to number theoretic aspects in connection with the Skolem-Mahler-Lech theorem on the zero sets of linear recursive sequences, see for instance Everest et al. [6] and Lagarias [20].

1.3.3. **Algebraic structures.** Important developments in mathematics were achieved by combining algebra and number theory with deep methods from geometry and dynamics (a paradigm for this development is Margulis' fundamental result [5, 24] concerning the Oppenheim conjecture on quadratic forms). This interplay between analytic, dynamical and stochastic methods with ideas from algebra and number theory is important for several subprojects of the present doctoral program. In this range, problems concerning non-unique factorisations (Project 03; here we mention the standard monographs [11, 16]) and additive problems in algebraic number fields (Projects 03 and 09) are investigated. Questions from combinatorial, geometric and topological group theory are further themes associated with this topic (Projects 01 and 10), see also the next item.

1.3.4. **Graph theory.** The geometric structure of fractals can often be described by properties of certain graphs (Projects 05 and 08). Moreover, post critically finite fractals can be approximated by graphs in order to study their topological and analytic properties (Project 04). For tight relations between number theory and graph theory we refer to the monograph by Peter Sarnak [28] and for connections with the theory of point distributions to Lubotzky et al. [22]. A

delightful presentation of the interplay between graph theory, number theory, harmonic analysis, spectral theory and other fields is given by the award-winning book of Lubotzky [21]. Algebraic and algorithmic aspects of graphs are considered in Project 07. Infinite vertex transitive graphs, their growth, isoperimetry, compactifications and other algebraic-geometric features play an important role not only in pure graph theory, but also in combinatorial and geometric group theory and in random walks (Project 01). In this context, we mention the recent solution (plus striking new ideas) by Eskin et al. [4] of a problem of Woess on quasi-isometry of homogeneous graphs.

1.3.5. **Methods from ergodic theory, harmonic analysis and spectral theory.** In modern mathematics frequently discrete problems could be solved by applying analytic tools. In particular, Furstenberg [9] gave deep insight into van der Waerden's problem concerning arithmetic progressions by using methods from ergodic theory. This approach was refined by Szemerédi (see [10] for a proof of Szemerédi's theorem using ergodic theory) and recently highlighted by a result of Green and Tao [13] who showed that there exist arbitrarily long arithmetic progressions in the set of prime numbers. For recent progress in the interplay of harmonic analysis and ergodic theory and applications to analytic number theory we refer to the fundamental results of Bourgain (see his survey [1]) as well as to Mauduit and Rivat [25] who obtained distribution results for digital sums over primes. In the context of our DK, harmonic analysis and spectral theory are also crucial in relation with random walks, geometric group theory and graph theory, where spectral properties are used as structural invariants. This goes back to classical work by Pólya [26] and Kesten [18]; for harmonic analysis on free groups, see Figá-Talamanca and Steger [8]. Ergodic theory, harmonic analysis and spectral theory play an important role in Projects 01, 02, 04, 08, 09.

1.3.6. **Discrete stochastics.** In the recent decades stochastic processes on graphs and fractals have been studied extensively. Regarding fractals, the more analytic approach via Dirichlet forms and the heat kernel is well documented in the focussed book of Kigami [19]. Graz is one of the centers of an approach to random walks which studies the interplay between algebraic, geometric and combinatorial structure theory on one hand and the probabilistic, spectral and potential theoretic properties of those stochastic processes on the other hand; see Woess' standard monograph [29]. These aspects are covered in Project 03, and Project 4 has partial links with these topics. Stochastic methods like limit theorems and laws of the iterated logarithms are part of Project 02.

1.3.7. **Analysis of algorithms, cryptography.** In Projects 02 and 09 point distributions are studied. It is well-known that distribution properties of point distributions can be used for error bounds in multivariate numerical integration which also has application in mathematical finance (see for instance Drmota and Tichy [3]). In particular, the discrepancy and related distribution measures are studied by analytic, probabilistic and Diophantine methods.

These methods can also be used for the investigation of special problems in cryptography (Project 05). Combinatorial optimisation algorithms are analysed in Project 07.

1.3.8. **Combinatorial optimisation.** In Project 07 special cases of hard combinatorial optimisation problems are treated. Such special cases typically result from putting further restrictions on the cost structure or on the underlying combinatorial structure of the problem. As the cost structure is often described by matrices, the investigation of combinatorial optimisation problems restricted to specially structured matrix classes plays an important role. Restrictions on the combinatorial structure often leads to the investigation of combinatorial optimisation problems restricted to certain graph classes which also provides a link to algorithmic graph theory. Costs for digit representations that are relevant in cryptography are studied in Project 05. Specific optimisation problems are also important in discrete differential geometry (see Project 10).

1.3.9. **Discrete applied geometry.** As outlined above many items are influenced by geometric ideas or are concerned with geometric objects such as fractals (Project 08), point distributions (Project 09), subdivision processes and transformation groups (Project 10). At the core of this item stands Project 10 which is devoted to smoothness analysis and discrete multi-scale analysis of data.

1.3.10. **Methodological interplay between the research projects.** The above brief description shows the large variety of strongly interacting research directions of our school. This wide range of research areas can only be covered by a suitably large critical mass of experienced researchers. The connection graph in Figure 1 of our faculty shows the intensive relations between the 10 projects. The relations between the various projects topics of the the respective topics are documented by the labelled edges of this graph.
In addition to the connection graph, we now outline several central features of the interplay between the research projects regarding the common features and the methods and tools used.

• **Analytic methods.** As outlined above, a big progress in Discrete Mathematics could be achieved by applying methods from ergodic theory to classical problems (such as van der Waerden's problem or Margulis' proof of the Oppenheim conjecture). Ergodic methods play an important role in several projects of the DK. For instance, in Project 04 this approach gives clarifies the arithmetic structure of general numeration systems, in this context we refer also to Project 01 where horocycles, group actions, random walks are studied. In this specific project also various tools from functional analysis are applied.
It is a classical procedure in Discrete Mathematics to apply the machinery of harmonic analysis. The classical convergence theory of Fourier series is used in Project 02 for the investigation for obtaining results in metric discrepancy theory. By combining these classical theorems with modern tools from martingale theory yields optimal probabilistic estimates for the discrepancy

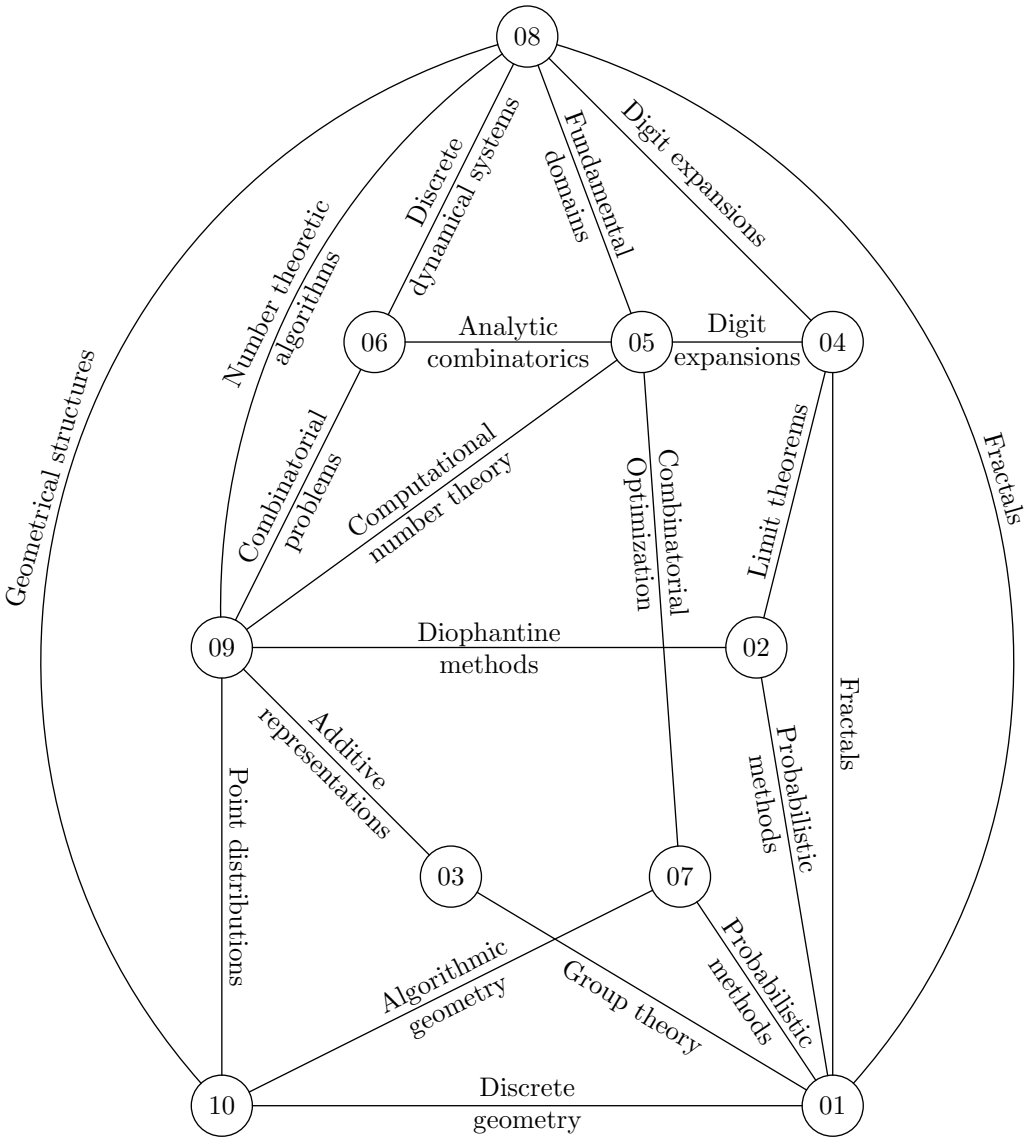

FIGURE 1. The connection graph

of sequences and related arithmetic functions. In Projects 08 and 09 character sum estimates, exponential sums and the circle method are used.

Harmonic analysis on discrete groups, in particular free groups and buildings, plays a central role in themes related Project 1 that go beyond that of a mere tool. We mention the introductory survey by Saloff-Coste [27]. For finite groups, see the celebrated book of Diaconis [2].

Methods from complex and asymptotic analysis are used to find the asymptotic behaviour of transition probabilities (see Projects 01 and 04), and the (related) complex dynamics in

the study of fractals (Project 04). The confluence of those directions became apparent in the collaboration of Grabner and Woess [12] and subsequent work.
Complex and asymptotic analysis is also a classical tool for dealing with questions of analytic number theory as well as in the analysis of digital sums and in the analysis of combinatorial algorithms (Project 05). In particular, we point out Mellin transforms and Hwang's quasi power theorem [17].
Methods from Diophantine analysis and approximation theory are important in at least four projects. Applications of Schmidt's subspace theorem [7] are used in metric discrepancy theory in Project 02. This approach combined with other techniques from Diophantine approximation (the hypergeometric method or Baker's method) are an essential ingredient for the study of Diophantine equations (Project 06), the investigation of additive problems and the description of pseudo random numbers (Project 09). Classical approximation theory is used in Project 10 for the computational modelling of surfaces.

• **Algorithmic aspects.** Algorithmic aspects appear in a combinatoric, a geometric and a number theoretic context. Combinatorial algorithms are studied from several points of view. In Project 05 they are used for the encryption of data in elliptic curve cryptosystems, in Project 07 they form a tool for solving optimisation problems and in Project 03 combinatorial word models for the description of non-unique factorisation are constructed. Geometric algorithms are important for the description of fractals (Project 08) as well as for the approximation of surfaces (Project 10). Number theoretic algorithms occur in the context of the solution of Diophantine equations (Project 06, 09), of factorisations (Project 03) or in specific applications in cryptography (Project 05).

• **Algebraic structures.** Groups and semigroups play a role in several research projects. In Project 07, costs of assignment problems are modelled by elements of a semigroup, in Project 03, block monoids are the essential algebraic structure for the description of non-unique factorisations. Groups are essential in graph theory as well as in the study of random walks (Project 01). The celebrated work of Gromov in Geometric Group Theory has been very essential here, e.g. [14], [15]. In this context also the structural theory of topological groups is of central interest.
Elliptic curves and global function fields over finite fields are the basic structures in Project 09. Elliptic curves, number fields and function fields are also highly relevant for applications in cryptography as studied in Project 05.

We end this subsection with a table containing the assignment of each of the faculty members to the respective "ÖSTAT" (Austrian Statistical Institute) research area.

Table 2: Assignment of the faculty members to the research area

Faculty member	Project number, title	Scientific discipline (ÖSTAT)
Wolfgang Woess	01: Random walk models on graphs and groups	1118, 1120, 1102, 1103
István Berkes	02: Probabilistic methods in combinatorial number theory	1118, 1119, 1103
Alfred Geroldinger	03: Additive Group Theory and Zero-Sum Theory	1119, 1102
Peter Grabner	04: Fractal analysis and combinatorics of digital expansions	1119, 1103, 1120
Clemens Heuberger	05: Digital expansions with applications in cryptography	1119, 1120, 1127
Peter Kirschenhofer	06: Polynomial Diophantine equations – combinatorial and number theoretic aspects	1119, 1120
Bettina Klinz	07: Structural Investigations on Combinatorial Optimisation Problems	1121, 1120
Jörg M. Thuswaldner	08: Number systems and fractal structures	1119, 1116, 1120
Robert F. Tichy	09: Diophantine approximation and combinatorial problems	1118, 1119, 1120, 1103
Johannes Wallner	10, Subdivision in nonlinear geometries	1107, 1135

Explanation of the ÖSTAT number codes:

1102 Algebra
1103 Analysis
1107 Geometrie (Geometry)
1116 Topologie (Topology)
1118 Wahrscheinlichkeitstheorie (Probability Theory)
1119 Zahlentheorie (Number Theory)
1120 Kombinatorik (Combinatorics)
1121 Operations Research
1127 Informationswissenschaft (Information Science)
1135 Differentialgeometrie (Differential Geometry)

1.4. **Scientific base of the DK-plus.**
Our faculty consists of 10 faculty members that are renowned in their fields of expertise. Six of them collaborate in the current national research network FWF-S96 "Analytic Combinatorics and Probabilistic Number Theory". The research done in this research network is strongly linked to the topic of the present doctoral program. It provides additional infrastructure and the PhD students of the present program will certainly benefit from this network in which also researchers from other Austrian universities (University of Vienna, TU Vienna, University of

Linz and University of Salzburg) are involved. In particular, it will help to initiate collaborations and mutual exchange on the national level. Also besides the mentioned FWF network the participating researchers have long standing and intensive contacts to many researchers of all Austrian Universities. In particular, the already tight contacts and cooperations with the research groups in Combinatorics at the University of Vienna (Prof. Christian Krattenthaler) and Vienna University of Technology (Prof. Michael Drmota) will be intensified further within the doctoral program. Furthermore, we plan to involve our retired colleagues Prof. Franz Halter-Koch from KFU Graz and Prof. Wilfried Imrich from MU Leoben in the teaching activities of the DK, where their profound knowledge in Algebra and Number Theory, resp. Graph Theory, will be a precious input.
The DK-plus is based on the research activities of the participating scientists. In what follows we give a brief outline of the research interests of the faculty members. More details can be found in Supplement 1 ("Beilage 1"), where also concrete showcases of PhD projects planned for the present program are presented by each of the faculty members.

1.4.1. **W. Woess (Speaker), Project 01.** The central topic of the research of W. Woess is "Random Walks on Infinite Graphs and Groups", which is also the title of the successful monograph [29]. Here, random walks are understood as Markov chains whose transition probabilities are adapted to an algebraic, geometric, resp. combinatorial structure of the underlying state space. The main theme is the interplay between probabilistic, analytic and potential theoretic properties of those random processes and the structural properties of that state space. From the probabilistic viewpoint, the question is what impact the particular type of structure has on various aspects of the behaviour of the random walk, such as transience/recurrence, decay and asymptotic behaviour of transition probabilities, rate of escape, convergence to a boundary at infinity and harmonic functions. Vice versa, random walks may also be seen as a nice tool for classifying, or at least describing the structure of graphs, groups and related objects. These competences are complemented by the expertise in Non-commutative Probability and Functional Analysis of the associated scientist F. Lehner.

1.4.2. **I. Berkes, Project 02.** The research interests of István Berkes comprise the following areas. Firstly, he is interested in probability theory, in particular, asymptotic theory of independent, exchangeable, stable and mixing processes, invariance principles and path-wise central limit theory play a role in his research. Moreover, he has several research papers in analysis dealing with random phenomena for lacunary series, discrepancy theory, applications of probability theory in analysis, pseudo-randomness, metric entropy. Besides that he works in mathematical statistics (limit theorems, statistical inference, empirical processes) and econometrics (nonlinear time series, long range dependence). Berkes is taking part in the FWF research network (NFN) S96 "Analytic combinatorics and probabilistic number theory".

1.4.3. **A. Geroldinger, Project 03.** One of the main areas of the research of A. Geroldinger is combinatorial and additive number theory. In particular, his research centers on zero-sum theory in abelian groups. Classical topics are the Theorem of Erdős-Ginzburg-Ziv and questions around the Davenport constant. Main methods in this area stem from additive group theory including all type of addition theorems. Another important topic in Geroldinger's research activities are non-unique factorisations in monoids and integral domains. The main objective here is a systematic treatment of phenomena related to the non-uniqueness of factorisations in structures of arithmetical interest. The main focus lies on principal and non-principal orders in algebraic number and function fields, Mori domains, Krull monoids and congruence monoids. The methods are algebraic, combinatorial and analytic. Moreover, Geroldinger is interested in the structure and ideal theory of commutative rings and monoids. At present he investigates the (arithmetic and ideal theoretic) structure of congruence monoids, C-monoids and their various generalisations.

1.4.4. **P. Grabner, Project 04.** The proposer of this project has been interested in the analysis and probability on fractals since the early 1990s. He has written several publications on this subject, as well as supervised three related PhD-theses. He is especially interested in the application of generating function techniques to this subject, which gives a close relation to the iteration theory of rational functions. Furthermore, the proposer has contributed to the study of digital expansions and their probabilistic and ergodic properties. Especially, he is interested in the study of distribution properties of arithmetic functions related to digital expansions, such as additive and multiplicative functions and representation numbers for redundant numeration, application of techniques from ergodic theory and probability theory to the study of systems of numeration, and applications of redundant numeration to cryptography. He has supervised one PhD-thesis on these subjects. Besides these subjects directly related to this proposal he is interested in the mathematical analysis of algorithms, where he also contributed several publications. The techniques used for the analysis of algorithms can also be applied for the study of transition probabilities on graphs, as well as for the derivation of asymptotic properties of additive and multiplicative functions. Grabner is a member of the abovementioned NFN S96.

1.4.5. **C. Heuberger, Project 05.** C. Heuberger's research interests include the asymptotic and probabilistic analysis of algorithms in the sense of D. Knuth (e.g., the expected occurrences of sub-blocks in certain digital expansions), cryptography (e.g., the design of sublinear scalar multiplication algorithms on Koblitz curves), graph theory (e.g. optimising some graph theoretical indices), number theory (e.g., algorithmic solution of Diophantine equations) and combinatorial optimisation (in particular, inverse optimisation). In cooperation with V. Rijmen (KU Leuven and TU Graz), he organised the Central European Conference on Cryptography in July 2008. A special issue of "Computing" has been edited on the occasion of this conference.

1.4.6. **P. Kirschenhofer, Project 06.** Kirschenhofer's main interests in Discrete Mathematics are Combinatorics (enumeration problems and "bijective" proofs, generating functions, combinatorial identities), combinatorial questions in number theory (combinatorial problems leading to Diophantine equations), Diophantine equations (polynomial Diophantine equations, relative Thue equations) and analysis of algorithms. He is a co-researcher of a project within the FWF-NFN S96 and therefore integrated within the corresponding network of research. The proposer is also in close contact with the combinatorial network "Séminaire Lotharingien de Combinatoire", which is one of the important places to bring together researchers working in enumerative Combinatorics in Austria, Germany, Italy and France. The proposer has served as member of the program committee of the Austrian-Slovak joint conference of mathematical societies held in 2007.

1.4.7. **B. Klinz, Project 07.** Both Bettina Klinz and the associated senior scientist Rainer Burkard work in the area of combinatorial optimisation. Among their research interests the following topics play a prominent role: assignment and transportation problems (including three-dimensional assignment problems and quadratic assignment problems), Monge properties, efficiently solvable special cases of hard combinatorial optimisation problems and location problems. A special focus in Bettina Klinz's research lies on the design of efficient algorithms on graphs, network flow problems, parametric optimisation and the investigation of efficiently solvable special cases of NP-hard combinatorial optimisation problems. These topics are at the heart of Project 07.

1.4.8. **J. Thuswaldner, Project 08.** Thuswaldner is interested in several aspects of numeration. In recent years he investigated arithmetic, analytic, geometric, fractal and topological properties of several kinds of number systems. Especially the interplay between numeration and fractal geometry is a central topic in Thuswaldner's research. Recently, together with his collaborators, he defined a general notion of dynamical systems, so-called "shift radix systems" that form a simultaneous generalisation of many well-known notions of number systems like β-expansions and canonical number systems. With help of this tool new characterization results of finiteness as well as periodicity properties of number systems could be established. Besides that, Thuswaldner gave systematic studies of topological properties of fractal tiles related to canonical number systems as well as of Rauzy fractals which are related to substitutional number systems. He is also interested in arithmetic functions related to number systems like the sum of digits function. For the study of these functions analytic tools as well as fractal properties of the underlying number systems are needed. Thuswaldner is a member of the research network (NFN) S96.

1.4.9. **R. Tichy, Project 09.** The research of Tichy (co-author of the book [3]) is focussed on the following topics: Diophantine equations and approximation, ergodic and dynamic methods in number theory, fractal structures in number theory, combinatorial and additive number

theory, algorithms and pseudo-randomness, stochastic methods, discrepancy and uniform distribution modulo 1, numeration systems, sequences and automata, harmonic analysis and exponential sums, multivariate approximation and energy functionals, asymptotic analysis of combinatorial algorithms, counting problems on graphs and applications in computational chemistry, stochastic modelling and simulations in finance and insurance. Tichy is also a member of the research network (NFN) S96.

1.4.10. **J. Wallner, Project 10.** A main focus of Wallner's work is nonlinear subdivision processes and discrete multiscale representations of data, which for started as a collaboration with Nira Dyn on proximity inequalities and their implications on the continuity and smoothness of limit curves produced by geometric, necessarily nonlinear, subdivision processes. Wallner's research group at TU Graz aims at a systematic theory of univariate and multivariate geometric subdivision rules defined in nonlinear geometries (Riemannian manifolds, Lie groups, Euclidean space minus obstacles). Other topics of research are the numerical processing of 3D geometry data, discrete differential geometry, semidiscrete surfaces, and lately especially the application of geometric methods to freeform architectural design. Wallner is a member of FWF research network S92 on industrial geometry.

1.4.11. **International collaborations.** The staff members of this doctoral program have numerous collaborations with scientists all over the world. In what follows we list some of the most important collaborators. In view of the large number of collaborators which can be seen from the publications of the faculty members it is clear that this list is by far not exhaustive.

Table 3: List of selected international collaborations of the faculty members

Project 01 : Woess						
Research area within the DK-plus	Country	Name	Sex	Email address	number of joint papers	Planned host lab for a PhD student
Probability on discrete structures	USA	Laurent Saloff-Coste	M	lsc@ math.cornell.edu	4	yes
Probability & Ergodic Theory	D	Vadim A. Kaimanovich	M	v.kaimanovich@ jacobs -university.de	4	yes
Group Theory, Spectral Theory	CH / D	Laurent Bartholdi	M	laurent.bartholdi @epfl.ch	2	yes
Group Theory, Spectral Theory	CH	Tatiana Smirnova -Nagnibeda	F	tatiana.smirnova -nagnibeda @math.unige.ch	1	yes

Project 02 : Berkes						
Research area within the DK-plus	Country	Name	Sex	Email address	number of joint papers	Planned host lab for a PhD student
Stochastics	USA	Lajos Horváth	M	horvath @math.utah.edu	31	yes
Stochastics	F	Michel Weber	M	weber@math .u-strasbg.fr	6	yes
Stochastics	H	Endre Csáki	M	csaki@renyi.hu	5	yes

Project 03 : Geroldinger						
Research area within the DK-plus	Country	Name	Sex	Email address	number of joint papers	Planned host lab for a PhD student
Commutative Algebra	USA	Scott Chapman	M	schapman @trinity.edu	2	no
Combinatorics	GB	Christian Elsholtz	M	christian.elsholtz @rhul.ac.uk	1	no
Number Theory	CDN	Weidong Gao	M	wdgao_1963 @yahoo.com.cn	15	no
Number Theory	F	Yahya Ould Hamidoune	M	yha@ccr.jussieu.fr	1	yes
Number Theory	F	Alain Plagne	M	plagne@math .polytechnique.fr	0	yes
Combinatorics	E	Oriol Serra	M	oserra @ma4.upc.edu	0	no

Project 04 : Grabner						
Research area within the DK-plus	Country	Name	Sex	Email address	number of joint papers	Planned host lab for a PhD student
Number Theory	F	Christiane Frougny	F	Christiane.Frougny @liafa.jussieu.fr	0	yes
Dynamical Systems	F	Pierre Liardet	M	liardet@gyptis .univ-mrs.fr	5	yes
Analysis on Fractals	USA	Robert Strichartz	M	str@math .cornell.edu	0	yes
Functional Equations	IL	Gregory Derfel	M	derfel@cs.bgu.ac.il	3	yes

Project 05 : Heuberger						
Research area within the DK-plus	Country	Name	Sex	Email address	number of joint papers	Planned host lab for a PhD student
Cryptography	D	Roberto Avanzi	M	roberto.avanzi @ruhr-uni-bochum.de	4	yes
Cryptography	CDN	James Muir	M	jamuir@cs.smu.ca	2	no
Combinatorics	ZA	Helmut Prodinger	M	hprodinger@sun.ac.za	16	yes

Project 06: Kirschenhofer						
Research area within the DK-plus	Country	Name	Sex	Email address	number of joint papers	Planned host lab for a PhD student
Combinatorics	F	Dominique Foata	M	foata@ math.u-strasbg.fr	1 in progress	yes
Combinatorial Identities	GB	Peter Larcombe	M	p.j.larcombe@ derby.ac.uk	1	yes
Diophantine Equations	H	Attila Pethő	M	pethoe@ math.klte.hu	2	yes

Project 07: Klinz						
Research area within the DK-plus	Country	Name	Sex	Email address	number of joint papers	Planned host lab for a PhD student
Combinatorial Optimisation	D	Horst Hamacher	M	hamacher @mathematik.uni-kl.de	6	yes
Combinatorial Optimisation	NL	Gerhard Woeginger	M	gwoegi@win.tue.nl	11	no
Combinatorial Optimisation	UK	Vladimir Deineko	M	Vladimir.Deineko @wbs.ac.uk	6	yes

Project 08: Thuswaldner						
Research area within the DK-plus	Country	Name	Sex	Email address	number of joint papers	Planned host lab for a PhD student
Number Theory, Fractals	JP	Shigeki Akiyama	M	akiyama@math.sc.niigata-u.ac.jp	10	yes
Fractals	F	Valérie Berthé	F	berthe@lirmm.fr	2	yes
Number Theory	F	Christian Mauduit	M	mauduit@iml.univ-mrs.fr	0	yes
Number Theory	H	Attila Pethő	M	pethoe @inf.unideb.hu	7	yes

Project 09 : Tichy						
Research area within the DK-plus	Country	Name	Sex	Email address	number of joint papers	Planned host lab for a PhD student
Metric Number Theory	USA	Roger Baker	M	baker @math.byu.edu	0	
Diophantine Approximation	F	Yuri F. Bilu	M	yuri@math .u-bordeaux1.fr	3	yes
Random Numbers	AUS	Igor Shparlinski	M	igor@comp .mq.edu.au	0	yes
Combinatorial Number Theory	NL	Robert Tijdeman	M	r.tijdeman @tiscali.nl	1	yes

Project 10: Wallner						
Research area within the DK-plus	Country	Name	Sex	Email address	number of joint papers	Planned host lab for a PhD student
Approximation Theory	D	Tomas Sauer	M	tomas.sauer@math .uni-giessen.de	0	yes
Approximation Theory	IL	Nira Dyn	F	niradyn@post .tau.ac.il	2	yes
Discrete Mathematics	USA	Leo Guibas	M	guibas@cs.stanford.edu	1	no
Discrete Geometry	D	Alexander I. Bobenko	M	bobenko@math .tu-berlin.de	2	no

For an excellent education of our students in advanced topics of Discrete Mathematics we plan to invite visiting professors on a regular basis. They will be financed partly by the FWF funding for the DK-plus and partly by Graz University of Technology. This enables the faculty to invite at least two visiting professors in each semester. Furthermore, the NAWI-Graz cooperation project between the two universities in Graz presently supports the research groups of this doctoral program by five PhD positions.
See the preliminary teaching program of the first 2 years in §1.8.2.

1.5. **Research Infrastructure.**
The necessary basic equipment (computers, rooms, etc.) will be provided by the three universities involved in the program. Furthermore, the universities guarantee that the faculty positions and the assigned university assistantships will be covered in case of vacancy on a postdoctoral level related to this doctoral program. In the case of Graz University of Technology this means that 4 positions at the Mathematics Departments will be covered on postdoc level in relation with this program. Furthermore, the full professor positions of Professors Berkes and Burkard will be replaced immediately after their retirement. It is also expected that the future successor of the retired Prof. Halter-Koch at KFU Graz will be involved in the DK at a later stage, contributing to the algebraic aspects. In addition, Graz University of Technology and the NAWI Graz cooperation project between the two Graz universities supports this doctoral program by visiting professorships.
The academic degrees will be awarded by that one of the three involved universities with which the respective principal investigator / advisor is affiliated. The teaching program is mainly based at both universities in Graz (which have a joint PhD program). The distance to Leoben is so close that commuting between the universities causes no problem for the students. (Indeed, it does not take longer than between different research institutions in Vienna.) Specific research seminars will be held at all involved universities.

1.6. **Added Value.**
The main aim of the present DK program is the education of PhD students and to provide them with an excellent environment to start their scientific career. The added value of this program in order to achieve this goal has many aspects which will be outlined in this section. Firstly, the critical mass of a faculty of ten experienced researchers plus two associated scientists enables us to cover a representative part of Discrete Mathematics which guarantees the students a broad education on a very high level. The organisational structure of the DK-plus outlined below in chapter 2 makes the whole program very transparent and ensures highest quality at any stage. Already in the recruitment process, which is organised according to the rules described in Section 2, all the faculty members are involved. This gives a unique possibility to rate the applicants for positions within the DK and to assure that only the best ones will be admitted. As mentioned below, this recruitment procedure does not only apply

to the 10 positions supported by the present program, but also to the 11 additional associated DK students ("associated collegiates"). This gives a total number of 21 excellent PhD students belonging to this program.

The mentoring system (see below in 2.3(ii)) as well as the joint Discrete Mathematics Seminar (see below in 1.8) will guarantee that the DK students will have frequent exchange with all faculty members in the framework of lectures, seminars and personal discussions. This will put them in a position to get acquainted with many branches of discrete mathematics during their studies. Besides that, the guest program of the DK enables us to invite many scientists from abroad in order to give lectures and to be available for scientific discussions with the DK students. In this way, together with the possibility to travel to research institutions all over the world, the DK students will get familiar with the scientific community and will have plenty of possibilities to gain international scientific contacts. These contacts shall be strengthened further during the obligatory six months stay abroad.

Also the interaction among the students themselves will be an important aspect of the life within DK. We plan to organise workshops and seminars where personal skills are trained as well as social events that strengthen the ability of the DK students to work in a team. To this end, we shall also encourage the DK students to initiate joint projects and write joint scientific papers.

The Discrete Mathematics Day at the end of each study year (see below in 2.4(v)) will be another platform of encounter in a festive atmosphere.

The mentoring system, Discrete Mathematics Seminar and Discrete Mathematics Day and the other joint activities also aim at creating a corporate identity of all members of the DK. This is going to be an obvious added value to the standard PhD education in Austria, where in many cases, PhD students carry out their individual work without much of an embedding in or interaction with a larger group.

As a matter of fact, one can observe that already now our PhD system, in particular with the recent, new PhD curricula at KFU and TU Graz, is in a phase of transition. One aspect of the new spirit might be viewed as the replacement of purely individual work with one where "peer control" and the embedding into, as well as exchange with, larger groups gain importance. Our DK-plus can add momentum to this renewal.

All in all, we expect that the framework of the present DK will unite and strengthen the doctoral education in the three participating universities enormously. We are confident that the synergy effects caused by the tight organisation of so many mathematicians interested in related areas of research will certainly result in a center of PhD education and research in Discrete Mathematics with considerable impact. This center will be internationally visible and a perfect starting ground for the scientific career of our DK students.

1.7. **Training goals.**

In Section 1.1, we have formulated our *mission and vision* as

Advanced mathematical education through advanced mathematical research
and
advanced mathematical research through advanced mathematical education.

In more detail, we believe that as a group of researchers that has achieved a considerable "critical mass" of competences in Discrete Mathematics, we can pass our knowledge and research abilities successfully to promising young people. Vice versa, we believe that this education will lead to new, advanced research achievements, whose significance will go beyond that of the sum of the single parts of the group; our diversity within Discrete Mathematics is an essential factor on the road towards success.

As we explain in Section 2.2 below, the recruitment of our DK students will be carried out after international advertisement of the postions, requesting a high standard. Thus we expect that they have graduated with a broad basic knowledge of Mathematics, and not only Discrete Mathematics. Indeed, on one hand the broad basic education is important for a flexible mastering of all the necessary mathematical tools that are needed during the dissertation work. On the other hand, additional specific education in the topics of Discrete Mathematics is part of the program of the DK. The work that they have to perform in the DK, and in which they have to be trained, is of course first of all scientific research in one of the sub-areas of Discrete Mathematics that are outlined in the single projects of the DK faculty members. This goes hand in hand with additional training and research activities regarding social competences, presentation techniques, group work and interaction, participation in workshops and conferences seen both as part of the typical activities of a scientist and as the initial training of firmness in those activities.

We mention here that we, as the DK on the whole, plan to establish an exchange with some of the international centers of Discrete Mathematics, both at the doc- and the postdoc-level. This complements the individual partners for such an exchange that are listed at the end of §1.3 (Table 2), as well as in the Supplement ("Beilage") 1, where the faculty members describe their CV, planned activities, cooperations, etc., in the DK-plus.

One comparable, very large and impressive program is the "Berlin Mathematical School", funded by DFG, which is a joint effort of the three major Berlin universities hosting Mathematics departments. This very successful enterprise is a kind of role model for our proposal, although their scope is wider than ours and spans over the entire width of mathematical subjects; accordingly, the number of participating research groups is larger. In a smaller city like Graz, together with Leoben, the human and logistic resources are of course more limited. Therefore we are focussing on a field which comprises a broad range of "contiguous" subjects and where we have a critical mass of local expertise, ongoing active research, and very good international network relations.

Geographically closest are the groups working in Combinatorics and Discrete Mathematics in Vienna, and a platform for exchange and competitive encounters is the Schrödinger Institute. Another natural partner besides the Berlin School of Mathematics is the Rényi Institute of the Hungarian Academy of Sciences. Slovenia, with the Universities of Ljubljana and Maribor very close to Graz, has a traditionally strong school of graph theory which has been in close contact with Leoben. This cooperation will be intensified for the purpose of the DK. In particular, the traditional Leoben-Ljubljana Graph theory Seminar will be extended to comprise also TU Graz as an organisational seat.

On the other side of the Atlantic, we already have contacted some of our colleagues at Waterloo University (Department of Combinatorics and Optimisation) and at Rutgers University (Mathematics Department, DIMACS and RUTCOR) for the purpose of future exchange. The basic education goals of the doctoral school in RUTCOR are similar to ours. However, the focus of our program lies on Discrete Mathematics emphasizing the relations to other branches of Mathematics such as analysis, geometry, number theory and probability. The focus at RUTCOR is operations research with an interdisciplinary flavor. This aspect of discrete mathematics is covered by our research group on combinatorial optimisation. The graduate school in Discrete Mathematics at the University of Waterloo is organised by the Department of Combinatorics and Optimization which hosts currently 30 full time faculty members. The Waterloo program is a broad program with some overlap with ours mainly in the areas of discrete optimisation, cryptography and graph theory.

Here we also mention the tight multiple contacts between Graz and Marseille on the level of joint research, Erasmus exchange, etc. In particular, the connection with the "École Doctorale Mathématiques et Informatique de Marseille" is strengthened by the recent "cotutelle" contract (the first in Styria, probably also the first in Austria) for the joint PhD supervision of DI Christoph Temmel.

In short, the principal training goal is the formation of young scientists whose self-confidence is based on solid learning and research. Their background in our DK-plus "Discrete Mathematics" shall be such that they can compete successfully within all international centers where research abilities in Mathematics play a major role.

The primary carreer opportunities for our PhDs will be in academic environments. Already in the past, the carreer paths of many among our former PhD students were successful in this respect. We mention some selected names: Günter Rote (full prof., FU Berlin), Gerhard Wöginger (full prof., TU Eindhoven), Clemens Fuchs (Dozent at ETH Zürich), Sara Brofferio (tenured position at Univ. Paris-Orsay), Ali Ünlü (Dozent at Augsburg Univ.), Thomas Stoll (tenured position at Univ. Marseille-Luminy), Hansjörg Albrecher (full prof., Univ. Lausanne), Stephan Wagner (lecturer, Univ. Stellenbosch) and Siegfried Hörmann (Univ. Utah; current offers from CUNY and Univ. Bruxelles). Of course, within the range of the more

"applied mathematical" projects, there are also good opportunities in industrial applications or finance (e.g., of combinatorial optimisation techniques, resp. financial modelling).
All in all, we plan the present doctoral program in a way that grants the participating students an optimal start for their scientific carreers. In particular, after this high level PhD education they will be well prepared in order to apply for positions at distinguished research institutions.

1.8. **DK-plus specific training program.**
Our training program as well as the rules for supervision and quality management (see 2.3 and 2.4 below) are designed so that it follows the principles of the European Charta for Researchers and the Code of Conduct for the Recruitment of Researchers, as put forward by the EU Commission in March 2005, see `http://ec.europa.eu/eracareers/pdf/am509774CEE_EN_E4.pdf`
The program is compatible with the PhD regulations (curricula) at each of the three participating universities. This is facilitated by the so-called NAWI Graz cooperation of the two Graz Universities in the Natural Sciences.
Our DK-plus is intended as a guiding landmark inside that frame that aims at a scientific level significantly above the average of the general doctoral programme. One additional benefit is that we also involve very active and competent mathematicians from Montanuniversität Leoben. The requirements of our specific curriculum go beyond the maxima of the general PhD requirements of each of the three partner universities. The fact that our DK students have to fulfil those higher requirements is founded on their application for the funded PhD positions and the resulting contract, of which those requirements are an integral part.
As already mentioned at the end of 1.1, the embedding of the DK-plus into the general framework of the PhD-curricula of the three universities is facilitated by the fact that our speaker is the chairman of the committee ("Curriculakommission") which is responsible for the regulations of all PhD studies at TU Graz and which has redesigned those programs on the global level of TU Graz recently. Note that the organisation of doctoral studies in Austria, like in many European countries, is such that students admitted for PhD must have taken a master degree or equivalent, of 5 years' duration. When admitted, they start right away with a PhD research project, while taking advanced courses during the work on that project. Thus, typically, the compulsory number of lecture hours to be taken within our PhD curricula is somewhat smaller than in countries where the preceding studies do not last as long and are organised differently. Of course, also brilliant candidates can be admitted to PhD studies at our universities who come from countries with different (maybe shorter) pre-doctoral curricula. According to the PhD regulations at our universities, this may be coupled with requiring additional course work.

1.8.1. **DK-plus Curriculum.** The official language of the DK is English. All courses will be given in English.

During the whole program the DK students have to attend successfully a variety of high level courses and seminars; the overall number required to be admitted to the thesis defense is 240 hours. In the Austrian way of quantifying that teaching load, it corresponds to 16 "Semesterwochenstunden" (hours per week in a semester, that is, 1 "Semesterwochenstunde" corresponds to a total of 15 hours). A typical course will have an average of 3 "Semesterwochenstunden", that is, 45 hours on the whole.
As mentioned above, this is compatible with the new PhD programs at the Graz University of Technology, the University of Graz and the University of Leoben.

Detailed description of the curriculum.
In the first semester of the four years, a course "Advanced Topics in Discrete Mathematics" is obligatory for all DK-students. It will present the main topics of the program and is accompanied by a learning seminar with the same title, where the students strengthen their skills in the techniques of Discrete Mathematics.
The direct follow-up to this course, starting with the second semester of the first year and continuing during the subsequent years, is the weekly joint Discrete Mathematics Seminar. Participation is obligatory for the DK students. It is also expected that the whole faculty attends this seminar. Senior DK students, faculty members and external visitors are supposed to contribute to this seminar.
Besides this, it appears highly desirable to have a platform of encounter with all the other PhD students in Mathematics which are not members of or associated with the DK: indeed, the presence and activities of the DK-plus should have a positive impact on the whole mathematical landscape here. Such a platform is a joint seminar & colloquium series to be held throughout the years, where guests as well as selected PhD students from all branches of mathematics give talks at a broader level, with a good share of talks on topics of Discrete Mathematics. Besides that, there are smaller research seminars on specific research issues, held by faculty members as well as visitors, related to the single projects as well as their interplay as described by the connection graph in Figure 1.
In addition, the students can choose from a variety of lectures, which are determined by the expertise of our faculty and complemented by visiting professors. The invitation of distinguished visitors aims not only at supporting the research activities of the DK students, but also at transferring their knowledge to the students in the teaching of advanced topics.
We plan to offer courses in the following fields:

- Advanced number theory (algebraic, algorithmic, additive)
- Group theory (additive, combinatorial, geometric)
- Graph theory (algebraic, algorithmic)
- Discrete dynamical systems, fractals, ergodic theory
- Random walks, Markov chains and discrete potential theory

- Limit theorems in probability theory, probabilistic number theory
- Analytic enumeration methods
- Analysis of algorithms
- Elliptic curves and cryptography
- Combinatorial optimisation
- Discrete and algorithmic geometry

(Several lectures of this type have been offered at both universities in Graz in the last years and were attended by excellent students. A good number of those students obtained post doc positions at distinguished universities very soon after finishing their doctoral studies.)
All those courses are open to other interested PhD students as well as advanced master students, while of course the level and the selection of the topics is going to be dictated by the needs of the DK.
It is essential for the success of the training of our students that they do not exclusively follow courses that are related directly to their own thesis work, but take part in at least one or two courses and seminars from the other areas that are present within the DK program as well as outside. In addition, the participation in international summer schools and workshops at other distinguished universities is very strongly recommended; a documented, active participation will be considered as fulfilment of a part of the curricular duties.
Besides advanced lectures in different fields of mathematics we intend to prepare the students to scientific life also concerning their presentation skills. Following previous, very positive experiences, we shall organise a course/seminar entitled "Scientific communication", obligatory for all DK students, together with an expert coming from Linguistics. Within that course, the DK students shall learn actively how to give presentations (in English) on topics of mathematical research. Those presentations shall be screened for critical control and improvement of their skills.
Another important ability to be achieved is the writing of scientific papers. This topic will be the content of a seminar within our doctoral program that takes place at least once within each cycle of 4 years. In addition, TU Graz offers short courses on "Scientific Proposal and Paper Writing" which will also be recommended to the DK students.
Along with the profound mathematical education, these "soft-skills" will be a crucial ingredient for a successful beginning of the scientific careers of the DK students. These courses–seminars are held by experienced supervisors who provide detailed expertise in the respective topics; such courses are offered by all three participating universities.
Although our faculty covers a broad spectrum of topics, it is important in many respects that we complement our teaching staff by external visiting professors.
During the recruitment phase for the 10 funded DK student positions, we plan a "kickoff–semester" (February–June 2010) for associated DK students (i.e., students who take part in

the DK, but whose salaries are not funded by the present programme; see below in section 1.9).

In addition to the standard courses and seminars we intend to organise each year one or two workshops combined with a summer school for the doctoral students. This enables the faculty to meet worldwide leading experts in their field as well as to guide the doctoral students to the frontiers of research.

In particular, already for the mentioned "kickoff" phase in 2010 we are considering to organise a 3 days' workshop on "Analytic enumeration methods in combinatorics and probability" with minicourses by Michael Drmota (Vienna), Steve Lalley (Chicago), Jean-Françis Marckert (Bordeaux) and Stephan Wagner (Stellenbosch) and someone from the French group of analytic combinatorics. This workshop should be held close to the hearing of the PhD candidates for the DK, who can be given to opportunity to participate.

Moreover, a workshop on "Numeration and fractals" will be organised by G. Dorfer and J. Thuswaldner within the FWF research network S96 in 2010 in close interaction with the initial DK activities.

We also want to organise meetings at the Erwin Schrödinger Institute in Vienna, which will be a good occasion of encounter and fruitful competition with our Viennese colleagues working in areas of Discrete Mathematics. In 2011, there will be two ESI programs where members of our group are invovled: "Bialgebras in Free Probability" (Feb.–April 2011; organisers: M. Aguiar, F. Lehner, R. Speicher, D. Voiculescu) and "Combinatorics, Number theory, and Dynamical Systems" (Oct.–Dec. 2011; organisers: M. Einsiedler, P. Grabner, Ch. Krattenthaler, T. Ziegler).

Furthermore, the traditional yearly Leoben-Ljubljana graph theory seminar will be extended to include also TU Graz, and one of the next meetings (probably 2011) will be organised at Graz and possibly coupled with a summer school. A conference on "Surfaces, Meshes, Geometric Structures" will be organised in summer or early autumn 2011 by J. Wallner.

Thus, 2011 will be a year of multiple offers of workshop and conference activites that are (co-)organised by members of our faculty and are thematically related with our DK-plus.

It is also required that the DK students spend at least one semester abroad. (This may be a complete semester, or also a sum of shorter stays.)

Their documented participation in training programs during these stays will be accredited as part of the fulfilment of the curricular duties. Centers with which we plan such an exchange (which should be bilateral) are listed above in 1.7 and below in the Appendix where the faculty members describe their CVs, planned activities and international partners.

1.8.2. **Preliminary (incomplete) teaching program of the first 2 years – overview of basic courses and seminars.**

Summer semester 2010 ("kickoff")

- Visiting Prof. Steven P. Lalley (Univ. Chicago), Prof. Wolfgang Woess (TU Graz): course on *Random processes on hyperbolic groups*
- Prof. Johannes Wallner (TU Graz): course on *Differential geometry*[1]
- Visiting Prof. Brigitte Servatius (Worcester Polytechnic): course on *Discrete geometry*
- Retired Prof. Franz Halter-Koch (KFU Graz): *Ideals in commutative rings and monoids*

Winter semester 2010/11 (1st regular DK semester)

- All faculty members: inaugural course on *Advanced Topics in Discrete Mathematics*
- Dr. Katherine Tiede, Prof. Wolfgang Woess (TU Graz): seminar on *Scientific Communication*
- Retired Prof. Wilfried Imrich (MU Leoben) and Prof. Wolfgang Woess (TU Graz): course on *Groups and graphs*
- Visiting Prof. Helmut Prodinger (Univ. Stellenbosch) and Prof. Clemens Heuberger (TU Graz): course on *Analytic combinatorics*
- Visiting Prof. Michael Barnsley (Australian Nat. Univ., Canberra) and Prof. Jörg Thuswaldner (MU Leoben): course on *Iterated function systems and fractals*

Summer semester 2011

- All faculty members & guests: *Discrete Mathematics Seminar*
- Prof. Peter Kirschenhofer (MU Leoben): course on *Advanced combinatorial enumeration methods*
- Profs. Rainer Burkard and Bettina Klinz (TU Graz): *Topics in combinatorial optimisation*
- Visiting Profs. Attila Pethő (Univ. Debrecen) and Clemens Fuchs (ETH Zürich): course on *Algorithmic methods in Diophantine equations*
- Visiting Prof. Christian Mauduit (CNRS Marseille) and Prof. Peter Grabner (TU Graz): course on *Arithmetics of pseudo-random sequences*

Winter semester 2011/12

- All faculty members & guests: *Discrete Mathematics Seminar*
- Prof. Robert Tichy (TU Graz): seminar on *Writing of Scientific Publications*
- Prof. Alfred Geroldinger (KFU Graz): course on *Non-unique factorisations*
- Prof. István Berkes (TU Graz), visiting Prof. Lajos Horváth (Univ. Utah): course on *Limit theorems in probability*

[1] Even though this is not discrete mathematics in itself, it is important for the DK subjects in several respects, such as the relation between Brownian motion on manifolds and random walks, and as the background for discrete differential geometry.

1.9. **Associated DK students.** ("Associated Collegiates")
As can be seen from the precontract signed by the rector of Graz University of Technology, this doctoral program is supported by several additional PhD positions funded by this University. We mention that one of these additional positions is exclusively reserved for female scientists, which emphasises the concern of gender aspects in our program. (Of course, we expect to have many more female DK students, and it should also be clear that in the recruiting process the scientific requirements will be the same for all candidates.) Moreover, at each of the three universities we expect to have further doctoral positions that are paid from other sources and that are suitable for employing associated DK students for the present DK. Individual fund raising efforts in this respect will be undertaken by faculty members.
Therefore we apply for support of **11 associated DK students**. All the positions designated for associated DK students are advertised internationally. Moreover, the selection procedure of the applicants is equivalent to the one for "internal" DK students; compare with §2.2. This guarantees that the qualification of the associated DK students has the same level as for the internal ones. Thus the environment of the DK will not only enable us to educate the fully paid 10 DK students on the high standard of a DK. It will also provide the framework for 11 additional PhD students to enjoy the advantages of this distinguished program. This forms a definite added value of the present program.

1.10. **Visibility and Dissemination.**
Each of the faculty members has a considerable record of publications. Most of them appeared in international mathematical journals. In addition, several of our faculty members (like W. Woess, A. Geroldinger and R. Tichy) authored books that are by now considered as standard monographs in their fields. Moreover, all participating scientists gave many talks at international conferences and have numerous collaborators all over the world. Several faculty members have a lot of experience in organising international conferences, workshops and summer schools. All in all, each of the faculty members is well visible in the international mathematical community. Therefore, we are confident that the present doctoral program which is jointly proposed by 10+2 very active scientists will lead to considerable synergy effects which will give us a strong visibility in the scientific community. As the total number of PhD students within our program is about twenty, a considerable number of young scientists will profit from this visibility on the one hand and will make their contribution in order to its further increase on the other hand.
In the following we want to specify the strategies that we are pursuing in order to guarantee high visibility of the education and research planned for this DK and to disseminate our scientific results.
The doctoral program will maintain a homepage which will be updated on a regular basis. This homepage shall contain personal information on the faculty members as well as on the

DK students. Moreover, we shall advertise all upcoming events like conferences, workshops, summer schools as well as lectures and colloquia that will take place in the framework of the present DK. Besides that, the publications written within the present program shall be put on the homepage as early as possible in order to guarantee that new results are available in public as soon as possible.

Another indispensable feature for increasing our visibility and disseminating the research results achieved in the DK is the exchange of scientists. We (faculty members as well as DK students) shall attend international conferences, workshops and summer schools on a regular basis in order to learn about new results, present own results, and in order to get in contact with leading experts of the respective field. On the other hand, an important task of the present program consists in the invitation of international experts for lectures and seminars as well as in the organisation of international conferences and summer schools. Summarizing, the proposed DK will provide an environment in which PhD students and faculty members maintain permanent exchange with experts all over the world.

Another task which is important is the fast and efficient dissemination of the scientific research performed by the DK students as well as by the faculty members. Besides putting new results on the DK-plus homepage we plan to make all our papers available at preprint servers (like `arXiv.org`, and also on the DK webpage) which guarantee a far reaching dissemination of the results.

In all those publications, the line

Doctoral Program "Discrete Mathematics"

will be added to the address header, and (as usual) the support by FWF will be mentioned clearly in a footnote.

As for the publication of the upcoming research papers, the proposers of this program aim at publishing the results obtained within this program in high quality mathematical journals, and (to a smaller extent) in conference proceedings. It is absolutely necessary that all work undergoes an ordinary peer reviewing procedure. According to the *Berlin Declaration on Open Access to Knowledge in the Sciences and Humanities* signed by the FWF, the proposers will take care that all the research results achieved within the DK will be published in a way that makes it possible to access them freely in the internet.

(As a maybe slightly less standard tool for displaying corporate identity as well as publicizing our DK, we want to produce a T-shirt with our DK logo.)

We would also like to put some emphasis on aspects of our research that are of interest for non-specialists in order to increase the public awareness of our program. Here we mention for instance fractal geometry or cryptography which are topics that are fascinating also for non-scientists. In this context it is our goal to make (at least some parts of) the homepage attractive for a broad mass of people in order to advertise the importance of mathematics to many people and to attract them to the things we are working on. Occasional publications

of popular articles in newspapers are planned in order to make our program visible to non-specialists. We also want to interact with "math.space" at the Museum Quarter in Vienna (whose mission consists in popularising mathematics) by contributing to the regular program of this institution, as well as bringing some part of this program and further popular talks to Graz. This can be organised in collaboration with the Austrian Mathematical Society, whose current president is R. Tichy from our faculty. In this context, we mention that 2012 is going to be a "Year of Mathematics" in Austria, of which we shall take advantage.

2. Rules of the DK-plus

2.1. Application and selection criteria for faculty members.

According to the FWF guidelines, the composition of the group of faculty members of the DK-plus who propose and supervise PhD projects can be altered in the two renewal phases after 4 and 8 years.

(i) The most essential requirements for joining the DK faculty are excellent publication and citation records in peer-reviewed journals and international collaborations that show a strong interplay with the wide domain of Discrete Mathematics, its applications and interplay with other disciplines. Candidates must of course also meet the formal requirements for supervising PhD theses and exams (in the involved universities, this is typically "Habilitation" or equivalent qualification).

(ii) Primarily, new faculty members shall be persons who have joined the staff of the participating institutions very recently, typically as new professors who have a record of excellent performance abroad, and who can widen and intensify the range of topics of Discrete Mathematics covered by the DK.[2]

(iii) For newly opened, topically relevant professor positions at the participating institutions, the aspects of their possible contributions to the DK shall be strongly emphasised already in the advertisements as well at the candidates' hearing and selection for those positions.

(iv) Candidates for becoming new faculty members are proposed to the Council of the DK by the Speaker (see below in 2.5 for the organisational structure of the DK and the definition of its panels). The Council checks the fulfilment of the requirements postulated in (i) and (ii). The International Board of Advisors has to be invited to express their opinion. After a public hearing, the faculty of the DK-plus votes for the admission of the new member, for which a majority of 2/3 of the present persons is required.

[2]We expect in particular that the successors of Professors Halter-Koch (KFU) and Burkard (TUG) will become part of the faculty and will add further substance in the fields of algebra, combinatorics and optimisation.

(v) In case of significant misconduct of a faculty member, such as plagiarism, non–obeying of the guidelines of quality control, the supervision duties or other grave violations of good scientific conduct, that member can be removed, after verification of the facts, by the Council in unanimous decision, which has to be confirmed by the International Board of Advisors.

2.2. **Recruitment of DK students.**

(i) The PhD positions within this DK shall be internationally advertised by the following tools.

- A professionally designed poster with a brief description of the DK, the application deadlines and a hint at the DK webpage.
- Worldwide distribution of the poster among mathematical institutes and centers.
- Information on webpages that collect mathematical job opportunities such as Luchsinger Mathematics and the respective webpages of various mathematical societies.
- Posting on the European Researcher's Mobility Portal of the European Commission.
- Advertisements in the notices of various mathematical societies.
- Personal contacts of the faculty members with colleagues all over the world.
- Direct contacts with talented and advanced students at all possible occasions such as summer (& winter) schools and workshops.

Relating to the last item, this doctoral program will have a strong interaction with the current FWF research network (NFN) S96 "Analytic combinatorics and probabilistic number theory" and several other research projects in this area. Within those projects various summer schools are organised. On those occasions the faculty members can get into contact with outstanding students from all over the world that may become candidates for our doctoral program. The connection of the research network S96 with this program yields an added value already at the stage of the recruitment of the DK students ("collegiates").

(ii) The rules of the "Working Group on Issues of Equal Opportunity Treatment" of TU Graz (see `http://www.akgleich.tugraz.at/`) will be implemented in the recruiting process, and we plan to directly involve a member of this group. The general aim is to recruit a considerable number of excellent female mathematicians in order to help mending the gender imbalance in the mathematical sciences. The representative for gender issues of the DK supervises the recruitment procedure of the DK in this respect.

(iii) Applicants must have finished or be close to finishing their studies at the pre-doctoral level. They have to present their CV and track record of their studies as well as two reference letters by distinguished researchers. A proof of written and oral command of English must be submitted. Each candidate should specify which is the sub–area and PhD project advisor of her/his preference.

(iv) The applications are screened by the faculty members. Each faculty member is entitled to produce a ranking that is specific to her/his PhD projects, taking into account the preferences expressed by the applicants. On this basis, the assembly of all faculty members discusses and elaborates a list of selected applicants, no more than twice the final number to be recruited, which are then invited for a hearing.

(v) The hearing is attended by all faculty members and to be held a few months before the scientific work of the program can start. In the hearing, each candidate has to give a short mathematical presentation of a topic of her/his choice and is then interviewed with the aim of displaying her/his background, knowledge, abilities and intentions for the future research work.

(vi) After the hearing, the faculty elaborates a final ranking, taking into account all available informations, specifically the CV, track record, recommendation letters as well as the performance at the hearing. This ranking also has to take into account the initial individual rankings by all faculty members; the assignment of a DK student to one of the research projects requires the consent of the respective faculty member.

(vii) The International Board of Advisors is informed about the final ranking and invited to comment upon it. Eventual objections have to be discussed by the General Assembly of the faculty members.

(viii) Before the final list is made public, the Universities' administration checks for each of those candidates the fulfilment of the formal admission criteria to PhD studies according to the Universities' general regulations. After that, the candidates are informed about their admission.

(ix) It is not mandatory to fill all positions immediately; some of them can be re-advertised at a later stage, after which another selection procedure following the above criteria will take place.

2.2.1. *Draft for the call for applications for the DK.*

Call for Applications

The newly founded Doctoral College (DK) in

Discrete Mathematics

offers an advanced PhD training and research program which is run jointly by Graz University of Technology, University of Graz and Montanuniversität Leoben, Austria. It is funded by the Austrian Science Fund (FWF) and the three supporting universities.
The range of topics in Discrete Mathematics comprises

- Commutative Algebra
- Number Theory
- Discrete Dynamics and Fractals
- Graph Theory
- Combinatorial Group Theory
- Discrete Stochastics
- Combinatorial Optimisation and Algorithmic Geometry
- Analysis of Algorithms, Cryptography

We offer 10 PhD positions for up to 4 years and a salary of €25600 per year, starting at the earliest with October 1, 2010. These positions are assigned to 10 specific research projects within the above areas. For details as well as the formal application criteria, see

`www.discrete.math.tugraz.at/`

The official language of the DK is English. The future college students must exhibit a strong track record of mathematical studies that is equivalent to a master degree which must be valid by the beginning of the contract. Graduates from excellent bachelor programs can be also admitted, subject ot additional curricular requirements. All candidates may present confidential reference letters by internationally recognized researchers and should indicate in their application the projects of their principal interest.
Applications must be sent by mail or email until March 31, 2010 to ... .
After a screening, a selection of candidates will be invited for a hearing in June, 2010. With a view toward increasing the number of female mathematicians, female candidates are especially encouraged to apply.

2.3. **Supervision.**

(Rules of supervision, training and PhD examination)

(i) Every advisor interacts regulary with the DK student(s) assigned to her/his project. During the regular teaching period, this has to follow a schedule that is at least bi-weekly. In any case, the advisor has to respond to the students' request for an exchange of ideas as soon as possible, and conversely, the advisor may summon the students to get in touch with her/him for the same purposes.

(ii) To each doctoral student three faculty members are assigned as "mentors", including the advisor, who guide her/him during the doctoral studies. The mentors are nominated by the Council and are expected to coordinate their mentoring activities. It is an aim of the mentoring

system to stimulate and support the transfer of the multiple expertise of the faculty members in combinatorics, number theory, algebra, discrete probability theory, combinatorial optimisation and discrete differential geometry, as well as techniques from harmonic and complex analysis, ergodic theory, etc. to the DK students.

(iii) Meetings between the DK students and the mentors have to take place at least twice per semester. The specific form in which those meetings take place (individually or in groups, in the framework of seminars or personal encounters) is left to the involved persons, as long as the interaction between the mentors of each graduate student is assured.

(iv) The mentors give advice to the DK students regarding which courses and seminars they should follow.
At the latest at the end of the first year of studies, each graduate student should participate in an international summer school or workshop with a more educational character, of at least one week duration. In each of the subsequent years, the participation in at least one international workshop, summer school or conference is obligatory. The mentors, in particular the advisor, are obliged to support the assigned DK students with regard to these activities.

(v) Each advisor interacts with the administrative head of the research units where her/his DK students are employed in order to ensure that those DK students can also take part in teaching activities, which is considered as an integral part of their training.

(vi) The DK students have to publish parts of their research already before their graduation in peer-reviewed journals. As generally usual in mathematical research, co-authors can only be persons who have taken an active and concrete part in the work. In particular, advisors' names are not automatically added to the list of authors of the publications. The Intellectual Property Rights are shared by the authors of each publication.

(vii) The mentors shall encourage the DK students to search international collaboration with other persons and to learn to seize such opportunities during contacts established at workshops, conferences, etc.

(viii) The PhD thesis of each DK student has to be evaluated by the thesis supervisor as well as by two external referees from different universities. At least one of those referees has to come from outside the three universities that take part in this DK. The proposals for the selection of the external referees have to be approved by the Council, as well as the responsible dean of graduate studies of the respective university, and are communicated to all faculty members. The external referees shall be contacted and will be sent a preliminary version of the thesis at least 2 months before the official submission. Their eventual response will be taken into account by the mentors and the candidate in the final phase. Each of the three final reports should express a grade for the thesis according to the Austrian grading system.
All faculty members shall be informed and given access to viewing the three referee reports.

(ix) The requirements for admission to the final exam, in accordance with the general regulations of the university, comprise the following.

- Fulfilment of the curricular requirements (240 hours, see above).
- Documented active participation in workshops, conferences and summer schools.
- Publication of at least two papers resulting from the thesis work in peer-reviewed journals (communications of acceptance for publication are in general sufficient). [3]
- The 3 referee reports on the thesis with positive grades.

(x) The final examination ("Rigorosum") is public and takes part within the Discrete Mathematics Seminar. It comprises a presentation of the research work by the candidate and a defense of the thesis. The external referees are invited to act as members of the examination committee. The examination itself follows the general regulations of the university.

(xi) Besides the official PhD document issued by the university, each successful candidate will receive a document signed by the Speaker and Deputy Speakers of the DK which certifies that the degree was achieved within the DK "Discrete Mathematics", specifying the title of the thesis and the names of the advisor and the external referees.

2.4. **Internal Monitoring.**

(i) Each DK student has to present talks on her/his work in the joint Discrete Mathematics Seminar, attended by the faculty members.

(ii) Towards the end of each academic year, at a date to be specified by the Council, every DK student submits a progress report (including a shorter abstract) on her/his work. This report has to be complemented by the advisor, and the other mentors are invited to add a written comment to that report. All the reports are collected and screened by the Council and are made accessible to all DK-plus members (faculty as well as DK students).

(iii) If there is evidence of problems in the progress of the studies, research and supervision, the Council can undertake steps to rectify those problems and/or summon according action from the supervisor, the mentors and the DK students.

(iv) The collected abstracts of progress reports, together with an overall summary prepared by the Council, are sent to the International Board of Advisors with an invitation to comment upon it.

(v) At the end of each summer semester, the Discrete Mathematics Day is held. It is open to the public, all DK members are expected to participate, and invitations are sent to the rectorates and other important personalities and institutions. The programme consists of short ($\approx$ 10 minutes) presentations by the DK students and one plenary talk by a distinguished

[3] In case of outstanding publications, already one paper can suffice, in accordance with a respective opinion expressed by the thesis referees. If no publication is presented then an additional external referee report is required. The aim to have published work already before the final exam is also formulated in the general PhD regulations of TU Graz, in force since 2007.

visitor, and possibly also a "popular" public talk for a general audience. The purpose of this event goes beyond internal monitoring, and also aims at underlining the DK's "corporate identity", at displaying the achievements of the DK-plus, and at promoting it visibility.

2.5. **Order of the DK-plus.**

(i) The DK-plus "Discrete Mathematics" is a joint programme of mathematicians at Graz University of Technology (TUG), the University of Graz (KFU) and Montanuniversität Leoben (MUL). It is based and administered at TU Graz. A specific agreement, which does not involve the FWF, will clarify the relation with the partners of TUG at KFU and MUL with regards to the issues of the DK.

(ii) Organisational structure of the DK-plus.
The DK is led and administered by the *Speaker* (from TU Graz) and two *Deputy Speakers* (one from KFU Graz and one from MU Leoben) with the support of the *Coordinator* (employed at TU Graz).
The *Council* consists of the Speaker, the Deputy Speakers, the *representative for gender issues* and a *representative of the DK students.* It can be augmented by *senior faculty members* in cases of necessity.
The *Faculty* consists of all scientists from TUG, KFU and MUL that lead and supervise PhD projects within this DK.
Associated scientists are excellent scientists from TUG, KFU and MUL that are not faculty members but support the research activities externally.
The *DK Assembly* consists of all faculty members, associated scientists and DK students.
The *International Advisory Board* consists of 5 internationally renowned experts in the sub-areas of Discrete Mathematics that are present in the DK.

(iii) The Speaker as the contract partner of the FWF is the chief executive organ of the DK who is primarily responsible for the interaction with the universities and the FWF and the correct performance of the work within the DK. In particular, she/he directs the correct financial management. She/he is supported in these activities by the Deputy Speakers and the Coordinator. The Deputy Speakers replace the Speaker in case she/he is prevented. They are in particular responsible for the interaction of the DK with their respective universities (KFU and MUL) and take care of the local financial issues.
The speaker may be exchanged in the 2nd or 3rd four-year-phase of the DK after unanimous decision of the faculty members. The International Advisory Board has to be informed.

(iv) The *Coordinator* is an employee at TU Graz with academic education and a scientific background in the realm of Discrete Mathematics. Her/his tasks are to support the administrative and organisational agenda of the DK. In particular, she/he maintains the DK web page, helps the DK students from abroad with administrative problems such as visa, staying permit, etc. She/he takes care of the announcements of seminars and talks and summons,

collects and reviews the different reports and accounting statements. She/he also takes care of organisational activities related to workshops, conferences research visits and trips. In these activities, she/he can rely on the practical support by the secretaries of the involved institutes, which will be formalized in her/his contract. Within TU Graz, she/he is subordinate to the Speaker. She/he is nominated by the Council.[4]

(v) The representative for gender issues is a faculty member who is nominated for each 4 year period in the first official faculty meeting after approval, resp. renewal of the DK. She/he belongs to the Council and takes part in its meetings and work. In particular, she/he supervises the correct handling of all gender issues according to the implemented rules of the "Working Group on Issues of Equal Opportunity Treatment".

(vi) The representative of the DK students in the Council is elected by all the DK students at the beginning of the regular work of each 4 year period. She/he takes part in the meetings and work of the Council.

(vii) The council meets regularly, at least twice per semester, in order to discuss and prepare the operative and scientific agenda of the DK. Its decisions are communicated to the DK members. The Council also supports the interaction between the DK and the universities. In cases of delicate issues or when the council does not reach a decision, it can appeal to the International Advisory Boardfor their opinion and support.

(viii) The International Advisory Board consists of

- Herbert Edelsbrunner (Duke University, Durham, NC, USA)
- Dominique Foata (Université de Strasbourg I, France)
- Laurent Saloff-Coste (Cornell University, Ithaca, NY, USA)
- Vera Sós (Hungarian Academy of Sciences, Budapest, Hungary)
- Robert Tijdeman (University of Leiden, The Netherlands)[5]

The boards' members support the activities of the DK with their expertise. They are are invited to take part in the candidates' hearings and are in any case informed about the selection of the DK students and about the scientific programme. Eventual objections by board members to decisions taken by the Speakers or the Council have to be discussed in a meeting of the Council. The Advisory Board has an important role in the internal monitoring (see above in 2.4).
The nomination of new Advisory Board members, if it becomes necessary, is part of the duties of the Council, which has to search feedback from the faculty members on this issue.

[4]The described role of the coordinator is based on the FWF guidelines. The salary is equivalent to that of an FWF psotdoc. Ideally, this is a person with full academic training who plans to direct her/his professional activities towards "scientific management."

[5]All 5 have confirmed.

(ix) The faculty members' rights include their votes in the nomination of the representative for gender issues, as well as in the ranking of the candidates for the DK and the nomination of associated scientists.
The faculty members obey the highest ethical and scientific standards in carrying out the work related to this DK-plus. They follow the rules of good scientific practice as formulated by each of the rectorates of TUG, KFU and MUL. They agree with their signature to pursue the goals of the DK as formulated in the present book, to fully accept its order and to implement its rules in every respect.

(x) Associated scientists can be nominated by the faculty. They are themselves not members of the faculty, but support the activities of one or more faculty members in the scientific work related to the DK. They may come from TUG, KFU or MUL and have to exhibit a strong scientific track record.

(xi) The PhD students of the DK ("collegiates") agree, via their employment contracts, to perform their studies and other activities according to the rules and requirements formulated in this booklet, which are also reproduced on the DK webpage. These requirements comprise, but go significantly beyond those of the official PhD curricula of the involved universities.

(xii) The General Assembly meets once per semester. The meetings of the General Assembly can be subdivided in a public and a private part. The public part is in particular open to representatives of the Rectorates and Faculties. The reserved part is restricted to faculty members and the representative of the DK students.
In the meetings, the General Assembly is informed about current issues and decisions of the Speakers and Council.

(xiii) Financial support for travel, conference and workshop participation by the DK students, as well as short visits of guest scientists, are handled by the Speaker (at TUG), resp. the Deputy Speaker (at KFU, resp. MUL) and will – compatibly with the regulations of the respective university and the FWF – be conferred to the direct responsibility of each faculty member for a yearly amount up to €3.000,- per advised DK student. The faculty members are responsible to provide the necessary documentation for the financial reports.
The use of larger sums for specific purposes (organisation of workshops, longer stays of guest scientists, etc.) has to be preceded in due time (at least 4 months before the respective event) by a financial plan that has to be discussed and approved by the Council.

(xiv) Each faculty member and associated scientist confirms by her/his signature that she/he agrees and will obey the rules and order of the DK. This also applies to new faculty members according to 2.1, as well as newly nominated associated scientists.

The faculty members and associated scientists declare with their signature that they commit themselves to implement the rules and order of the DK-plus "Discrete Mathematics".

Istvan Berkes

Alfred Geroldinger

Peter Grabner

Clemens Heuberger

Peter Kirschenhofer

Bettina Klinz

Jörg Thuswaldner

Robert Tichy

Johannes Wallner

Wolfgang Woess

Rainer Burkard

Franz Lehner

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List of abbreviations, terminology.

DK, DK-plus — Doctoral Program (Doktoratskolleg)
DK student (FWF terminology: collegiate) — PhD student enrolled in DK
KFU — Karl-Franzens-Universität Graz (University of Graz)
MUL — Montanuniversität Leoben (University of Leoben)
NFN — national research network of the FWF
TUG — Technische Universität Graz (Graz University of Technology)

Authors of the proposal text.

The general proposal text was prepared in intensive cooperation by

- Jörg Thuswaldner (MUL)
- Robert Tichy (TUG)
- Wolfgang Woess (TUG)

The individual parts in Supplement 1 were prepared by the respective faculty members.

All participating scientists have read and approved the text.