EIGENVALUES OF THE HOMOGENEOUS TREE AND THE DISCRETE WEYL THEOREM

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Abstract. This is joint work with Flavia Colonna and David Singman. We look at the homogeneous tree with two different geometric structures: the hyperbolic and the Euclidean. The hyperbolic structure is where the tree is unbounded. It corresponds to the hyperbolic 1-ball. In both cases, we are looking for eigenfunctions which vanish on a ball of finite distance from the origin. But in the hyperbolic case, a ball of finite radius is finite, whereas in the Euclidean case, we are looking for eigenfunctions that vanish at the boundary, which is a finite distance from the origin.

In the hyperbolic case, once we find an eigenvalue whose eigenfunctions vanish on the sphere of radius N, we can extend the eigenfunction to the whoile tree. It wll then have the property that for some number K, the function will vanish on all spheres of radius $N + mK$. These functions can be radial or non-radial.

With the Euclidean metric the tree is bounded, that is, the boundary is a finite distance from the origin. and so the tree plus boundary becomes a compact metric space.

The classical Weyl's theorem on the eigenvalues of the Laplacian regards a bounded manifold embedded in Euclidean n-space. It says that the number of eigenvalues of the Laplacian whose eigenfunctions vanish at the boundary satisfy the following property: If $E(x)$ is the number of such eigenvalues less than x, then there is a constant A, depending on n and the volume of the manifold, such that $E(x) = Ax^{n/2} + o(x)$. For the Euclidean tree of homogeneity $q + 1$, let $E(x)$ be the number of positive eigenvalues of eigenfunctions that vanish at the bounday and whose *logarithm* to the base q^2 is less than x, then $E(x) = x+O(1)$, in fact it seems likely that for an integer n, $E(n) = n-1$. The proof is not complete, but we have a partial proof and great many numerical verifications.

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