LEOBEN-LJUBLJANA GRAPH THEORY SEMINAR 2016

Dedicated to Norbert Seifter's 60th Birthday

Special session on the occasion of Bojan Mohar's 60th Birthday

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Titles and abstracts

Vladimir Batagelj (Ljubljana - Koper): *Widespread index*

Abstract. On January 12, 2016 Clement Levallois posted at SocNet a message asking for a measure of how widespread is the distribution of a node attribute in a network. In the paper we propose and study two indices for measuring the widespread of a node attribute value in a network: the (simple) widespread index and the domination index. A computation of the proposed indices using the program Pajek is illustrated on two networks: the Class network and the US Airports network.

Chris Godsil (Waterloo): *Graph isomorphism and quantum walks*

Abstract. It has been suggested that graph isomorphism may be one of the problems that can be solved faster on a quantum computer than on a clasical computer. Consequently physicists have devoted some effort to find algorithms for graph isomorphism. I will report on work undertaken by me and my colleagues, using tools from graph spectral theory and finite geometry, to show that the algorithms proposed to date do not work.

Veronica Hernández Martinez (Madrid): *On the diameter, minimum and maximum degree and hyperbolicity constant*

Abstract. In this work, we obtain good upper bounds for the diameter of any graph in terms of its minimum degree and its order, improving a classical theorem due to Erdös, Pach, Pollack and Tuza. We use these bounds in order to study hyperbolic graphs (in the Gromov sense). If X is a geodesic metric space and $x_1, x_2, x_3 \in X$, a geodesic triangle $T = \{x_1, x_2, x_3\}$ is the union of the three geodesics $[x_1x_2]$, $[x_2x_3]$ and $[x_3x_1]$ in X. The space X is δ-hyperbolic in the Gromov sense if any side of T is contained in a δ-neighborhood of the union of the two other sides, for every geodesic triangle T in X. If X is hyperbolic, we denote by $\delta(X)$ the sharp hyperbolicity constant of X, i.e.

$\delta(X) = \inf\{\delta \geq 0 : \quad X$ is δ-hyperbolic$\}$. To compute the hyperbolicity constant is an almost intractable problem, thus it is natural to try to bound it in terms of some parameters of the graph. Let $\mathcal{H}(n, \delta_0)$ be the set of graphs G with n vertices and minimum degree δ_0, and $\mathcal{J}(n, \Delta)$ be the set of graphs G with n vertices and maximum degree Δ. We study the four following extremal problems on graphs: $a(n, \delta_0) := \min\{\delta(G) \mid G \in \mathcal{H}(n, \delta_0)\}$, $b(n, \delta_0) := \max\{\delta(G) \mid G \in \mathcal{H}(n, \delta_0)\}$, $\alpha(n, \Delta) := \min\{\delta(G) \mid G \in \mathcal{J}(n, \Delta)\}$ and $\beta(n, \Delta) := \max\{\delta(G) \mid G \in \mathcal{J}(n, \Delta)\}$. In particular, we obtain bounds for $b(n, \delta_0)$ and we compute the precise value of $a(n, \delta_0)$, $\alpha(n, \Delta)$ and $\beta(n, \Delta)$ for all values of n, δ_0 and Δ, respectively.

Wilfried Imrich (Leoben): *On the infinite motion conjecture*

Abstract. A graph G is called k-distinguishable if there exists a k-coloring of its set of vertices that is only preserved by the identity automorphism. The smallest k for which G is k-distinguishable is the distinguishing number $D(G)$. The definition is due to Mike Albertson (1996) and gave rise to a wealth of papers, although automorphism breaking was not new at that time.
For infinite graphs an outstanding problem in symmetry breaking is the Infinite Motion Conjecture by Tom Tucker (2007). It conjectures that any connected, infinite, locally finite graph G is 2-distinguishable if all non-trivial automorphism of G move infinitely many vertices.
Despite many deep results towards its solution the conjecture is still open. This talk presents a summary of new results and new classes of graphs for which the conjecture holds. For some of them the motion requirement is relaxed.

Aleksandar Jurišić (Ljubljana): *Flowers at pentagon and Moore graphs*

Abstract. Combinatorial structures of two famous graphs with large girth, namely the Coxeter graph (28 vertices) and the Sylvester graph (36 vertices), are studied. Simple techniques, such as two-way counting, partitions, circuit chasing and covers are used to identify smaller structures (such as pentagons and a double cover of the cube) and to show that there are no other graphs that share a small number of regularity properties with them. We notice that some other graphs with a few hundred vertices (and some with possibly even a few thousands vertices) and large girth have similar properties, and could be studied using the above mentioned techniques. In particular, we show that just as the Hoffman-Singleton graph contains the Sylvester graph, a Moore graph of valency 57, whose existence is a famous open problem, must contain a subgraph with a structure that is similar to the one we derived for the Sylvester graph.
This is joint work with Janos Vidali.

Christoph Koch (Graz): *Jigsaw percolation on random hypergraphs*

Abstract. Jigsaw percolation was introduced by Brummit, Chatterjee, Dey, and Sivakoff as a model for interactions within a social network. It was inspired by the idea of collectively solving a puzzle. The premise is that each individual of a group of people has a

piece of a puzzle all of which must be combined in a certain way to solve the puzzle. The compatibility of different puzzle pieces and the information which pairs of people meet are stored in two graphs (on a common vertex set), the puzzle graph and the people graph. Bollobs, Riordan, Slivken, and Smith studied the process when both graphs are given by independent binomial random graphs. More abstractly the process can be seen as a notion of simultaneous connectedness of a pair of (random) graphs. We transfer the process to hypergraphs in the context of high-order connectedness. We provide the asymptotic order of the critical threshold probability for percolation when both hypergraphs are chosen binomially at random, extending the result of Bollob'as, Riordan, Slivken, and Smith. The evolution of the process is closely related to the presence of traversable triples in the pair of random hypergraphs.
This is joint work with Béla Bollobás, Oliver Cooley, and Mihyun Kang.

Jurij Kovič (Ljubljana-Koper): *Energy of geometrical structures*

Abstract. We introduce the concept of the energy of a geometrical structure, a parameter depending on the angles between its adjacent arcs.
This parameter may be used for the comparison of various solids, tilings, configurations, graphs realised with straight edges and other structures with angles.
We obtain some upper and lower bounds for the average vertex energy of various classes of geometrical structures.

Arnold R. Kräuter (Leoben): *Permanents of matrices of signed ones*

Abstract. The permanent of an n-square matrix $A = [a_{ij}]$ is defined by

$$\mathrm{per}(A) = \sum_{\sigma \in S_n} a_{1,\sigma(1)} \cdot \ldots \cdot a_{n,\sigma(n)},$$

where S_n denotes the set of all permutations of the integers $1, \ldots, n$.
Due to their various applications in different fields, permanents of (0,1)-matrices and, more general, nonnegative matrices matrices have been studied extensively. Although of combinatorial interest, too, permanents of matrices of signed ones need more subtle investigations.
The talk gives a comprehensive survey on more or less recent research results in this topic. They all have in common that they are based on and/or motivated by the papers [1] and [2] published three decades ago.

[1] A. R. Kräuter and N. Seifter, On some questions concerning permanents of $(1, -1)$-matrices, *Israel Journal of Mathematics* **45**, 53 - 62 (1983).

[2] A. R. Kräuter and N. Seifter, Some properties of the permanent of $(1, -1)$-matrices, *Linear and Multilinear Algebra* **15**, 207 - 223 (1984).

Florian Lehner (Hamburg): *The reconstruction problem for infinite graphs*

Abstract. An important open question in the theory of finite graphs is whether it is possible to reconstruct any large enough finite graph from the family of subgraphs which

can be obtained by removing individual vertices. The same problems for various classes of infinite graphs, such as trees or locally finite connected infinite graphs, has also remained open for the last few decades. We resolve these questions about infinite graphs by exhibiting locally finite trees which are not reconstructible.
(joint with N. Bowler, J. Erde, P. Heinig and M. Pitz)

Aleksander Malnič (Ljubljana): *Graph Covers – Thirty Years Later*

Abstract. Analyzing structural properties of graphs with nice symmetry properties, classification, enumeration, construction of infinite families, and building catalogs of particular classes of interesting graphs up to a certain reasonable size – these topics have become an area of intense research in the last thirty years. One of the most widely used tools in this context is that of covering space techniques. In the talk, some old as well as some recent results related to lifting automorphisms will be reviewed, with focus on algorithmic and complexity aspects that have largely been overlooked in the past.

Babak Miraftab (Hamburg): *Algebraic graph theory of infinite graphs*

Abstract. A problem proposed by Diestel is to extend algebraic flow theory of finite graphs to infinite graphs. In this talk, we introduce a new definition of flows of infinite graphs (not necessarily locally finite graphs) which is compatible with the notation of flow for finite graphs. By the compactness method, we can extend almost all theorems about finite graphs to infinite locally finite graphs. But this method generally does not enable us to generalize even further to arbitrary infinite graphs. We will present a new technique to over come this obstacle. This enables us to extend to the main theorems of finite flow theory to infinite graphs.

Bojan Mohar (Vancouver): *Normal graph covers*

Abstract. A graph is normal if it admits a clique cover C and a stable set cover S such that each clique in C and each stable set in S have a vertex in common. The pair (C, S) is a normal cover of the graph. We present the following extremal property of normal covers. For positive integers c, s, if a graph with n vertices admits a normal cover with cliques of sizes at most c and stable sets of sizes at most s, then $c + s \geq \log_2(n)$. For infinitely many n, we also give a construction of a graph with n vertices that admits a normal cover with cliques and stable sets of sizes less than $0.87 \log_2(n)$. Furthermore, we show that for all n, there exists a normal graph with n vertices, clique number $\Theta(\log_2(n))$ and independence number $\Theta(\log_2(n))$.
When c or s are very small, we can describe all normal graphs with the largest possible number of vertices that allow a normal cover with cliques of sizes at most c and stable sets of sizes at most s. However, such extremal graphs remain elusive even for moderately small values of c and s.
Joint work with David Gajser.

Rögnvaldur G. Möller (Reykjavik): *Highly-arc-transitive digraphs with prime out-valency and examples*

Abstract. The concept of a highly-arc-transitive digraph was defined by Cameron, Praeger and Wormald in a paper that appeared in 1993. Examples constructed by Seifter (+ co-workers) and Mohar (+ co-workers) have shown that suggestions put forward in that paper are wrong. But if it assumed that the highly-arc-transitive digraph has prime out-valency then some of the suggestions of Cameron, Praeger and Wormald are correct. The second part of the talk is about a general method to construct k-arc-transitive digraphs that are not (k+1)-arc-transitive. This construction gives examples that limit the possibilities of extending the results in the first part and also give examples of digraphs with polynomial growth that are k-arc-transitive but not (k+1)-arc-transitive. (As everybody know graphs with polynomial growth are very close to Seifter's heart.)
Joint work with Primož Potočnik and Norbert Seifter.

Marko Orel (Koper): *On preserver problems, graph homomorphisms, and finite geometry*

Abstract. Preserver problems is a research area in matrix theory, where a typical problem demands a characterization of all maps on certain set of matrices that preserve some function, subset or a relation. In the talk I will present some results in this area, where the proofs rely on the known theory about graph homomorphisms and cores. Sometimes, the existence of particular preservers turns out to be equivalent to the equality between the chromatic and the clique number of an associated graph, and to some of the open and well-known problems in finite geometry.

Iztok Peterin (Maribor): *A characterization of graphs with disjoint total dominating sets*

Abstract. A set S of vertices in a graph G is a total dominating set of G if every vertex is adjacent to a vertex in S. A fundamental problem in total domination theory in graphs is to determine which graphs have two disjoint total dominating sets. We provide a constructive characterization of the graphs that have two disjoint total dominating sets. More detailed, every graph whose vertices can be partition into two total dominating sets can be obtain from one of four basic graphs with a finite sequence of 19 (simple) operations. (Joint work with Michael A. Henning)

Primož Potočnik (Ljubljana): *Cubic vertex-transitive graphs*

Abstract. A connected graph is called cubic if each of its vertices has valence 3, and is vertex-transitive if for any two vertices there exists an automorphism of the graph that maps one to the other. Studying finite cubic vertex-transitive graphs has a long and venerable tradition, going back to the work of Tutte (dealing with the arc-transitive case) and Foster, Coxeter, Frucht and Powers (considering the non arc-transitive case). Due to the classical result of Tutte on the order of the automorphism group of a finite arc-transitive

cubic graph, it is possible to use known group theoretical tools to compile a complete list of arc-transitive graphs up to a reasonably small number of vertices. However, the non arc-transitive case has eluded this type of analysis until very recently. In my talk I will describe some recent results, obtained in collaboration with Gabriel Verret and Pablo Spiga, that enabled us to obtain a complete list of all connected cubic vertex-transitive graphs of order at most 1280.

Franz Rendl (Klagenfurt): *Semidefinite programming relaxations for bandwidth and vertex-separators in graphs*

Abstract. A fundamental problem in numerical linear algebra consists in rearranging the rows and columns of a (symmetric) n times n matrix in such a way that either the nonzero entries appear within a band of small width along the main diagonal (bandwidth problem), or such that the matrix decomposes into two pieces after the removal of a few rows and columns (vertex separator problem).
Such problems can be approached using graph partition techniques. We recall several formulations of this problem and investigate relaxations based on eigenvalue optimization and on semidefinite programming.
It turns out that a semidefinite programming relaxation in matrices of order 2n provides rather tight bounds on the size of a vertex separator, and can be computed efficiently for problems of medium sizes. We present the theoretical framework for this relaxation and also provide computational results on instances from several matrix libraries.
(Joint work with Renata Sotirov, Tilburg University, Netherlands)

Philipp Sprüssel (Graz): *Homological connectivity of random hypergraphs*

Abstract. Linial and Meshulam introduced a model of random simplicial 2-complexes as follows. Start with n 0-simplices (or vertices) and let each pair of vertices form a 1-simplex (or edge). Finally, each triple of vertices forms a 2-simplex (or face) with probability p independently. Such a complex is called $\mathbb{F}_2$-*homological 1-connected* (or *hom-connected* for short) if its first homology group with coefficients in $\mathbb{F}_2$ is trivial. They showed that this model undergoes a phase transition with respect to hom-connectivity at around $p = \frac{2\log n}{n}$, and that the critical obstruction to hom-connectivity is the presence of an edge which is not contained in any face.
We take a different approach by defining our random simplicial 2-complex “top down” rather than “bottom up”: in our model, each pair of vertices forms an edge only if it is part of a face. Thus the complex is generated by a random 3-uniform hypergraph by taking the down-closure. The critical obstruction to hom-connectivity in the previous model no longer exists in our model by definition. We show that in this model, the phase transition for hom-connectivity occurs at around $p = \frac{\log n + \frac{1}{2}\log\log n}{n}$ and give a characterisation of the new critical obstruction. The arguments are complicated by the fact that in this setting, hom-connectivity is not a monotone property.
This talk is based on joint work with Oliver Cooley, Penny Haxell, and Mihyun Kang.

Peter F. Stadler (Leipzig): *Axioms for undirected and directed connectivity*

Abstract. The idea to define connectedness axiomatically dates back to the 1940s. In the undirected case, a natural starting point is to consider a set system $\mathcal{C}$ satisfying (c1) $C_i \in \mathcal{C}$ implies $\bigcup_i C_i \in \mathcal{C}$. Equivalently, one may start from *connected components* so that (i) $x \in\notin A$ implies $A[x] = \emptyset$, (ii) $A[x] \subseteq A$, (iii) $A \subseteq B$ implies $A[x] \subseteq B[x]$, (iv) $(A[x])[x] = A[x]$, and (v) $y \in A[x]$ implies $y \in A[y]$. A third, equivalent, axiom system can be formulated in terms of separations: (S0) $\emptyset \mid B$; (S1) $A' \subseteq A$, $B' \subseteq B$, and $A \mid B$ implies $A' \mid B'$; (SR1) $A \mid C$ and $B \mid A \cup C$ implies $A \cup B \mid C$; (SR2) $A_i \cup B_i = Z_i$ and $A_i \mid B_i$ implies $\bigcup_i A_i \mid \bigcap_i B_i$; (S2) $A \mid B$ implies $B \mid A$. (Ronse 1998, Stadler & Stadler 2015). We will discuss some interesting types of abstract connective spaces, including graphs. The concept of connective spaces can be generalized further to considering "generalized reaches", oriented components, and separations that lack the symmetry propery. These naturally encompass directed hypergraphs. Again we will cover some useful special classes.
Joint work with Bärbel M. R. Stadler.

C. Ronse. Set-theoretical algebraic approaches to connectivity in continuous or digital spaces. *J. Math. Imaging Vision* **8**: 41-58, 1998.
C. Ronse. Axiomatics for oriented connectivity. *Pattern Recognition Let.* **47**: 120128, 2014.
B.M.R. Stadler & P.F. Stadler. Connective spaces. *Math. Comp. Sci.* **9**: 409-436, 2015.

Janoš Vidali (Ljubljana): *Restrictions on classical distance-regular graphs*

Abstract. Let Γ be a distance-regular graph of diameter $d \geq 2$. It is said to have *classical parameters* (d, b, α, β) when its intersection array $\{b_0, b_1, \ldots, b_{d-1}; c_1, c_2, \ldots, c_d\}$ satisfies $b_i = ([d] - [i])(\beta - \alpha[i])$ and $c_{i+1} = [i+1](1 + \alpha[i])$ $(0 \leq i \leq d-1)$, where $[i] := 1 + b + \cdots + b^{i-1}$. Many well-known families of distance-regular graphs have classical parameters. There are, however, also sets of classical parameters for which existence of a corresponding graph is not known. It turns out that in most such cases we have either $\alpha = b - 1$ or $\alpha = b$. For each of these two conditions we derive bounds on the parameter β, which give us complete classifications in the case $b = -2$.
One can say that a distance-regular graph Γ with $d \geq 2$ and eigenvalues $k = \theta_0 > \theta_1 > \cdots > \theta_d$ is *tight* if and only if each local graph is connected strongly-regular, with nontrivial eigenvalues $-1 - b_1(1 + \theta_1)^{-1}$ and $-1 - b_1(1 + \theta_d)^{-1}$. We study tight graphs with classical parameters. The known examples include Johnson graphs $J(2d, d)$, halved $(2d)$-cubes and the Gosset graph. We show that a distance-regular graph with classical parameters is tight if and only if $\beta = 1 + \alpha[d-1]$ and $b, \alpha > 0$. For such graphs, we find closed formulas for the parameters of the CAB partitions and the distance partition corresponding to an edge. Finally, we construct a new two-parameter feasible family of tight distance-regular graphs with classical parameters $(d, b, b-1, b^{d-1})$, primitive iff $b \neq 1$, and show that it is satisfied only by d-cubes $(b = 1)$.
This is joint work with Aleksandar Jurišić.

Wolfgang Woess (Graz): *Horocyclic products of trees - a quasi-isometry problem, its solution, and further ramifications*

Abstract. This talk is concerned with infinite, connected, locally finite graphs which admit transitive group actions. In 1989/1990 the speaker posed the following question: is there such a graph which is not quasi-isometric with (= "does not look vaguely like") a Cayley graph of a fintely generated group. A few years later, Diestel and Leader proposed a construction of what was conjectured to be such an example. It is a horocyclic product of two regular trees with different degrees. It was only in 2012+2013 that the complete answer was given in two impressive papers by Eskin, Fisher and Whyte in the Annals of Mathematics, along with further related results.
On the other hand, the horoycyclic product of two trees with the same degree *is* a Cayley graph, of a lamplighter group (wreath product of the infinite cyclic group with a finite group). This has lead to a body of work by the speaker plus coauthors concerning random walks and harmonic functions on those graphs.
Further interesting ramifications concern horocyclic products of more than two trees.
The talk will review some of the main features of this development.

Arjana Žitnik (Ljubljana): *Chiral astral realizations of cyclic 3-configurations*

Abstract. A combinatorial (n_k) configuration is *cyclic* if it admits an automorphism of order n that permutes its points and lines, respectively. A geometric configuration (n_3) is *astral* if both points and lines form two orbits under the group of symmetries of the configuration. This is the largest amount of symmetry any geometric (n_3) configuration can possess. Branko Grünbaum has shown that in the plane, only cyclic and dihedral symmetry come into question.
We consider the problem of geometric realization of combinatorial cyclic $(2m_3)$ configurations as astral configurations with cyclic symmetry. We provide methods for producing such geometric realizations for many classes of cyclic $(2m_3)$ configurations and we also show that there are infinitely many cyclic $(2m_3)$ configurations which cannot be geometrically realized as astral configurations with cyclic symmetry.
