On a class of random walks on locally finite groups.

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Introduction.

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- G is unimodular & amenable.
- $\{X_i\}_{i=1}^{\infty}$ are G - valued i.i.d., $X(n) = X(0) \cdot X_1 \cdot X_2 \cdot \ldots \cdot X_n$ is a random walk on G starting at $X(0) = x$.

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- Assumption: $\mu = \mathbb{P}_{X_1}$ has the following form,

$$\mu = \sum_{k=0}^{\infty} c_k m_k,$$

where m_k is the normalized Haar measure on G_k, $\{c_k\}_{k=0}^{\infty}$ is a sequence of positive reals such that $\sum_k c_k = 1$.

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   In particular,
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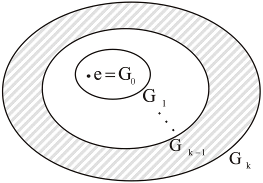
3. Put $\mu_t(x) := \mu_t(\{x\})$, then for $x \in G_k \setminus G_{k-1}$,
   $$\mu_t(x) = \sum_{n \geq k} \frac{C_n(t)}{|G_n|}, \quad C_n(t) = (\sum_{i \leq n} c_i)^t - (\sum_{i \leq n-1} c_i)^t, \quad C_0(t) = c_0^t.$$



Basic properties of μ.

In particular,

- $$\mu_t(e) = \sum_{n\geq 0} C_n(t)/|G_n|.$$
- For any finite $B \subset G$,

$$\mathbb{P}(X(t) \in B | X(0) = e) = \mu_t(B) \sim \mu_t(e)|B| \quad \text{at} \quad \infty.$$

Theorem.

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Example: Let $G = S_\infty = \bigcup_{n>1} S_n$. Put $\sigma(n) = \sum_{k>n} c_k$, then $\{X(n)\}$ is recurrent if and only if

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In particular, let $\sigma(n) \asymp n^\alpha/n!$, then,

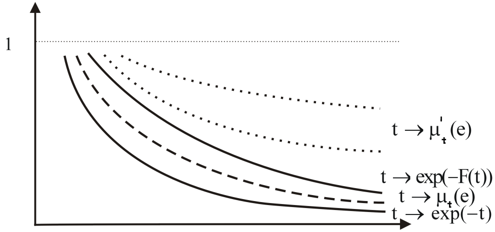
- $X(n)$ is recurrent if $\alpha \leq 1$,
- $X(n)$ is transient if $\alpha > 1$.

Recurrence/transience of random walks.

- Brofferio, S., Woess, W.: *On transience of card shuffling*, Proc. Amer. Math. Soc 129 (2001), No. 5, 1513-1519.
- Lawler, G.F.: *Recurrence and transience for a card shuffling model*, Combinatorics, Probability and Computing 4 (1995), 133-142.
- Flatto, L., Pitt, J.: *Recurrence criteria for random walks on countable abelian groups*, Illinois J. Math. 18 (1974), 1-19.
- Darling, D., Erdös, P.: *On the recurrence of a certain chain*, Proc. Am. Math. Soc. 19 (1968), 336-338.

Theorem 2.

Let $F : \mathbb{R}_+ \to \mathbb{R}_+$, $F(t) = o(t)$ at ∞. Then $\exists \mu_t, \mu'_t$ such that:



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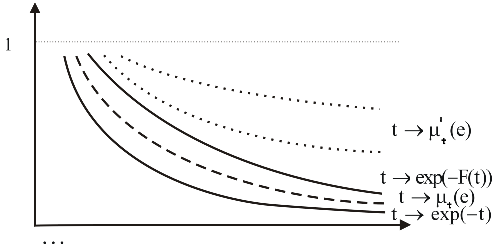

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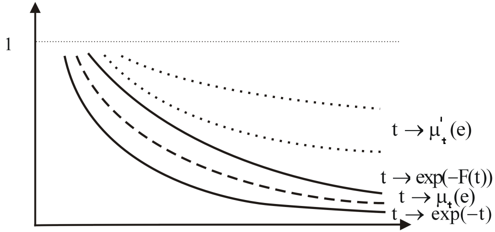

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1) G is amenable, hence $\mu_n(e) = \exp(-n \cdot o(1))$, can be made as close as possible to $n \to \exp(-n)$ by an appropriate choice of $\{c_k\}$.
2) G is not finitely generated, hence $\mu'_n(e) \to 0$ at ∞ can be made as slow as possible by an appropriate choice of $\{c_k\}$.