DOCTORAL PROGRAM
DISCRETE MATHEMATICS
TU & KFU GRAZ • MU LEOBEN
AUSTRIA

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DK: Doctoral Program in Discrete Mathematics

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Graz, June 2012

Branching Random Walks and Related Topics

July 9-11, 2012

Lecture hall **C307**, Steyrgasse 30/III, TU Graz

Monday, July 9, 2012

$10^{00} - 10^{45}$	Talk by Maura Salvatori (Milan) *Harmonic functions and Brownian motion on* $\mathrm{Sol}(p, q)$
$10^{45} - 11^{00}$	**Coffee and tea break**
$14^{00} - 16^{00}$	Ph.D Defence: Elisabetta Candellero (TU Graz) *Limit behaviors for random walks and branching random walks on some products of groups*

Tuesday, July 10, 2012

$09^{30} - 10^{15}$	Talk by Fabio Zucca (Milan) *Can frogs survive locally?*
$10^{15} - 10^{30}$	**Coffee and tea break**
$10^{30} - 11^{15}$	Talk by Steve Lalley (Chicago) *Branching Brownian Motion and Branching Random Walk in Dimension 1: Supercritical and Critical*
$14^{30} - 15^{15}$	Talk by Daniela Bertacchi (Milan) *Contact and voter processes on the infinite percolation cluster*
$15^{15} - 15^{30}$	**Coffee and tea break**
$15^{30} - 16^{15}$	Talk by Sebastian Müller (Marseille) *Branching random walks on groups - some soft proofs*

Wednesday, July 11, 2012

$09^{30} - 10^{15}$	Talk by Francois Ledrappier (Paris) *Regularity of the entropy for some random walks*
$10^{15} - 10^{30}$	**Coffee and tea break**
$10^{30} - 11^{15}$	Talk by Konrad Kolesko (Wroclaw) *Inhomogeneous smoothing transform in the critical case*

Abstracts of the talks

- **Maura Salvatori, Milan**

  Title: Harmonic functions and Brownian motion on $\mathrm{Sol}(p,q)$

  Abstract: The Lie group $\mathrm{Sol}(p,q)$ is the semidirect product induced by the action of $\mathbb{R}$ on $\mathbb{R}^2$ which is given by $(x,y) \to \big(\mathrm{e}^{pz}x, \mathrm{e}^{-qz}y\big)$, $z \in \mathbb{R}$. It can be also described as the horocyclic product of two hyperbolic planes with curvatures $-p^2$ and $-q^2$, respectively. Viewing $\mathrm{Sol}(p,q)$ as a 3-dimensional manifold, it carries a natural Riemannian metric and Laplace-Beltrami operator. We add a linear drift term in the vertical variable, and study the associated Brownian motion with drift. We obtain a central limit theorem and compute the rate of escape. We also explain how Brownian motion converges almost surely to the boundary of the natural geometric compactification of $\mathrm{Sol}(p,q)$. Moreover, we study all positive harmonic functions for the Laplacian with drift, and determine explicitly all minimal harmonic functions.

  This talk is based on a joint work with S. Brofferio and W. Woess.

- **Elisabetta Candellero, Graz**

  Title: Limit behaviors for random walks and branching random walks on some products of groups

  Abstract:

- **Fabio Zucca, Milan**

  Title: Can frogs survive locally?

  Abstract: A frog model is an interacting random walk system on $\mathbb{Z}$ where at time 0 there is an active particle at 0 and one inactive particle on each site $n \geq 1$. Particles become active when hit by another active particle. Once activated they perform an asymmetric nearest neighbor random walk which depends only on the starting location of the particle. We give conditions for global survival, local survival and infinite activation both in the case where all particles are immortal and in the case where particles have geometrically distributed lifespan (with a parameter depending on the starting location of the particle).

  This is a joint work with Daniela Bertacchi and Fábio Prates Machado.

- **Steve Lalley, Chicago**

  Title: Branching Brownian Motion and Branching Random Walk in Dimension 1: Supercritical and Critical

  Abstract: In a branching Brownian motion, individual particles execute Brownian motions and independently, following exponentially distributed gestation times, reproduce according to the law of a Galton-Watson process. Of interest is the location R_t of the rightmost particle at time t, and, in the critical case (when the mean number of offspring per particle is 1) the furthest displacement $M := \sup_t R_t$ from the origin. I will discuss some recent and some not-so-recent development in the study of these objects, both in the critical and supercritical cases.

  (A) In the simplest supercritical case, the reproduction law is *simple binary fission*: each particle produces two offspring. McKean (1975) showed that in this case the cumulative distribution function of R_t satisfies the *Fisher-KPP* equation, and therefore the Kolmogorov-Piscunov-Petrovski theorem implies that

(1) $$\lim_{t\to\infty} P\{R_t - m_t \leq x\} = w(x),$$

  where m_t is the median of the distribution of R_t and $w(x)$ is the steady-state wave solution of the Fisher equation. Bramson (1978) showed that $m_t = \sqrt{2}t - (3/2\sqrt{2})\log t + O(1)$, and Lalley and Sellke (1987) showed that the steady-state wave solution $w(x)$ is a mixture of Gumbel (extreme value) distributions. Lalley and Sellke's arguments led them to conjecture that the point process of particle locations $\mathcal{P}_t$, centered at R_t, should converge weakly to a limit point process. This conjecture has recently been proved by Arguin, Bovier, and Kistler.

  (B) In the critical case, the branching process dies out with probability one, and hence the maximal displacement M is finite. Fleischman and Sawyer (1979) proved that for the $0/2$ offspring distribution (each particle produces either dies or fissions, each with probability 1/2),

(2) $$P\{M \geq x\} \equiv C/x^2 \quad \text{as } x \to \infty,$$

by exploiting the fact that the distribution function of M satisfies a simple second-order ODE. It is natural to conjecture that a similar result should hold for critical branching random walk on $\mathbb{Z}$. Together with Yuan Shao I have recently proven this conjecture: precisely, if the step distribution of the random walk has mean zero and finite $(4+\varepsilon)$th moment, and if the offspring distribution has finite third moment, then the asymptotic relation (2) remains valid, for a positive constant C depending on only the second moments of the step and offspring distributions.

- **Daniela Bertacchi, Milan**
  Title: Contact and voter processes on the infinite percolation cluster
  Abstract: We study two stochastic processes on the infinite percolation cluster $\mathcal{C}_\infty$ of $\mathbb{Z}^d$ $(d \geq 3)$. The first one is a contact process which admits at most N particles per site and where the more particles are already present at a site, the more difficult it becomes to add particles on that site. This process is a model for reproductions in a patchy habitat, and its behaviour depends on both the intra-patch reproduction rate α and the inter-patch reproduction rate β. We provide results on survival and extinction of the population. The second process is a voter model where there are N particles per site, chosen among two species. Two scenarios are possible as time tends to ∞: either only one species colonizes the whole space (clustering) or the two species both persist (coexistence). We prove that clustering occurs iff random walks on $\mathcal{C}_\infty$ have the infinite collision property, which is true if $d=2$ and false if $d \geq 3$.
  This is based on joint work with N.Lanchier and F.Zucca.

- **Sebastian Müller, Marseille**
  Title: Branching random walks on groups - some soft proofs
  Abstract: We give an introduction on Branching random walks (BRW) on groups and present some natural qualitative questions. In particular, we will stress on connections between percolation theory and BRW.

- **Francois Ledrappier, Paris**
  Title: Regularity of the entropy for some random walks
  Abstract: We consider random walks with finite support on a Gromov hyperbolic group. We let the directing measure change with a constant support. Then, the entropy depends Lipschitz on the measure.

- **Konrad Kolesko, Wroclaw**
  Title: Inhomogeneous smoothing transform in the critical case
  Abstract: We study fixed points of an inhomogeneous smoothing transform, i.e. solutions of the equation
  $$X \stackrel{d}{=} C + \sum_{i=1}^{N} T_i X_i,$$
  where $(C, T_1, \ldots, T_N)$ is a given sequence of non-negative random variables and $X_1, \ldots, X_N$ independent copies of X. Assuming that there exists a positive α such that $\sum_i T_i^\alpha = 1$, and $\sum_i T_i^\alpha \log T_i = 0$ (the critical case) we describe the tail of X.