

TRIMMED STABLE AR(1) PROCESSES

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ABSTRACT. In this paper we investigate the distribution of trimmed sums of dependent observations with heavy tails. We consider the case of autoregressive processes of order one with independent innovations in the domain of attraction of a stable law. We show if the d largest (in magnitude) terms are removed from the sample, then the sum of the remaining elements satisfies a functional central limit theorem with random centering provided $d = d(n) \geq n^\gamma$ (for some $\gamma > 0$) and $d(n)/n \rightarrow 0$. This result is used to get asymptotics for the widely used CUSUM process in case of dependent heavy tailed observations.

1. Introduction

Let X_1, X_2, \dots , be independent, identically distributed random variables in the domain of attraction of a stable law with index $0 < \alpha < 2$. Lévy [38] and Darling [22] noted that the order of magnitude of the sum $S_n = \sum_{k=1}^n X_k$ is the same as that of its largest term and the contribution of a fixed, but large number of extremal terms is essentially responsible for the distribution of S_n . The asymptotic distribution of the trimmed sum $S_n^{(d)}$ obtained from S_n by discarding the d smallest and d largest summands was determined by LePage et al. [37] and Csörgő et al. [20] proved that in case of moderate trimming, i.e. $d(n) \rightarrow \infty$, $d(n)/n \rightarrow 0$ the trimmed sum $S_n^{(d)}$ satisfies the central limit theorem. Heavy trimming, i.e. when $0 < \lim_{n \rightarrow \infty} d(n)/n < 1$, is studied in [17]–[19]. Arov and Bobrov [2], Mori [43], Hall [33], Teugels [48], Griffin and Pruitt [31], [32] and Kesten [35] considered a different type of trimming of the sample. Let $\eta_{n,d}$ denote the d -th largest element of $|X_1|, \dots, |X_n|$. These authors were interested in the asymptotic behavior of the modulus trimmed sum ${}^{(d)}S_n = \sum_{k=1}^n X_k I\{|X_k| \leq \eta_{n,d}\}$, i.e. when from the sum we remove the d elements with the largest absolute values. Griffin and Pruitt [31] proved that the trimmed central limit theorem of Csörgő et al. [20] remains valid for modulus trimmed sums provided the distribution of X_1 is symmetric, but it generally fails for nonsymmetric variables and it can happen that ${}^{(d)}S_n$ is asymptotically normal for some $d(n)$, but not for another $d'(n) \geq d(n)$. This is somewhat unexpected, since removing more large elements from the sample should result in better behavior. Sufficient conditions for the asymptotic normality of ${}^{(d)}S_n$ in the nonsymmetric case were given in [9]. On the other hand, Berkes et al. [10] showed that if $d(n) \rightarrow \infty$,

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$d(n)/n \rightarrow 0$, a functional central limit theorem always holds for ${}^{(d)}S_n$ with a random centering factor. Some of these results are extended in [36] to long range dependent sequences.

Trimming also has important applications in statistics. As an example, we consider the detection of possible changes in the location model

$$X_j = c_j + e_j, \quad 1 \leq j \leq n,$$

where e_1, \dots, e_n are random errors. Under the null hypothesis

$$H_0 : c_1 = c_2 = \dots = c_n$$

of no change in the location parameter we have

$$(1.1) \quad X_j = c + e_j, \quad 1 \leq j \leq n,$$

with some constant c . Under the alternative there are r changes:

H_A : there is $r \geq 1$ and $1 < k_1 < k_2 < \dots < k_r < n$ such that

$$\begin{aligned} c_1 = \dots = c_{k_1-1} &\neq c_{k_1} = c_{k_1+1} = \dots = c_{k_2-1} \neq c_{k_2} = c_{k_2+1} = \dots \\ &= c_{k_r-1} \neq c_{k_r} = \dots = c_n. \end{aligned}$$

The most popular methods to test H_0 against H_A (cf. [5] and [21]) are based on the CUSUM process

$$(1.2) \quad U_n(x) = \sum_{i=1}^{\lfloor nx \rfloor} X_i - \frac{\lfloor nx \rfloor}{n} \sum_{i=1}^n X_i,$$

where $\lfloor \cdot \rfloor$ denotes the integer part. Clearly, if H_0 is true, then $U_n(t)$ does not depend on the common but unknown location parameter c_1 . It is well known if X_1, \dots, X_n are independent and identically distributed random variables with a finite second moment, then

$$\frac{1}{(n\text{var}(X_1))^{1/2}} U_n(x) \xrightarrow{\mathcal{D}[0,1]} B(x),$$

where $B(x)$ is a Brownian bridge and $\xrightarrow{\mathcal{D}[0,1]}$ means weak convergence in the space $\mathcal{D}[0,1]$ of càdlàg functions equipped with the Skorokhod J_1 topology (cf. [12]). Assuming that X_1, X_2, \dots, X_n are independent and identically distributed random variables in the domain of attraction of a stable law of index $\alpha \in (0, 2)$, Aue et al. [3] showed that

$$\frac{1}{n^{1/\alpha} \hat{L}(n)} U_n(x) \xrightarrow{\mathcal{D}[0,1]} B_\alpha(x),$$

where \hat{L} is a slowly varying function at ∞ and $B_\alpha(x)$ is an α -stable bridge. (The α -stable bridge is defined as $B_\alpha(x) = W_\alpha(x) - xW_\alpha(1)$, where W_α is a Lévy α -stable motion.) Since nothing is known on the distributions of the functionals of α -stable bridges, Berkes et al. [10] suggested the trimmed CUSUM process

$$(1.3) \quad T_{n,d}(x) = \sum_{i=1}^{\lfloor nx \rfloor} X_i I\{|X_i| \leq \eta_{n,d}\} - \frac{\lfloor nx \rfloor}{n} \sum_{i=1}^n X_i I\{|X_i| \leq \eta_{n,d}\}.$$

Assuming that the X_i 's are independent and identically distributed and are in the domain of attraction of a stable law, they proved

$$(1.4) \quad \frac{1}{\sigma_n} T_{n,d}(x) \xrightarrow{\mathcal{D}[0,1]} B(x),$$

where

$$\sigma_n^2 = \frac{\alpha}{2-\alpha} (H^{-1}(d/n))^2 d,$$

$B(t)$ is a Brownian bridge and H^{-1} denotes the generalized inverse of H , the survival function of X_1 . The CUSUM process has also been widely used in case of dependent variables but it is nearly always assumed that the observations have high moments and the dependence in the sequence is weak. For a review we refer to [5]. However, very few papers consider the instability of time series models with heavy tails.

Fama [28] and Mandelbrot [40, 41] pointed out that the distributions of commodity and stock returns are often heavy tailed with possible infinite variance and their research started the investigation of time series models where the marginal distributions have regularly varying tails. Davis and Resnick [24, 25] investigated the properties of moving averages with regularly varying tails and obtained non-Gaussian limits for the sample covariances and correlations. Their results were extended to heavy tailed ARCH in [23]. The empirical periodogram was studied by Mikosch et al. [42]. Andrews et al. [1] estimated the parameters of autoregressive processes with stable innovations.

2. Main results

In this paper we study trimmed sums of AR(1) sequences with heavy tails. Let e_i be a non-anticipative (i.e. $\sigma\{\varepsilon_j, j \leq i\}$ measurable) solution of

$$(2.1) \quad e_i = \rho e_{i-1} + \varepsilon_i \quad -\infty < i < \infty.$$

We assume throughout this paper that

$$(2.2) \quad \varepsilon_j, -\infty < j < \infty \text{ are independent and identically distributed,}$$

$$(2.3) \quad \varepsilon_0 \text{ belongs to the domain of attraction of a stable random variable } \xi^{(\alpha)} \text{ with parameter } 0 < \alpha < 2,$$

and

$$(2.4) \quad \varepsilon_0 \text{ is symmetric when } \alpha = 1.$$

Assumption (2.3) means that

$$(2.5) \quad \left(\sum_{j=1}^n \varepsilon_j - a_n \right) / b_n \xrightarrow{\mathcal{D}} \xi^{(\alpha)}$$

for some numerical sequences a_n and b_n . The necessary and sufficient condition for this is

$$(2.6) \quad \lim_{t \rightarrow \infty} \frac{P\{\varepsilon_0 > t\}}{L_*(t)t^{-\alpha}} = p \quad \text{and} \quad \lim_{t \rightarrow \infty} \frac{P\{\varepsilon_0 \leq -t\}}{L_*(t)t^{-\alpha}} = q$$

for some numbers $p \geq 0$, $q \geq 0$, $p + q = 1$, where L_* is a slowly varying function at ∞ . It is known that (2.1) has a unique stationary non-anticipative solution if and only if

$$(2.7) \quad -1 < \rho < 1.$$

Under assumptions (2.2)–(2.7), $\{e_j\}$ is a stationary sequence and $E|e_0|^\kappa < \infty$ for all $0 < \kappa < \alpha$ but $E|e_0|^\kappa = \infty$ for all $\kappa > \alpha$. AR(1) processes with stable innovations were considered by Chan and Tran [15], Chan [14], Aue and Horváth [4] and Zhang and Chan [51] who investigated the case when ρ is close to 1 and provided estimates for ρ and the other parameters when the observations do not have finite variances.

The convergence of the finite dimensional distributions of $U_n(x)$ in the AR(1) case is an immediate consequence of Phillips and Solo [44] representation. Let \xrightarrow{fdd} denote the convergence of the finite dimensional distributions. If (1.1)–(2.4) and (2.7) hold, then we have

$$(2.8) \quad \frac{1 - \rho}{n^{1/\alpha} L_*(n)} U_n(x) \xrightarrow{fdd} B_\alpha(x),$$

where $B_\alpha(x)$, $0 \leq x \leq 1$ is an α -stable bridge and L_* is defined in (2.6). It has been pointed out in [6, 7] that the *fdd* convergence in (2.8) cannot be replaced with weak convergence in $\mathcal{D}[0, 1]$. However, Avram and Taquq [7] proved that $U_n(x)$ converges in the weak- M_1 sense under some additional regularity conditions. Some of their regularity conditions were removed by Tyran-Kamińska [49]. For further results on the weak convergence of dependent sequences with infinite variance in the M_1 topology we refer to [8].

We formulate now our main results. On the truncation parameter $d = d(n)$ we will assume

$$(2.9) \quad \lim_{n \rightarrow \infty} d(n)/n = 0$$

and

$$(2.10) \quad d(n) \geq n^\delta \quad \text{with some } 0 < \delta < 1.$$

Let $F(x) = P\{X_0 \leq x\}$, $H(x) = P\{|X_0| > x\}$ and let $H^{-1}(t)$ be the (generalized) inverse of H . Our last condition will be used to establish the weak law of large numbers for $\eta_{n,d}$. We assume that ε_0 has a density function $p(t)$ which satisfies

$$(2.11) \quad \int_{-\infty}^{\infty} |p(t+s) - p(t)| dt \leq C|s| \quad \text{with some positive constant } C.$$

(Here, and the sequel, all constants will be finite and positive.) Let

$$(2.12) \quad A_n = d^{1/2} H^{-1}(d/n)$$

and

$$(2.13) \quad m(t) = EX_1 I\{|X_1| \leq t\}.$$

Theorem 2.1. *If (1.1), (2.1)–(2.4), (2.7) and (2.9)–(2.11) hold, then we have that*

$$\left(\frac{2-\alpha}{\alpha}\right)^{1/2} \left(\frac{1-\rho}{1+\rho}\right)^{1/2} \frac{1}{A_n} \sum_{k=1}^n [X_k I\{|X_k| \leq \eta_{m,d}\} - m(\eta_{m,d})] \xrightarrow{\mathcal{D}[0,1]} W(x),$$

where $W(x)$ is a Wiener process.

The result in Theorem 2.1 uses the the random centering factor $m(\eta_{m,d})$. This factor is characteristic for the asymptotic distribution of the modulus trimmed partial sums process, as first observed in [10]. Since a random translation of the terms in the CUSUM process cancels out, the next result is an immediate consequence of Theorem 2.1.

Theorem 2.2. *If (1.1), (2.1)–(2.4), (2.7) and (2.9)–(2.11) hold, then we have that*

$$\left(\frac{2-\alpha}{\alpha}\right)^{1/2} \left(\frac{1-\rho}{1+\rho}\right)^{1/2} \frac{T_{n,d}(x)}{A_n} \xrightarrow{\mathcal{D}[0,1]} B(x),$$

where $B(x)$ is a Brownian bridge.

Statistical applications of Theorem 2.2 require the estimation of the norming factor from the observations. We suggest a kernel type estimator for the norming factor in Theorem 2.2 which is computed from the trimmed observations $X_i^* = X_i I\{|X_i| \leq \eta_{m,d}\}$. Let

$$\hat{s}_n^2 = \hat{\gamma}_0 + 2 \sum_{j=1}^{n-1} \omega(j/h) \hat{\gamma}_j,$$

where

$$\hat{\gamma}_j = \frac{1}{n} \sum_{i=1}^{n-j} (X_i^* - \bar{X}_n^*)(X_{i+j}^* - \bar{X}_n^*) \quad \text{with} \quad \bar{X}_n^* = \frac{1}{n} \sum_{i=1}^n X_i^*.$$

For the kernel $\omega(t)$ we assume the following regularity conditions: (i) $\omega(0) = 1$ (ii) $\omega(t) = 0$ if $t > a$ with some $a > 0$ (iii) ω is Lipschitz continuous (iv) the Fourier transform of ω is Lipschitz continuous and integrable on the real line. Assuming that $h = h(n) \rightarrow \infty$ and $h(n)/n \rightarrow 0$, the method in [34] and [39] can be used to establish that

$$\frac{\hat{s}_n}{A_n} \xrightarrow{P} \left(\frac{\alpha}{2-\alpha}\right)^{1/2} \left(\frac{1+\rho}{1-\rho}\right)^{1/2}.$$

Hence Theorem 2.2 yields

$$\frac{T_{n,d}(x)}{\hat{s}_n} \xrightarrow{\mathcal{D}[0,1]} B(x),$$

where $B(x)$ is a Brownian bridge.

In this paper we considered a stationary AR(1) sequence with stable innovations. Theorems 2.1 and 2.2 could be extended to linear processes but this extension would require nontrivial modifications of our method or a completely different approach.

3. Preliminary results

The proofs of Theorems 2.1 and 2.2 are based on several technical lemmas.

In the sequel we can and will assume without loss of generality that

$$(3.1) \quad E\varepsilon_0 = 0, \text{ if } 1 < \alpha < 2.$$

Under these conditions, in (2.5) we can choose $a_n = 0$ and b_n can be chosen any sequence satisfying

$$(3.2) \quad \frac{n}{b_n^\alpha} L_*(b_n) \rightarrow 1.$$

According to Theorem 2.3 of Cline [16] (cf. also Davis and Resnick [25]), $H(x)$, the survival function of $|X_0|$ satisfies

$$(3.3) \quad H(x) = x^{-\alpha} L(x),$$

where $L(x)$ is a slowly varying function at ∞ and

$$(3.4) \quad \lim_{x \rightarrow \infty} \frac{H(x)}{P\{|\varepsilon_0| > x\}} = \lim_{x \rightarrow \infty} \frac{L(x)}{L_*(x)} = \frac{1}{1 - |\rho|^\alpha}.$$

Let

$$u_{k,n}(t) = X_k I\{|X_k| \leq tH^{-1}(d/n)\} \quad \text{and} \quad m_n(t) = E[X_0 I\{|X_0| \leq tH^{-1}(d/n)\}].$$

The main goal of this section is to get bounds for $E u_{0,n}(t) u_{k,n}(s)$ and $\text{cov}(u_{0,n}(t), u_{k,n}(s))$.

Lemma 3.1. *We assume that (1.1), (2.1)–(2.4), (2.7) and (3.1) hold. Let $\mathbf{Y}^{(k)} = (X_0, X_k)$ and let $\mathbf{Y}_i^{(k)}, i = 1, 2, \dots$ be independent and identically distributed copies of $\mathbf{Y}^{(k)}$. Then*

$$\frac{\mathbf{Y}_1^{(k)} + \dots + \mathbf{Y}_n^{(k)}}{n^{1/\alpha} L_*(n)} \xrightarrow{\mathcal{D}} \mathbf{Z}^{(k)} \quad \text{as } n \rightarrow \infty,$$

where $\mathbf{Z}^{(k)} = (Z_1^{(k)}, Z_2^{(k)})$ with

$$Z_1^{(k)} = \sum_{\ell=0}^{\infty} \rho^\ell \xi_{-\ell}^{(\alpha)} \quad \text{and} \quad Z_2^{(k)} = \sum_{\ell=0}^{\infty} \rho^\ell \xi_{k-\ell}^{(\alpha)}$$

and $\xi_\ell^{(\alpha)}, -\infty < \ell < \infty$ are independent and identically distributed copies of $\xi^{(\alpha)}$.

Proof. It follows from (2.1) that

$$(3.5) \quad X_k - c = \sum_{\ell=0}^{\infty} \rho^\ell \varepsilon_{k-\ell} = \sum_{\ell=0}^{k-1} \rho^\ell \varepsilon_{k-\ell} + \rho^k X_0, \quad 1 \leq k < \infty.$$

Let $\varepsilon_\ell^{(i)}$, $-\infty < \ell < \infty$, $i = 1, 2, \dots$ be independent and identically distributed copies of ε_0 . Clearly

$$\mathbf{Y}_i^{(k)} = (Y_{i,1}^{(k)}, Y_{i,2}^{(k)}) \quad \text{with} \quad Y_{i,1}^{(k)} = \sum_{\ell=0}^{\infty} \rho^\ell \varepsilon_{-\ell}^{(i)} \quad \text{and} \quad Y_{i,2}^{(k)} = \sum_{\ell=0}^{\infty} \rho^\ell \varepsilon_{k-\ell}^{(i)}$$

are independent and identically distributed copies of $\mathbf{Y}^{(k)}$. Elementary algebra gives

$$\sum_{i=1}^n Y_{i,1}^{(k)} = \sum_{\ell=0}^{\infty} \rho^\ell \sum_{i=1}^n \varepsilon_{-\ell}^{(i)} \quad \text{and} \quad \sum_{i=1}^n Y_{i,2}^{(k)} = \sum_{\ell=0}^{k-1} \rho^\ell \sum_{i=1}^n \varepsilon_{k-\ell}^{(i)} + \rho^k \sum_{\ell=0}^{\infty} \rho^\ell \sum_{i=1}^n \varepsilon_{-\ell}^{(i)}.$$

For every $L \geq 0$ by (2.5) we have that (recall that under our conditions the centering factors a_n in (2.5) can be chosen 0)

$$\frac{1}{b_n} \left(\sum_{i=1}^n \varepsilon_\ell^{(i)}, -L \leq \ell \leq L \right) \xrightarrow{\mathcal{D}} \left(\xi_\ell^{(\alpha)}, -L \leq \ell \leq L \right),$$

where $\xi_\ell^{(\alpha)}$, $-\infty < \ell < \infty$ are independent and identically distributed copies of $\xi^{(\alpha)}$. Let $0 < \kappa < \alpha$. It follows from Theorem 6.1 of de Acosta and Giné [27], p. 225 that

$$E \left| \frac{1}{b_n} \sum_{i=1}^n \varepsilon_\ell^{(i)} \right|^\kappa \leq C_1,$$

and therefore for every $x > 0$ we have that

$$\lim_{L \rightarrow \infty} \limsup_{n \rightarrow \infty} P \left\{ \sum_{\ell=L+1}^{\infty} \rho^\ell \left| \frac{1}{b_n} \sum_{i=1}^n \varepsilon_\ell^{(i)} \right| > x \right\} = 0$$

and similarly

$$\lim_{L \rightarrow \infty} P \left\{ \sum_{\ell=L+1}^{\infty} \rho^\ell |\xi_\ell^{(\alpha)}| > x \right\} = 0.$$

This completes the proof of the lemma. □

Let \mathbf{i} denote the imaginary unit.

Lemma 3.2. *Let \mathbf{Y} be a stable vector variable with characteristic function $\psi(s, t)$. Then there exists a measure ν on the Borel sets of R^2 such that for some $\mathcal{C}_1, \mathcal{C}_2$ and any $\gamma > 0$*

$$\psi(s, t) = \exp \left\{ \mathbf{i}(\mathcal{C}_1 s + \mathcal{C}_2 t) + \int_{|\mathbf{u}| > \gamma} (e^{\mathbf{i}(s u_1 + t u_2)} - 1) \nu(du_1, du_2) \right\}$$

$$+ \int_{0 < |\mathbf{u}| \leq \gamma} \left(e^{\mathbf{i}(su_1 + tu_2)} - 1 - \mathbf{i}(su_1 + tu_2) \right) \nu(du_1, du_2) \Big\},$$

where $\mathbf{u} = (u_1, u_2)$.

The result can be found, for example, in Gikhman and Skorohod ([29], Chapter 5). ν is called the Lévy measure in the canonical representation of the characteristic function of \mathbf{Y} . The stable vectors in our paper will be centered, i.e. $\mathcal{C}_1 = \mathcal{C}_2 = 0$.

Lemma 3.3. *If (1.1), (2.1)–(2.4), (2.7) and (3.1) hold, then we have*

$$\lim_{T \rightarrow \infty} \frac{T^{\alpha-2}}{L_*(T)} EX_0 I\{|X_0| \leq vT\} X_k I\{|X_k| \leq wT\} = \frac{\alpha}{2-\alpha} \frac{\rho^k}{1-|\rho|^\alpha} (\min(v, w|\rho|^{-k}))^{2-\alpha}.$$

Proof. It follows from Theorem 4 of Resnick and Greenwood [46] that

$$(3.6) \quad \lim_{n \rightarrow \infty} nP \left\{ \frac{(X_0, X_k)}{b_n} \in \mathbf{A} \right\} = \nu(\mathbf{A}),$$

where b_n is defined in (3.2) and \mathbf{A} is any Borel set of R^2 , not containing $(0, 0)$, $\nu(\mathbf{A}) < \infty$ and the ν -measure of the boundary of \mathbf{A} is 0. Since $nL_*(b_n)/b_n^\alpha \rightarrow 1$, with the choice of $n = \lfloor T^\alpha/L_*(T) \rfloor$ we get from (3.6) that

$$(3.7) \quad \lim_{T \rightarrow \infty} \frac{T^\alpha}{L_*(T)} P\{(X_0, X_k)/T \in \mathbf{A}\} = \nu(\mathbf{A}),$$

where ν is the Lévy measure in the canonical representation of the characteristic function of $\mathbf{Z}^{(k)}$. Denoting the joint distribution of X_0/T and X_k/T by $\nu_k^{(T)}$, relation (3.7) says

$$\lim_{T \rightarrow \infty} \frac{T^\alpha}{L_*(T)} \mu_k^{(T)}(\mathbf{A}) = \nu(\mathbf{A})$$

for any Borel-set $\mathbf{A} \subset R^2$ not containing $(0, 0)$ and having Lebesgue measure 0 for its boundary. Since the function $f(x, y) = xy$ equals 0 at the origin, using the weak convergence of $T^\alpha \mu_k^{(T)}/L_*(T)$ over $D([-v, v] \times [-w, w] \setminus [-s, s] \times [-s, s])$ and letting $s \downarrow 0$ we get

$$\lim_{T \rightarrow \infty} \frac{T^\alpha}{L_*(T)} \int_{-v}^v \int_{-w}^w xy \nu_k^{(T)}(dx, dy) = \int_{-v}^v \int_{-w}^w xy \nu(dx, dy)$$

which can be written equivalently as

$$\lim_{T \rightarrow \infty} \frac{T^{\alpha-2}}{L_*(T)} EX_0 I\{|X_0| \leq vT\} X_k I\{|X_k| \leq wT\} = \int_{-v}^v \int_{-w}^w xy \nu(dx, dy).$$

Since $\xi^{(\alpha)}$ is a stable random variable, its characteristic function can be written as $\exp(-\psi(t))$ and with this notation we get

$$E \exp(\mathbf{i}(sZ_1^{(k)} + tZ_2^{(k)})) = \exp \left(- \sum_{\ell=0}^{\infty} \psi(s\rho^\ell + t\rho^{k+\ell}) - \sum_{\ell=0}^{k-1} \psi(t\rho^\ell) \right).$$

If $\hat{\nu}_\ell$ denotes the Lévy measure associated with the characteristic function $\exp(-\psi(s\rho^\ell + t\rho^{k+\ell}))$ and $\tilde{\nu}_\ell$ corresponds to $\exp(-\psi(t\rho^\ell))$, then we have

$$\nu(\mathbf{A}) = \sum_{\ell=0}^{\infty} \hat{\nu}_\ell(\mathbf{A}) + \sum_{\ell=0}^{k-1} \tilde{\nu}_\ell(\mathbf{A}).$$

Hence

$$\int_{-v}^v \int_{-w}^w xy\nu(dx, dy) = \sum_{\ell=0}^{\infty} \int_{-v}^v \int_{-w}^w xy\hat{\nu}_\ell(dx, dy).$$

Next we note that there is a positive constant A^* such that

$$\lim_{x \rightarrow \infty} \frac{P\{|\xi^{(\alpha)}| > x\}}{x^{-\alpha}} = A^*$$

and therefore by Bingham et al. ([13], p. 346) we obtain that

$$\lim_{x \rightarrow \infty} \frac{E(\xi^{(\alpha)})^2 I\{|\xi^{(\alpha)}| \leq x\}}{x^2 P\{|\xi^{(\alpha)}| > x\}} = \frac{\alpha}{2 - \alpha}$$

resulting in

$$\lim_{x \rightarrow \infty} \frac{E(\xi^{(\alpha)})^2 I\{|\xi^{(\alpha)}| \leq x\}}{x^{2-\alpha}} = A^* \frac{\alpha}{2 - \alpha}.$$

The last relation implies

$$\begin{aligned} & \lim_{T \rightarrow \infty} T^{\alpha-2} E \left[\rho^{2\ell+k} (\xi^{(\alpha)})^2 I\{|\xi^{(\alpha)}| \leq T \min(v|\rho|^{-\ell}, w|\rho|^{-(\ell+k)})\} \right] \\ &= A^* \frac{\alpha}{2 - \alpha} \rho^{2\ell+k} (\min(v|\rho|^{-\ell}, w|\rho|^{-(\ell+k)}))^{2-\alpha}. \end{aligned}$$

We note that $\exp(-\psi(s\rho^\ell + t\rho^{k+\ell}))$ is the characteristic function of the vector $(\rho^\ell \xi^{(\alpha)}, \rho^{k+\ell} \xi^{(\alpha)})$, so repeating the arguments leading to (3.6) and (3.7) for this vector instead of (X_0, X_k) we get

$$\lim_{T \rightarrow \infty} \rho^{k+2\ell} \frac{T^{\alpha-2}}{A^*} E \xi^{(\alpha)} I\{|\rho^\ell \xi^{(\alpha)}| \leq vT\} \xi^{(\alpha)} I\{|\rho^{\ell+k} \xi^{(\alpha)}| \leq wT\} = \int_{-v}^v \int_{-w}^w xy\hat{\nu}_\ell(dx, dy),$$

and therefore

$$\int_{-v}^v \int_{-w}^w xy\hat{\nu}_\ell(dx, dy) = \frac{\alpha}{2 - \alpha} \rho^k |\rho|^{\alpha\ell} (\min(v, w|\rho|^{-k}))^{2-\alpha}.$$

Summing for $\ell = 0, 1, \dots$, we get Lemma 3.3. \square

Lemma 3.4. *If (1.1), (2.1)–(2.4), (2.7), (2.9), (2.10) and (3.1) hold, then for every $k = 0, 1, 2, \dots$*

$$(3.8) \quad \lim_{n \rightarrow \infty} \frac{nE(u_{0,n}(s) - m_n(s))(u_{k,n}(t) - m_n(t))}{A_n^2} = \frac{\alpha}{2 - \alpha} \rho^k (\min(s, t|\rho|^{-k}))^{2-\alpha}.$$

Proof. If $1 < \alpha < 2$, then

$$(3.9) \quad \lim_{n \rightarrow \infty} m_n(t) = EX_0 \quad \text{for any } t > 0.$$

If $0 < \alpha < 1$, then

$$(3.10) \quad \begin{aligned} |m_n(t)| &\leq \int_{-tH^{-1}(d/n)}^{tH^{-1}(d/n)} |x| dF(x) \\ &= - \int_0^{tH^{-1}(d/n)} x dH(x) = -xH(x) \Big|_0^{tH^{-1}(d/n)} + \int_0^{tH^{-1}(d/n)} H(x) dx. \end{aligned}$$

By (3.3) and Bingham et al. ([13], p. 26) we have for $0 < \alpha < 1$

$$(3.11) \quad \lim_{y \rightarrow \infty} \frac{\int_0^y H(x) dx}{yH(y)/(1-\alpha)} = 1,$$

and therefore

$$(3.12) \quad m_n(t) = O\left(H^{-1}(d/n) \frac{d}{n}\right).$$

If $\alpha = 1$, by assumption e_0 is symmetric, so under (1.1) we have that $X_1 = e_1 + c_1$ and therefore

$$(3.13) \quad \begin{aligned} m_n(t) &= O(1) + E[e_0 I\{|X_0| \leq tH^{-1}(d/n)\}] \\ &= O(1) + \int_{tH^{-1}(d/n)-c_1}^{tH^{-1}(d/n)+c_1} x dP\{e_1 \leq x\} \\ &= O\left(H^{-1}(d/n) \frac{d}{n}\right) + \int_{tH^{-1}(d/n)-c_1}^{tH^{-1}(d/n)+c_1} P\{e_1 \leq x\} dx \\ &= O\left(H^{-1}(d/n) \frac{d}{n} \log H^{-1}(d/n)\right). \end{aligned}$$

Thus we get from (3.9)–(3.13) for all $0 < \alpha < 2$ that

$$(3.14) \quad \frac{nm_n(s)m_n(t)}{A_n^2} \rightarrow 0.$$

Lemma 3.3 yields

$$\begin{aligned} &\lim_{n \rightarrow \infty} \frac{n}{A_n^2} \frac{L(H^{-1}(d/n))}{L_*(H^{-1}(d/n))} EX_0 I\{|X_0| \leq sH^{-1}(d/n)\} X_k I\{|X_k| \leq tH^{-1}(d/n)\} \\ &= \frac{\alpha}{2-\alpha} \frac{\rho^k}{1-|\rho|^\alpha} (\min(s, t|\rho|^{-k}))^{2-\alpha}. \end{aligned}$$

By (3.4) we have

$$\lim_{n \rightarrow \infty} \frac{L(H^{-1}(d/n))}{L_*(H^{-1}(d/n))} = \frac{1}{1 - |\rho|^\alpha},$$

which completes the proof of the lemma. \square

Lemma 3.5. *If (1.1), (2.1)–(2.4), (2.7), (2.9), (2.10) and (3.1) hold, we have for all $1/2 \leq s \leq t \leq 3/2$ and $0 \leq x \leq 1$ that*

$$\begin{aligned} & \lim_{n \rightarrow \infty} \frac{1}{A_n^2} E \left(\sum_{k=1}^{\lfloor nx \rfloor} (u_{k,n}(s) - m_n(s)) \right) \left(\sum_{k=1}^{\lfloor nx \rfloor} (u_{k,n}(t) - m_n(t)) \right) \\ &= x \frac{\alpha}{2 - \alpha} \left(s^{2-\alpha} + \sum_{k=1}^{\infty} \rho^k [(\min(s, t|\rho|^{-k})^{2-\alpha} + \min(t, s|\rho|^{-k})^{2-\alpha}] \right). \end{aligned}$$

Proof. We note that

$$\begin{aligned} & E \left(\sum_{k=1}^{\lfloor nx \rfloor} (u_{k,n}(s) - m_n(s)) \right) \left(\sum_{k=1}^{\lfloor nx \rfloor} (u_{k,n}(t) - m_n(t)) \right) \\ &= \lfloor nx \rfloor E(u_{0,n}(s) - m_n(s))(u_{0,n}(t) - m_n(t)) \\ &\quad + \sum_{k=1}^{\lfloor nx \rfloor - 1} (\lfloor nx \rfloor - k) E(u_{0,n}(s) - m_n(s))(u_{k,n}(t) - m_n(t)) \\ &\quad + \sum_{k=1}^{\lfloor nx \rfloor - 1} (\lfloor nx \rfloor - k) E(u_{0,n}(t) - m_n(t))(u_{k,n}(s) - m_n(s)). \end{aligned}$$

Let

$$(3.15) \quad e_k^* = \sum_{\ell=0}^{k-1} \rho^\ell \varepsilon_{k-\ell} \quad \text{and} \quad X_k^* = c_1 + e_k^*.$$

Using again Theorem 2.3 of Cline [16] (cf. also [25]), it follows that there is a constant C_1 such that

$$(3.16) \quad P\{|X_k^*| > x\} \leq C_1 x^{-\alpha} L(x) \quad \text{for all } k \text{ and } 0 \leq x < \infty.$$

Clearly as in (3.5),

$$(3.17) \quad X_k - X_k^* = e_k - e_k^* = \sum_{\ell=k}^{\infty} \rho^\ell \varepsilon_{k-\ell} = \sum_{j=0}^{\infty} \rho^{k+j} \varepsilon_{-j} = \rho^k (X_0 - c_1).$$

Next we write

$$\begin{aligned} & |E(u_{0,n}(s) - m_n(s))(u_{k,n}(t) - m_n(t))| \\ &= |E u_{0,n}(t) u_{0,n}(s) - m_n(t) m_n(s)| \end{aligned}$$

$$\begin{aligned}
&\leq |E(X_{0,n}(X_k - X_k^*)I\{|X_0| \leq sH^{-1}(d/n)\}I\{|X_k| \leq tH^{-1}(d/n)\})| \\
&\quad + |E(X_0X_k^*I\{|X_0| \leq sH^{-1}(d/n)\}I\{|X_k| \leq tH^{-1}(d/n)\}) - m_n(s)m_n(t)| \\
&\leq A_{1,k,n} + A_{2,k,n} + A_{3,k,n}
\end{aligned}$$

with

$$\begin{aligned}
A_{1,k,n} &= E|X_0(X_k - X_k^*)I\{|X_0| \leq sH^{-1}(d/n)\}I\{|X_k| \leq tH^{-1}(d/n)\}|, \\
A_{2,k,n} &= E[|X_0X_k^*I\{|X_0| \leq sH^{-1}(d/n)\} \\
&\quad \times |I\{|X_k| \leq tH^{-1}(d/n)\} - I\{|X_k^*| \leq tH^{-1}(d/n)\}|]
\end{aligned}$$

and

$$A_{3,k,n} = |E(X_0X_k^*)I\{|X_0| \leq sH^{-1}(d/n)\}I\{|X_k^*| \leq tH^{-1}(d/n)\} - m_n(s)m_n(t)|.$$

Using (3.14) and (3.17) we conclude

$$\begin{aligned}
(3.18) \quad A_{1,k,n} &\leq |\rho|^k E|X_0||X_0 - c_1|I\{|X_0| \leq sH^{-1}(d/n)\} \\
&\leq C_2|\rho|^k (H^{-1}(d/n))^2 d/n
\end{aligned}$$

with some constant C_2 . Next we note that

$$\begin{aligned}
(3.19) \quad A_{2,k,n} &\leq E[|X_0X_k^*I\{|X_0| \leq sH^{-1}(d/n)\} \\
&\quad \times I\{tH^{-1}(d/n) - |\rho|^k|X_0| \leq |X_k^*| \leq tH^{-1}(d/n)\}] \\
&\quad + E[|X_0X_k^*I\{|X_0| \leq sH^{-1}(d/n)\} \\
&\quad \times I\{tH^{-1}(d/n) \leq |X_k^*| \leq tH^{-1}(d/n) + |\rho|^k|X_0|\}] \\
&= A_{2,k,n}^{(1)} + A_{2,k,n}^{(2)}.
\end{aligned}$$

Using the independence of X_0 and X_k^* we get

$$\begin{aligned}
A_{2,k,n}^{(1)} &\leq E|X_0|I\{|X_0| \leq sH^{-1}(d/n)\} \\
&\quad \times E|X_k^*|I\{tH^{-1}(d/n) - |\rho|^k H^{-1}(d/n) \leq |X_k^*| \leq tH^{-1}(d/n)\}.
\end{aligned}$$

By (3.16) we have that

$$\begin{aligned}
(3.20) \quad &E|X_k^*|I\{tH^{-1}(d/n) - |\rho|^k H^{-1}(d/n) \leq |X_k^*| \leq tH^{-1}(d/n)\} \\
&= -xP\{|X_k^*| > x\} \Big|_{tH^{-1}(d/n) - |\rho|^k H^{-1}(d/n)}^{tH^{-1}(d/n)} + \int_{tH^{-1}(d/n) - |\rho|^k H^{-1}(d/n)}^{tH^{-1}(d/n)} P\{|X_k^*| > x\} dx \\
&\leq \int_{tH^{-1}(d/n) - |\rho|^k H^{-1}(d/n)}^{tH^{-1}(d/n)} P\{|X_k^*| > x\} dx \\
&\leq C_3|\rho|^k H^{-1}(d/n) d/n,
\end{aligned}$$

where C_3 is a constant. Hence, on account of (3.9), (3.12) and (3.13) we obtain that with some constant C_4

$$A_{2,k,n}^{(1)} \leq C_4\rho^k (H^{-1}(d/n))^2 d/n$$

and similarly

$$A_{2,k,n}^{(2)} \leq C_4 \rho^k (H^{-1}(d/n))^2 d/n,$$

resulting in

$$(3.21) \quad A_{2,k,n} \leq C_5 \rho^k (H^{-1}(d/n))^2 d/n.$$

Using again the independence of X_0 and X_k^* we get

$$A_{3,k,n} = |m_n(s)| |EX_k^* I\{|X_k^*| \leq tH^{-1}(d/n)\} - m_n(t)|.$$

It is easy to see that

$$\begin{aligned} & EX_k^* I\{|X_k^*| \leq tH^{-1}(d/n)\} \\ &= EX_k^* I\{|X_k^*| \leq tH^{-1}(d/n)\} I\{|X_0| > |\rho|^{-k/2} H^{-1}(d/n)\} \\ &\quad + EX_k^* I\{|X_k^*| \leq tH^{-1}(d/n)\} I\{|X_0| \leq |\rho|^{-k/2} H^{-1}(d/n)\} \end{aligned}$$

and by the independence of X_0 and X_k^* and (3.16) we have

$$\begin{aligned} |EX_k^* I\{|X_k^*| \leq tH^{-1}(d/n)\} I\{|X_0| > |\rho|^{-k/2} H^{-1}(d/n)\}| &\leq C_5 |m_n(t)| H(|\rho|^{-k/2} H^{-1}(d/n)) \\ &\leq C_6 |m_n(t)| \rho^{k\alpha/2} d/n. \end{aligned}$$

Next we note that

$$\begin{aligned} & |E[X_k^* I\{|X_k^*| \leq tH^{-1}(d/n), |X_0| \leq |\rho|^{-k/2} H^{-1}(d/n)\}] \\ &\quad - E[(X_k^* + \rho^k(X_0 - c_1)) I\{|X_k^* + \rho^k(X_0 - c_1)| \leq tH^{-1}(d/n), \\ &\quad\quad\quad |X_0| \leq |\rho|^{-k/2} H^{-1}(d/n)\}]]| \\ &\leq |\rho|^k E[|X_0 - c_1| I\{|X_k^* + \rho^k(X_0 - c_1)| \leq tH^{-1}(d/n), \\ &\quad\quad\quad |X_0| \leq |\rho|^{-k/2} H^{-1}(d/n)\}]] \\ &+ E[|X_k^*| I\{|X_k^*| \leq tH^{-1}(d/n), |X_0| \leq |\rho|^{-k/2} H^{-1}(d/n)\} \\ &\quad - I\{|X_k^* + \rho^k(X_0 - c_1)| \leq tH^{-1}(d/n), |X_0| \leq |\rho|^{-k/2} H^{-1}(d/n)\}]] \\ &\leq |\rho|^k (|\rho|^{-k/2} H^{-1}(d/n) + |c_1|) \\ &\quad + E|X_k^*| I\{(t - |\rho|^{k/2})H^{-1}(d/n) - |c_1| |\rho|^k \leq |X_k^*| \leq tH^{-1}(d/n)\} \\ &\quad + E|X_k^*| I\{tH^{-1}(d/n) \leq |X_k^*| \leq (t + |\rho|^{-k/2})H^{-1}(d/n) + |c_1| |\rho|^k\} \\ &\leq C_7 (|\rho|^{k/2} H^{-1}(d/n) + |\rho|^k H^{-1}(d/n) d/n) \end{aligned}$$

by (3.20). Similarly

$$\begin{aligned} & |EX_k I\{|X_k| \leq tH^{-1}(d/n)\} - EX_k I\{|X_k| \leq tH^{-1}(d/n), |X_0| \leq |\rho|^{-k/2} H^{-1}(d/n)\}| \\ &\leq C_8 (|\rho|^{k/2} H^{-1}(d/n) + |\rho|^k H^{-1}(d/n) d/n). \end{aligned}$$

Hence

$$(3.22) \quad A_{3,k,n} \leq C_9 |\rho|^{\tau k} (H^{-1}(d/n))^2 d/n, \quad \text{where } \tau = \min\{1, \alpha\}/2.$$

Putting together (3.18), (3.21) and (3.22) we get that

$$(3.23) \quad \lim_{K \rightarrow \infty} \limsup_{n \rightarrow \infty} \frac{1}{A_n^2} \sum_{k=K}^{\lfloor nx \rfloor - 1} |(\lfloor nx \rfloor - k)E(u_{0,n}(s) - m_n(s))(u_{k,n}(t) - m_n(t))| = 0.$$

The lemma now follows from Lemma 3.4 and (3.23). \square

4. A weak convergence result

Define the two-parameter process

$$L_n(t, x) = \frac{1}{A_n} \sum_{i=1}^{\lfloor nx \rfloor} (X_i I\{|X_i| \leq tH^{-1}(d/n)\} - m_n(t)),$$

for $0 \leq x \leq 1$, $1/2 \leq t \leq 3/2$. First we show the tightness of $L_n(t, x)$. The proof is based on a generalization of [11]. We introduce

$$X_{i,1} = \max(X_i, 0), \quad X_{i,2} = \min(X_i, 0)$$

and

$$m_{n,1}(t) = EX_{0,1}I\{|X_0| \leq tH^{-1}(d/n)\}, \quad m_{n,2}(t) = EX_{0,2}I\{|X_0| \leq tH^{-1}(d/n)\}.$$

Similarly to $L_n(t, x)$, we define

$$L_{n,1}(t, x) = \frac{1}{A_n} \sum_{i=1}^{\lfloor nx \rfloor} (X_{i,1} I\{|X_i| \leq tH^{-1}(d/n)\} - m_{n,1}(t)),$$

and $L_{n,2}(t, x)$ is defined in a similar fashion. Clearly, if both $L_{n,1}$ and $L_{n,2}$ are tight, then $L_n(t, x)$ is tight as well. We prove only tightness of $L_{n,1}$, the same argument can be used in case of $L_{n,2}$. Let

$$g_n = \frac{1}{d^{1/2} \log \log n}.$$

Lemma 4.1. *If (1.1), (2.1)–(2.4), (2.7)–(2.9), (2.10) and (3.1) hold, then*

$$(4.1) \quad m_{n,1}(t) \text{ is a non-decreasing function on } [1/2, 3/2],$$

$$(4.2) \quad \frac{n}{A_n} \sup_{|t_2 - t_1| \leq g_n} |m_{n,1}(t_2) - m_{n,1}(t_1)| \rightarrow 0, \quad n \rightarrow \infty,$$

$$(4.3) \quad E|L_{n,1}(t_2, x) - L_{n,1}(t_1, x)|^6 \leq C_1 |t_2 - t_1|^\tau, \quad \text{if } |t_2 - t_1| \geq g_n,$$

and

$$(4.4) \quad E|L_{n,1}(t, x_2) - L_{n,1}(t, x_1)|^6 \leq C_1 |x_2 - x_1|^\tau, \quad \text{if } |x_2 - x_1| \geq g_n,$$

with some $\tau > 2$ and constant C_1 .

Proof. The definition of $m_{n,1}(t)$ implies immediately (4.1).

By the definition of $m_{n,1}(t)$ we have for all $1/2 \leq t_1 \leq t_2 \leq 3/2$ that

$$\begin{aligned}
0 &\leq m_{n,1}(t_2) - m_{n,1}(t_1) = EX_{0,1}(I\{t_1 H^{-1}(d/n) < |X_0| \leq t_2 H^{-1}(d/n)\}) \\
&\leq \int_{t_1 H^{-1}(d/n)}^{t_2 H^{-1}(d/n)} x dH(x) \\
&\leq C_2 \left(|t_2 H^{-1}(d/n) H(t_2 H^{-1}(d/n)) - t_1 H^{-1}(d/n) H(t_1 H^{-1}(d/n))| \right. \\
&\quad \left. + |t_2 - t_1| H^{-1}(d/n) H(t_1 H^{-1}(d/n)) \right) \\
&\leq C_3 |t_2 - t_1| \frac{d}{n} H^{-1}(d/n)
\end{aligned}$$

on account of integration by parts and (3.3), establishing (4.2).

Next we introduce

$$(4.5) \quad Y_i = \sum_{k=0}^{\lfloor K \log n \rfloor} \rho^k \varepsilon_{i-k} + c_1, \quad Y_{i,1} = \max(Y_i, 0)$$

and $\xi_i = \eta_i - E\eta_i$ with

$$\eta_i = \eta_i(t_1, t_2) = Y_{i,1} I\{t_1 H^{-1}(d/n) < |Y_i| \leq t_2 H^{-1}(d/n)\}.$$

Since $E|\varepsilon_0|^{\alpha/2} < \infty$, using Markov's inequality we see that for every $\beta > 0$ there is a constant $K = K(\beta)$ such that

$$(4.6) \quad \left| E(L_{n,1}(t_2, x) - L_{n,1}(t_1, x))^6 - \frac{1}{A_n^6} \sum_{1 \leq i_1, \dots, i_6 \leq \lfloor nx \rfloor} E\xi_{i_1} \dots \xi_{i_6} \right| \leq C_5 n^{-\beta}.$$

We note that by definition, $\{\xi_i\}$ is a stationary, $\lfloor K \log n \rfloor$ -dependent sequence with zero mean. Let us divide the indices i_1, \dots, i_6 into groups so that the difference between the indices within a group are less than $\lfloor K \log n \rfloor$ and between groups is larger than $\lfloor K \log n \rfloor$. Clearly $E\xi_{i_1} \dots \xi_{i_6} = 0$, if there is at least one group containing a single element. So it suffices to consider the cases when all groups contain at least two elements. This allows the cases of one single group with 6 elements (D_1), two groups with 3+3 (D_2) or 4+2 (D_3) elements and finally 3 groups with 2 elements in each (D_4). If there is only one group, then via Hölder's inequality we have

$$|E\xi_{i_1} \dots \xi_{i_6}| \leq E|\xi_0|^6 \leq 2^6 (E|\eta_0|^6 + |E\eta_0|^6)$$

Since the cardinality of D_1 is bounded by constant times $n(\log n)^5$ we conclude

$$\begin{aligned} & \left| \frac{1}{A_n^6} \sum_{D_1} E\xi_{i_1} \cdots \xi_{i_6} \right| \\ & \leq C_6 \left(\frac{n(\log n)^5}{A_n^6} [EX_0^6 I\{t_1 H^{-1}(d/n) \leq |X_0| \leq t_2 H^{-1}(d/n)\}] \right. \\ & \quad \left. + (EX_0 I\{t_1 H^{-1}(d/n) \leq |X_0| \leq t_2 H^{-1}(d/n)\})^6 + n^{-\beta} \right). \end{aligned}$$

Integration by parts and (3.3) yield

$$EX_0^6 I\{t_1 H^{-1}(d/n) \leq |X_0| \leq t_2 H^{-1}(d/n)\} \leq C_7 |t_2 - t_1| \frac{d}{n} (H^{-1}(d/n))^6,$$

resulting in

$$\left| \frac{1}{A_n^6} \sum_{D_1} E\xi_{i_1} \cdots \xi_{i_6} \right| \leq C_8 \left(\frac{(\log n)^5}{d^2} |t_2 - t_1| + n^{-\beta} \right).$$

Using again the $\lfloor K \log n \rfloor$ dependence of $\{\xi_i\}$ and the fact that the cardinality of D_2 is constant times $n^2(\log n)^4$ we conclude via Hölder's inequality

$$\begin{aligned} & \left| \frac{1}{A_n^6} \sum_{D_2} E\xi_{i_1} \cdots \xi_{i_6} \right| \\ & = \left| \frac{1}{A_n^6} \sum_{D_2} E\xi_{i_1} \xi_{i_2} \xi_{i_3} E\xi_{i_4} \xi_{i_5} \xi_{i_6} \right| \\ & \leq C_8 \left(\frac{n^2(\log n)^4}{A_n^6} [EX_0^3 I\{t_1 H^{-1}(d/n) \leq |X_0| \leq t_2 H^{-1}(d/n)\}] \right. \\ & \quad \left. + (EX_0 I\{t_1 H^{-1}(d/n) \leq |X_0| \leq t_2 H^{-1}(d/n)\})^3 \right]^2 + n^{-\beta} \\ & \leq C_9 \left(\frac{(\log n)^4}{d} (t_2 - t_1)^2 + n^{-\beta} \right). \end{aligned}$$

Similar arguments give

$$\left| \frac{1}{A_n^6} \sum_{D_3} E\xi_{i_1} \cdots \xi_{i_6} \right| \leq C_{10} \left(\frac{(\log n)^4}{d} (t_2 - t_1)^2 + n^{-\beta} \right).$$

Following the proof of Lemma 3.5 we obtain

$$\begin{aligned} \left| \frac{1}{A_n^6} \sum_{D_4} E\xi_{i_1} \cdots \xi_{i_6} \right| & \leq C_{11} \left(\frac{1}{A_n^6} \left(n \sum_{i=0}^{\infty} \xi_0 \xi_i \right)^3 + n^{-\beta} \right) \\ & \leq C_{11} (|t_2 - t_1|^3 + n^{-\beta}). \end{aligned}$$

Putting together our estimates and using the choice of g_n we conclude for all $|t_2 - t_1| \geq g_n$

$$\begin{aligned} E(L_{n,1}(t_2, x) - L_{n,1}(t_1, x))^6 &\leq C_{12} \left(\frac{(\log n)^5}{d^2} |t_2 - t_1| + \frac{(\log n)^4}{d} |t_2 - t_1|^2 + |t_2 - t_1|^3 + n^{-\beta} \right) \\ &\leq C_{13} |t_2 - t_1|^\tau \end{aligned}$$

with any $2 < \tau \leq 3$ on account of assumption (2.10). Hence the proof of (4.3) is complete. The proof of (4.4) goes along the lines of the arguments used to establish (4.3) and therefore it is omitted. \square

Lemma 4.2. *If (1.1), (2.1)–(2.4), (2.7), (2.9), (2.10) and (3.1) hold, then $L_n(t, x)$ is tight in $\mathcal{D}([1/2, 3/2] \times [0, 1])$.*

Proof. It follows from a minor modification of Lemma 6 in [11] that both $L_{n,1}$ and $L_{n,2}$ are tight. Since $L_n = L_{n,1} + L_{n,2}$, the result is proven. \square

Next we consider the convergence of the finite dimensional distributions. It is based in the following lemma:

Lemma 4.3. *We assume that (1.1), (2.1)–(2.4), (2.7), (2.9), (2.10) and (3.1) hold. Let $N = \lfloor (\log n)^\gamma \rfloor$ with some $\gamma > 0$. Then*

$$(4.7) \quad \begin{aligned} E \left(\sum_{i=1}^N (X_i I\{|X_i| \leq tH^{-1}(d/n)\}) - E[X_i I\{|X_i| \leq tH^{-1}(d/n)\}] \right)^4 \\ \leq C_{13} \left(N(\log N)^3 (H^{-1}(d/n))^4 \frac{d}{n} + N^2 (H^{-1}(d/n))^4 \left(\frac{d}{n} \right)^2 \right) \end{aligned}$$

with some constant C_{13} and

$$(4.8) \quad \begin{aligned} \lim_{n \rightarrow \infty} \frac{Nn}{A_n^2} E \left(\sum_{k=1}^N (u_{k,n}(s) - m_n(s)) \right) \left(\sum_{k=1}^N (u_{k,n}(t) - m_n(t)) \right) \\ = \frac{\alpha}{2 - \alpha} \left(s^{2-\alpha} + \sum_{k=1}^{\infty} \rho^k [(\min(s, t|\rho|^{-k})^{2-\alpha} + \min(t, s|\rho|^{-k})^{2-\alpha}] \right). \end{aligned}$$

Proof. We recall the definition of ξ_i from the proof of Lemma 4.1. For any $\beta > 0$, choosing K in the definition of Y_i in (4.5) we get that

$$E \left(\sum_{i=1}^N (X_i I\{|X_i| \leq tH^{-1}(d/n)\}) - E[X_i I\{|X_i| \leq tH^{-1}(d/n)\}] \right)^4 \leq C_{14} \left(E \left(\sum_{i=1}^N \xi_i \right)^4 + n^{-\beta} \right).$$

We write

$$E \left(\sum_{i=1}^N \xi_i \right)^4 = \sum_{i_1, \dots, i_4} E \xi_{i_1} \dots \xi_{i_4}.$$

We note again that the $\{\xi_i\}$ is a stationary $K \log n$ dependent sequence with 0 mean. Let us divide the indices i_1, \dots, i_4 into blocks so that the difference between the indices within a block is less than $K \log n$ and between blocks is larger than $K \log n$. Clearly $E\xi_{i_1} \dots \xi_{i_4} = 0$, if there is at least one block containing only a single element. So we need to consider the cases of one single block with 4 elements (D_1) and two blocks with 2+2 elements (D_2). The number of the elements in D_1 is not greater than constant times $N(\log N)^3$ and as we showed in the proof of Lemma 4.1

$$E\xi_0^4 \leq C_{14} \left((H^{-1}(d/n))^4 \frac{d}{n} + n^{-\beta} \right),$$

assuming that K in (4.5) is sufficiently large. Hence

$$\left| \sum_{D_1}^N E\xi_{i_1} \dots \xi_{i_4} \right| \leq C_{15} \left(N(\log N)^3 (H^{-1}(d/n))^4 \frac{d}{n} + n^{-\beta} \right).$$

As in the proof of Lemma 4.1 we get that

$$\left| \sum_{D_2}^N E\xi_{i_1} \dots \xi_{i_4} \right| \leq C_{16} N^2 \left(\sum_{i=0}^N |E\xi_0 \xi_i| \right)^2$$

and

$$\sum_{i=0}^{\infty} |E\xi_0 \xi_i| \leq \left(C_{17} (H^{-1}(d/n))^2 \frac{d}{n} + n^{-\beta} \right),$$

completing the proof of (4.7). The proof of (4.8) goes along the lines of the arguments used to establish Lemma 3.5. \square

Lemma 4.4. *If (1.1), (2.1)–(2.4), (2.7), (2.9), (2.10) and (3.1) hold, then*

$$L_n(t, x) \longrightarrow \Gamma(t, x) \text{ weakly in } \mathcal{D}([1/2, 3/2]) \times [0, 1],$$

where $\Gamma(t, x)$ is a Gaussian process with $E\Gamma(t, x) = 0$ and

$$\begin{aligned} & E\Gamma(t, x)\Gamma(s, y) \\ &= \min(x, y) \frac{\alpha}{2 - \alpha} \left((\min(s, t))^{2-\alpha} + \sum_{k=1}^{\infty} \rho^k [(\min(s, t|\rho|^{-k})^{2-\alpha} + \min(t, s|\rho|^{-k})^{2-\alpha}] \right). \end{aligned}$$

Proof. By Lemma 4.2, the process $L_n(t, x)$ is tight, so we need only to show the convergence of the finite dimensional distributions. By the Cramér–Wold device it is sufficient to prove the asymptotic normality of

$$Q_n = \sum_{j=1}^J \sum_{\ell=0}^L \mu_{j,\ell} (L_n(t_j, x_{\ell+1}) - L_n(t_j, x_\ell))$$

for all J, L , real coefficients $\mu_{j,\ell}$, $1/2 \leq t_j \leq 3/2$, $1 \leq j \leq J$, and $0 = x_0 < x_1 < \dots < x_L < x_{L+1} = 1$. We recall the definition of X_k^* from the proof of Lemma 3.5 (cf. (3.17)) and define

$$\bar{L}_n(t, x) = \frac{1}{A_n} \sum_{i=1}^{\lfloor nx \rfloor} (X_k^* I\{|X_k^*| \leq tH^{-1}(d/n)\} - EX_k^* I\{|X_k^*| \leq tH^{-1}(d/n)\}).$$

Choosing K large enough in the definition of X_k^* , we get from the arguments used in the proof of Lemmas 3.5, 4.1 and 4.3 that

$$E(L_n(t, x) - \bar{L}_n(t, x))^2 \rightarrow 0.$$

So we need to establish only the asymptotic normality of

$$\bar{Q}_n = \sum_{\ell=0}^L \sum_{j=1}^J \mu_{j,\ell} (\bar{L}_n(t_j, x_{\ell+1}) - \bar{L}_n(t_j, x_\ell)).$$

Let

$$z_{k,\ell} = \sum_{j=1}^J \mu_{j,\ell} (X_k^* I\{|X_k^*| \leq t_j H^{-1}(d/n)\} - E[X_k^* I\{|X_k^*| \leq t_j H^{-1}(d/n)\}]).$$

Since for all ℓ

$$E \left(\frac{1}{A_n} \sum_{k=1}^{\lfloor K \log n \rfloor} z_{k,\ell} \right)^2 \rightarrow 0,$$

by stationarity and the $\lfloor K \log n \rfloor$ -dependence of $z_{k,\ell}$ for any ℓ we get that the variables

$$\frac{1}{A_n} \sum_{k=\lfloor nx_\ell \rfloor + 1}^{\lfloor nx_{\ell+1} \rfloor} z_{k,\ell}, \quad 1 \leq \ell \leq L \text{ are asymptotically independent.}$$

By stationarity we have

$$\frac{1}{A_n} \sum_{k=\lfloor nx_\ell \rfloor + 1}^{\lfloor nx_{\ell+1} \rfloor} z_{k,\ell} \stackrel{\mathcal{D}}{=} \frac{1}{A_n} \sum_{k=1}^{\lfloor nx_{\ell+1} \rfloor - \lfloor nx_\ell \rfloor} z_{k,\ell}.$$

Let us divide the integers of $[1, \lfloor nx_{\ell+1} \rfloor - \lfloor nx_\ell \rfloor]$ into consecutive blocks $R_1, V_1, R_2, V_2, \dots, R_s, V_s$ such that for $1 \leq i \leq s-1$, R_i contains $\lfloor (\log n)^\gamma \rfloor$ integers, V_i contains $\lfloor K \log n \rfloor$ integers, the last two blocks might contain less elements. Let

$$\zeta_{i,1} = \sum_{k \in R_i} z_{k,\ell} \quad \text{and} \quad \zeta_{i,2} = \sum_{k \in V_i} z_{k,\ell}.$$

Due to the $\lfloor K \log n \rfloor$ dependence and stationarity, the variables $\zeta_{i,2}$, $1 \leq i < s$ are independent and identically distributed and the proof of Lemma 3.5 shows that

$$E \left(\frac{1}{A_n} \sum_{i=1}^s \zeta_{i,2} \right)^2 \rightarrow 0.$$

Using Lemma 4.3 we get that

$$E\zeta_{i,1}^2 \geq C_{18}(\log n)^\gamma (H^{-1}(d/n))^2 d/n$$

and

$$E\zeta_{i,1}^2 \leq C_{19} \left((\log n)^\gamma (\log \log n)^3 (H^{-1}(d/n))^4 \frac{d}{n} + (\log n)^{2\gamma} (H^{-1}(d/n))^4 \left(\frac{d}{n} \right)^2 \right).$$

Since s is proportional to $n/(\log n)^\gamma$, a simple calculation yields

$$\frac{\sum_{i=1}^s E\zeta_{i,1}^4}{\left(\sum_{i=1}^s E\zeta_{i,1}^2 \right)^2} \rightarrow 0,$$

Thus the central limit theorem with Lyapunov's remainder term (cf. [45], p. 154) implies the asymptotic normality of

$\sum_{1 \leq k \leq \lfloor nx_{\ell+1} \rfloor - \lfloor nx_\ell \rfloor} z_{k,\ell}$. This completes the proof of Lemma 4.4. \square

5. Proof of Theorems 2.1 and 2.2

We need the weak law of large numbers for $\eta_{d,n}$.

Lemma 5.1. *If (1.1), (2.1)–(2.4), (2.7), (2.9)–(2.11) and (3.1) hold, then we have*

$$\frac{\eta_{d,n}}{H^{-1}(d/n)} \xrightarrow{P} 1$$

Proof. Using Gorodetskii [30] and Withers [50] we get that X_k is a strongly mixing stationary sequence with mixing rate $\alpha(k) \leq C_1 \exp(-\lambda k)$ for some $C_1 > 0$ and $\lambda > 0$. Fix $1/2 < t < 2$ and let $T_k = I\{|X_k| \geq tH^{-1}(d/n)\}$, $1 \leq k \leq n$. Clearly, $ET_k = P\{|X_k| \geq tH^{-1}(d/n)\} = H(tH^{-1}(d/n))$ and due to the regular variation of H , $ET_k/(d/n) \rightarrow t^{-\alpha}$, as $n \rightarrow \infty$. On the other hand, by the correlation inequality of Davydov [26] we get for any $p > 2$ that

$$\begin{aligned} |ET_0 T_k - ET_0 ET_k| &\leq (\alpha(k))^{(p-1)/p} (ET_0^p)^{1/p} (ET_k^p)^{1/p} \\ &\leq C_1 \exp(-\lambda k(p-1)/p) (ET_0^p)^{2/p} \\ &= C_1 \exp(-\lambda k(p-1)/p) (ET_0)^{2/p} \\ &\leq C_2 \exp(-\lambda k(p-1)/p) (d/n)^{2/p}. \end{aligned}$$

Hence setting $\bar{T}_k = T_k - ET_k$ we conclude that

$$E \left(\sum_{k=1}^n \bar{T}_k \right)^2 = nE\bar{T}_0^2 + 2 \sum_{k=1}^{n-1} (n-k) E\bar{T}_0 \bar{T}_k$$

$$\begin{aligned}
&\leq n \left(E\bar{T}_k^2 + 2 \sum_{k=1}^{n-1} |E\bar{T}_0\bar{T}_k| \right) \\
&\leq n \left(ET_0^2 + C_3 \sum_{k=1}^{n-1} \exp(-\lambda k(p-1)/p)(d/n)^{2/p} \right) \\
&\leq n (ET_0 + C_5(d/n)^{2/p}) \\
&\leq n(d/n)^{2/p}.
\end{aligned}$$

Thus by Markov's inequality we have that

$$P \left\{ \sum_{k=1}^n \bar{T}_k \geq d^{2/p} \right\} \leq C_6 n^{(p-2)/p} / d^{2/p} \rightarrow 0,$$

provided that $d/n^{(2-p)/p} \rightarrow 0$. Since $d \geq n^\delta$, choosing p near 2, it follows that

$$\sum_{k=1}^n T_k = t^{-\alpha} d(1 + o_P(1)) + o_P(d^{2/p}) = t^{-\alpha} d(1 + o_P(1)).$$

In other words,

$$\frac{1}{d} \#\{k \leq n : |X_k| \geq tH^{-1}(d/n)\} \xrightarrow{P} t^{-\alpha}, \quad \text{as } n \rightarrow \infty.$$

This shows that

$$\lim_{n \rightarrow \infty} P\{\eta_{n,d} \geq tH^{-1}(d/n)\} = 1 \quad \text{for } t < 1$$

and

$$\lim_{n \rightarrow \infty} P\{\eta_{n,d} \geq tH^{-1}(d/n)\} = 0 \quad \text{for } t > 1,$$

completing the proof of Lemma 5.1. \square

Proof of Theorem 2.1. We note that $\Gamma(t, x)$ is a continuous process. Hence combining Lemmas 4.4 and 5.1 we conclude

$$L_n(\eta_{d,n}/H^{-1}(d/n), x) \xrightarrow{\mathcal{D}^{[0,1]}} \Gamma(1, x).$$

It is easy to see that

$$\{\Gamma(1, x), 0 \leq x \leq 1\} \stackrel{\mathcal{D}}{=} \left\{ \left(\frac{\alpha}{2-\alpha} \frac{1+\rho}{1-\rho} \right)^{1/2} W(x), 0 \leq x \leq 1 \right\},$$

where $W(x)$ is a Wiener process, which completes the proof. \square

Proof of Theorem 2.2. Since

$$\frac{1}{A_n} T_{n,d}(x) = L_n(\eta_{d,n}/H^{-1}(d/n), x) - \frac{\lfloor nx \rfloor}{n} L_n(\eta_{d,n}/H^{-1}(d/n), 1),$$

Theorem 2.1 yields

$$\frac{1}{A_n} T_{n,d}(x) \xrightarrow{\mathcal{D}[0,1]} \left(\frac{\alpha}{2-\alpha} \frac{1+\rho}{1-\rho} \right)^{1/2} (W(x) - xW(1)).$$

By definition, $B(x) = W(x) - xW(1)$, $0 \leq x \leq 1$ is a Brownian bridge, so the proof of Theorem 2.2 is complete. \square

REFERENCES

- [1] Andrews, B., Calder, M. and Davis, R.A.: Maximum likelihood estimation for α -stable autoregressive processes. *Ann. Stat.* **37**(2009), 1946–1982.
- [2] Arov, D.Z. and Bobrov, A.A.: The extreme terms of a sample and their role in the sum of independent variables. *Theor. Probab. Appl.* **5**(1960), 377–396,
- [3] Aue, A., Berkes, I. and Horváth, L.: Selection from a stable box. *Bernoulli* **14**(2008), 125–139.
- [4] Aue, A. and Horváth, L.: A limit theorem for mildly explosive autoregression with stable errors. *Economet. Theor.* **23**(2007), 201–220.
- [5] Aue, A. and Horváth, L.: Structural breaks in time series. *J. Time Ser. Anal.* **34**(2013), 1–16.
- [6] Avram, F.S. and Taqqu, M.S.: Weak convergence of moving averages with infinite variance. In: *Dependence in Probability and Statistics: A Survey of Recent Results* (E. Eberlain and M.S. Taqqu, eds.), 399–425, Birkhäuser, Boston, 1986.
- [7] Avram, F.S. and Taqqu, M.S.: Weak convergence of moving averages in the α -stable domain of attraction. *Ann. Probab.* **20**(1992), 483–503.
- [8] Basrak, B., Krizmanić, D. and Segers, J.: A functional limit theorem for partial sums of dependent random variables with infinite variance. *Ann. Probab.* **40**(2012), 2008–2033.
- [9] Berkes, I. and Horváth, L.: The central limit theorem for sums of trimmed variables with heavy tails. *Stoch. Proc. Appl.* **122**(2012), 449–465.
- [10] Berkes, I., Horváth, L. and Schauer, J.: Asymptotics trimmed CUSUM statistics. *Bernoulli* **17**(2011), 1344–1367.
- [11] Berkes, I., Horváth, L., Ling, S. and Schauer, J.: Testing for structural change of AR model to threshold AR model. *J. Time Ser. Anal.* **32**(2011), 547–565.
- [12] Billingsley, P.: *Convergence of Probability Measures*. Wiley, 1968.
- [13] Bingham, N.H., Goldie, C.M. and Teugels, J.L.: *Regular Variation*. Cambridge University Press, 1987.
- [14] Chan, N.H.: Inference for near-integrated time series with infinite variance. *J. Amer. Stat. Assoc.* **85**(1990), 1069–1074.
- [15] Chan, N.H. and Tran, L.T.: On the first-order autoregressive processes with infinite variance. *Economet. Theor.* **5**(1989), 354–362.
- [16] Cline, D.: *Infinite Series of Random Variables with Regularly Varying Tails*. Technical report 83-24. The Institute of Applied Mathematics and Statistics, The University of British Columbia, Vancouver, Canada, 1983.
- [17] Csörgő, M., Csörgő, S., Horváth, L. and Mason, D.M: Weighted empirical and quantile processes. *Ann. Probab.* **14**(1986), 31–85.
- [18] Csörgő, M., Csörgő, S., Horváth, L. and Mason, D.M: Normal and stable convergence of convergence of integral functions of the empirical distribution function. *Ann. Probab.* **14**(1986), 86–118.
- [19] Csörgő, S., Haeusler, E. and Mason, D.M: The quantile-transform approach to the asymptotic distribution of modulus trimmed sums. In: *Sums, trimmed sums and extremes*, 337–353, Progress in Probability **23**, Birkhäuser, Boston, MA, 1991.

- [20] Csörgő, S., Horváth, L. and Mason, D.M: What portion of the sample makes a partial sum asymptotically stable or normal? *Probab. Theory Rel. Fields* **72**(1986), 1–16.
- [21] Csörgő, M. and Horváth, L.: *Limit Theorems in Change-Point Analysis*. Wiley, 1998.
- [22] Darling, D.A.: The influence of the maximum term in the addition of independent random variables. *Trans. Amer. Math. Soc.* **73**(1952), 95–107.
- [23] Davis, R.A. and Mikosch, T.: The sample autocorrelations of heavy-tailed processes with applications to ARCH. *Ann. Stat.* **26**(1998), 2049–2080.
- [24] Davis, R. and Resnick, S.: Limit theory for moving averages of random variables with regularly varying tail probabilities. *Ann. Probab.* **13**(1985), 179–195.
- [25] Davis, R. and Resnick, S.: Limit theory for the sample covariance and correlation functions of moving averages. *Ann. Stat.* **14**(1986), 533–558.
- [26] Davydov, Y.A.: Convergence of distributions generated by stationary stochastic processes. *Theor. Probab. Appl.* **13**(1968), 691–696.
- [27] de Acosta, A. and Giné, E.: Convergence of moments and related functionals in the general central limit theorem in Banach spaces. *Z. Wahrsch. verw. Geb.* **48**(1979), 213–231.
- [28] Fama, E.: The behavior of stock market price. *J. Bus.* **38**(1965), 34–105
- [29] Gikhman, I. and Skorohod, A. V.: *Introduction to the Theory of Random Processes*. Saunders, Philadelphia.
- [30] Gorodetskii, V.V.: On the strong mixing property for linear sequences. *Theor. Probab. Appl.* **22**(1977), 411–413.
- [31] Griffin, P. and Pruitt, W.E.: The central limit problem for trimmed sums. *Math. Proc. Cambridge Phil. Soc.* **102**(1987), 329–349.
- [32] Griffin, P. and Pruitt, W.E.: Asymptotic normality and subsequential limits of trimmed sums. *Ann. Probab.* **17**(1989), 1186–1219.
- [33] Hall, P.: On the extreme terms of a sample from the domain of attraction of a stable law. *J. London Math. Soc.* **18**(1978), 181–191.
- [34] Horváth, L. and Reeder, R.: Detecting changes in functional linear models. *J. Multivariate Anal.* **111**(2012), 310–334.
- [35] Kesten, H.: Convergence in distribution of lightly trimmed and untrimmed sums are equivalent. *Math. Proc. Cambridge Phil. Soc.* **113**(1993), 615–638.
- [36] Kulik, R. and Ould Haye, M.: Trimmed sums of long range dependent moving averages. *Stat. Probab. Letters* **78**(2008), 2536–2542.
- [37] Lepage, R., Woodroffe, M. and Zinn, J.: Convergence to a stable distribution via order statistics. *Ann. Probab.* **9**(1981), 624–632.
- [38] Lévy, P.: Propriétés asymptotiques des sommes de variables aléatoires indépendentes en enchaines. *J. de Mathématiques* **14** (1935) 347–402.
- [39] Liu, W. and Wu, W.B.: Asymptotics of spectral density estimates. *Economet. Theor.* **26**(2010), 1218–1245.
- [40] Mandelbrot, B.B.: The variation of certain speculative price. *J. Bus.* **36**(1963), 394–419.
- [41] Mandelbrot, B.B.: The variation of other speculative prices. *J. Bus.* **40**(1967), 393–413.
- [42] Mikosch, T., Resnick, S. and Samorodnitsky, G.: The maximum of the periodogram for a heavy-tailed sequence. *Ann. Probab.* **28**(2000), 885–908.
- [43] Mori, T.: on the limit distributions of lightly trimmed sums. *Math. Proc. Cambridge Phil. Soc.* **96**(1984), 507–516.
- [44] Phillips, P.C.B. and Solo, V.: Asymptotics for linear processes. *Ann. Stat.* **20**(1992), 971–1001.
- [45] Petrov, V.V.: *Limit Theorems of Probability Theory*. Clarendon Press, Oxford.
- [46] Resnick, S. and Greenwood, P.: A bivariate stable characterization and domains of attraction. *J. Multivariate Anal.* **9**(1979), 206–221.

- [47] Skorokhod, A.V.: Limit theorems for stochastic processes. *Theor. Probab. Appl.* **1**(1956), 261–290.
- [48] Teugels, J.L.: Limit theorems on order statistics. *Ann. Probab.* **9**(1981), 868–880.
- [49] Tyran–Kamińska, M.: Functional limit theorems for linear processes in the domain of attraction of stable laws. *Stat. Probab. Letters* **80**(2010), 975–981.
- [50] Withers, C.S.: Conditions for linear processes to be strong–mixing. *Z. Wahrsch. verw. Geb.* **57**(1981), 477–480.
- [51] Zhang, R–M. and Chan, N.H.: Maximum likelihood estimation for nearly non–stationary stable autoregressive processes. *J. Time Ser. Anal.* **33**(2012), 542–553.

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