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title:

The Dirichlet problem, operator algebras and a Keldych-type theorem.

abstract:

Let U be a bounded open subset of the n-dimensional Euclidean space. A continuous function on the boundary of U is said to be regular if it encodes the unrestricted boundary values of a function harmonic on U. It is well known that in general not all the continuous functions on the boundary are regular. This fact prompted the search for an appropriate definition of "generalized solutions" of the Dirichlet problem. An answer to this question is given by the Perron-Wiener-Brelot (PWB) generalized solution which yelds a positive linear operator from the algebra of continuous functions on the boundary of U to the Banach space of bounded harmonic functions on U, which maps every regular function to the corresponding classical solution. A remarkable and important theorem by Keldych shows that the PWB operator can be characterized as the unique linear operator with the above properties. The known proofs of this result are rather subtle and rely on the deep notion of capacity. In this talk---a joint work with Sebastiano Carpi---we explain some recent results on a new version of the Keldych theorem which admits a much simpler proof. The central idea comes from the observation that the PWB operator has a stronger positivity property which we call harmonic positivity. Powerful methods from the theory of injective operators systems can be brought to bear on this topic. This approach naturally extends to cover much more general situations in both commutative and non-commutative settings.