Wolfgang Woess (Graz Univ. of Technology). Boundary representations of λ -harmonic and polyharmonic functions on trees

On a countable tree T, allowing vertices with infinite degree, we consider an arbitrary stochastic irreducible nearest neighbour transition operator P. We provide a boundary integral representation for general eigenfunctions of P with eigenvalue $\lambda \in \mathbb{C}$. This is possible whenever λ is in the resolvent set of P as a self-adjoint operator on a suitable ℓ^2 -space and the on-diagonal elements of the resolvent ("Green function") do not vanish at λ . We show that when P is invariant under a transitive group action, the latter condition holds for all $\lambda \neq 0$ in the resolvent set. These results extend and complete previous results by Cartier, by Figa-Talamanca and Steger, and by Woess. Furthermore, for those eigenvalues. we provide an integral representation of λ -polyharmonic functions of any order n, that is, functions $f: T \to \mathbb{C}$ for which $(\lambda \cdot I - P)^n f = 0$. This is a far-reaching extension of work of Cohen et al.

We can also provide an analogous result for polyharmonic functions on the unit disk with respect to the hyperbolic Laplacian, based on old results of Helgason.

This is joint work with Massimo Picardello (Rome).