Uniqueness of currents in a network of finite total resistance

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Graz, 2.7.09

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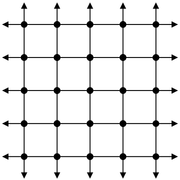

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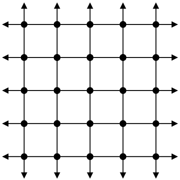
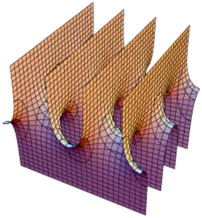
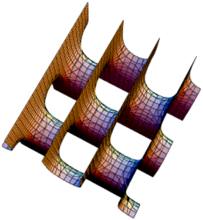




$$\begin{pmatrix} x[u, v] \\ y[u, v] \\ z[u, v] \end{pmatrix} = \begin{pmatrix} u \\ v \\ \frac{\mathrm{Log}[\mathrm{Cos}[a\,v]\,\mathrm{Sec}[a\,u]]}{a} \end{pmatrix}$$



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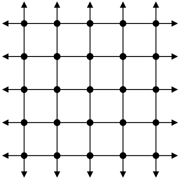

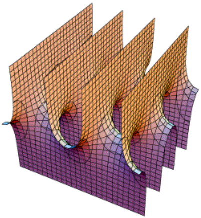

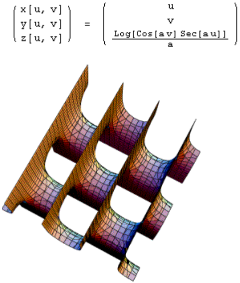

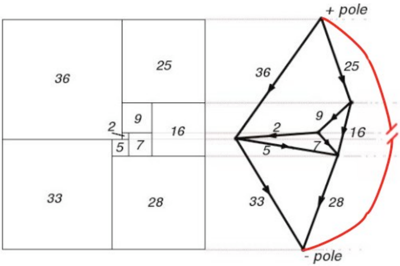

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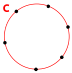

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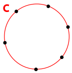

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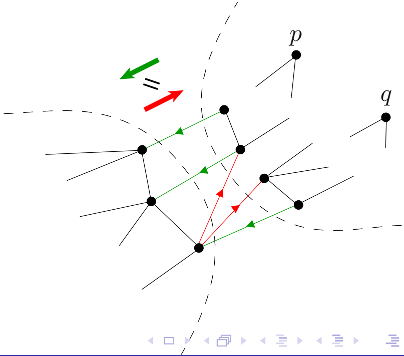

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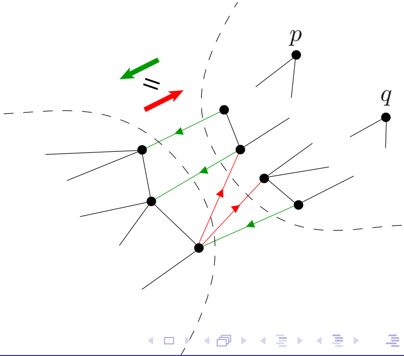

Non-elusive flow:
The net flow along any such cut must be zero:

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Theorem (G '08)

In a network with $\sum_{e \in E} r(e) < \infty$ *there is a unique non-elusive flow with finite energy that satisfies Kirchhoff's cycle law.*

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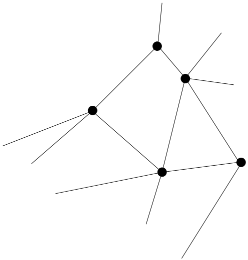

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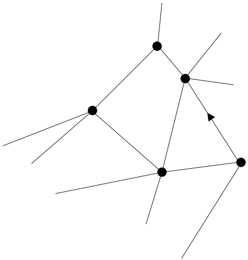

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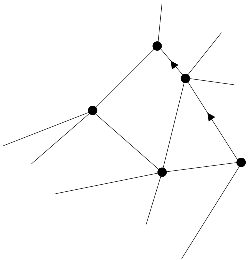

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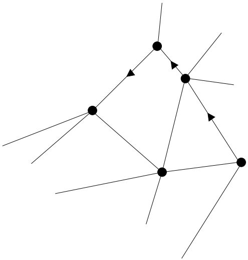

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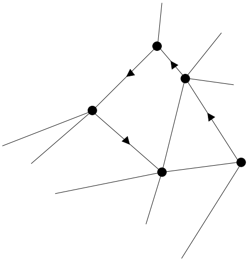

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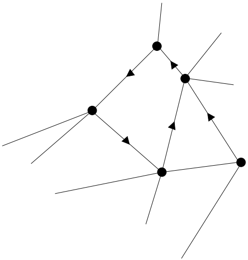

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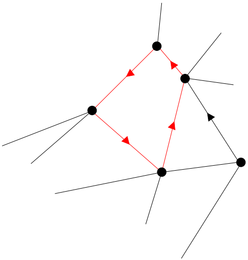

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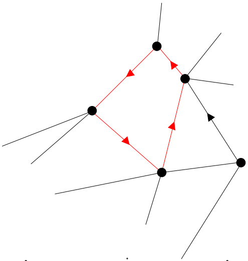

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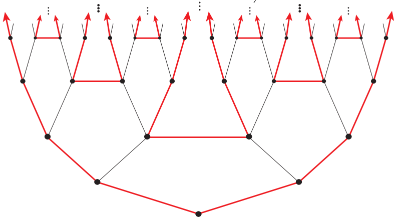
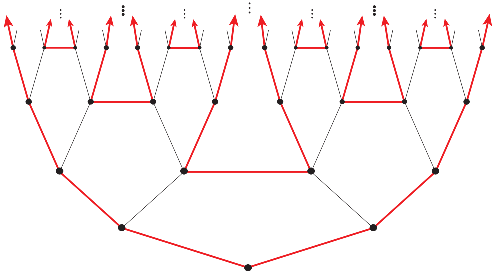


Wild circles



A wild circle

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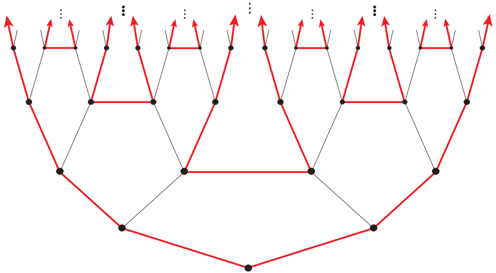

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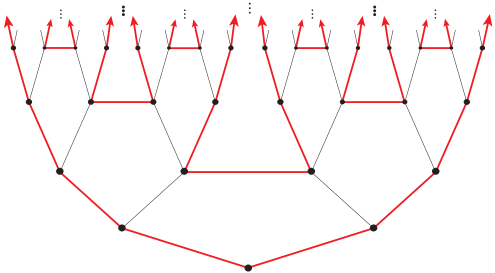

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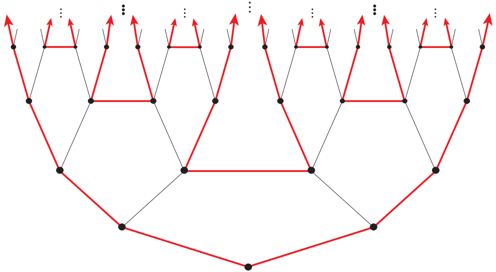

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The "gaps" between the double-rays are filled by a Cantor set of ends

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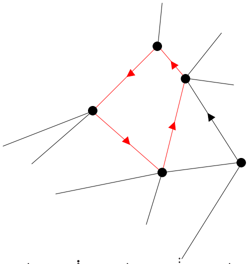

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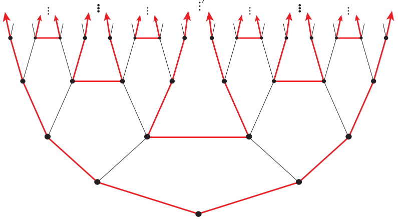

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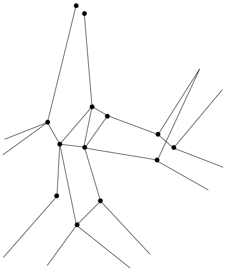

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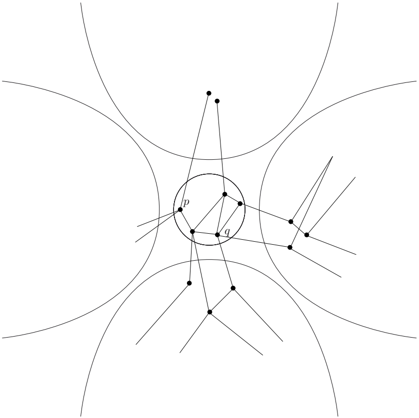

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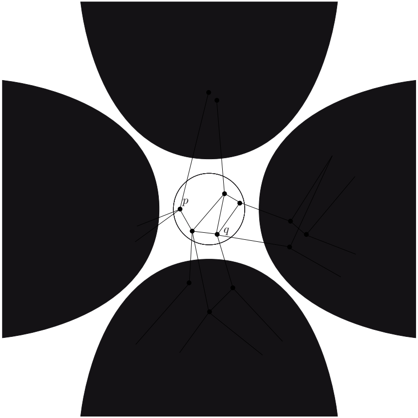

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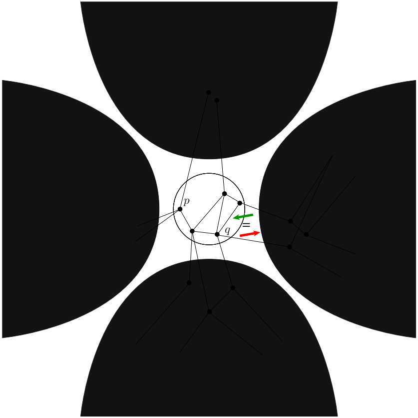

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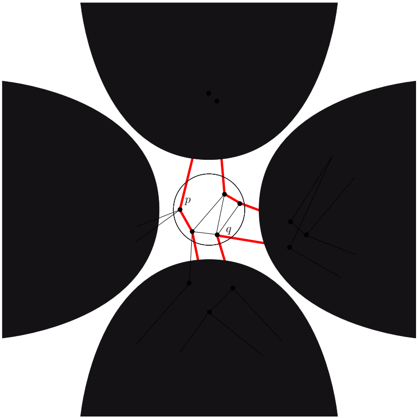

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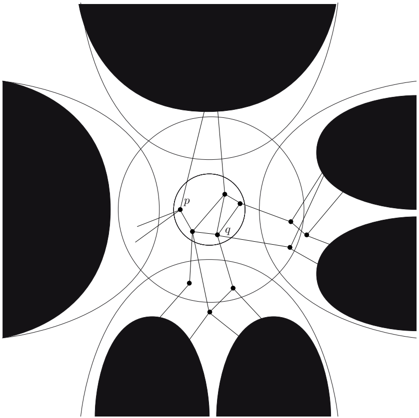

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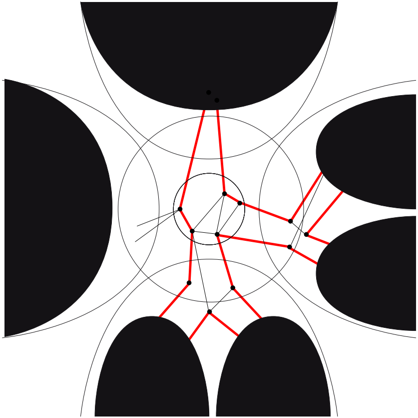

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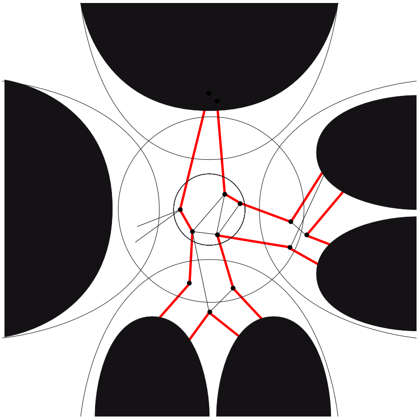

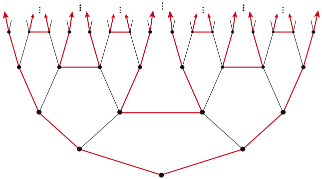

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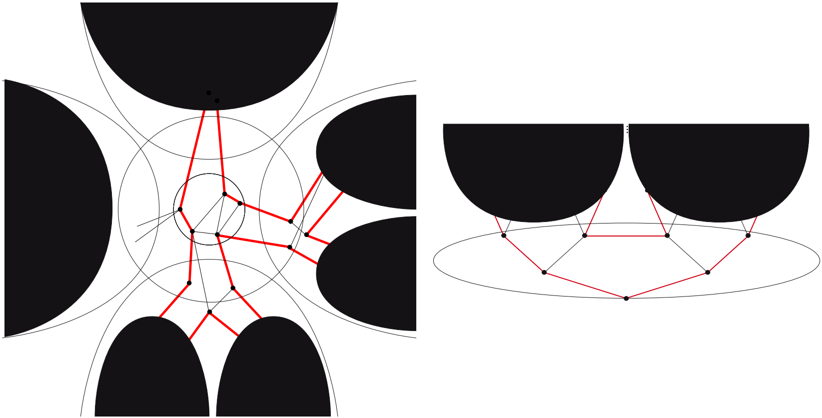

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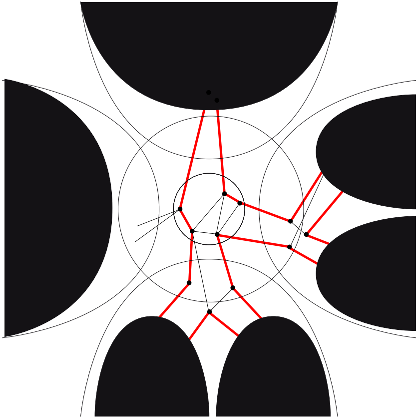

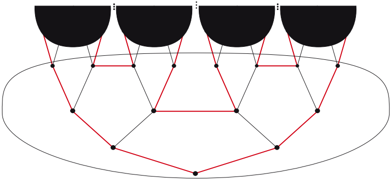

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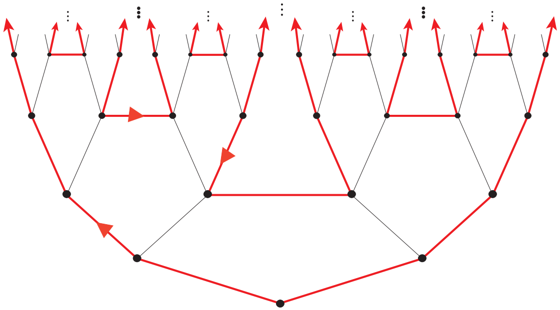

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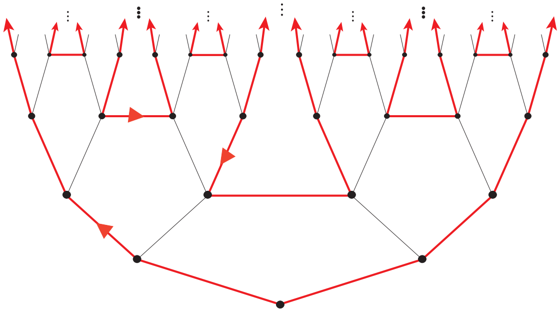

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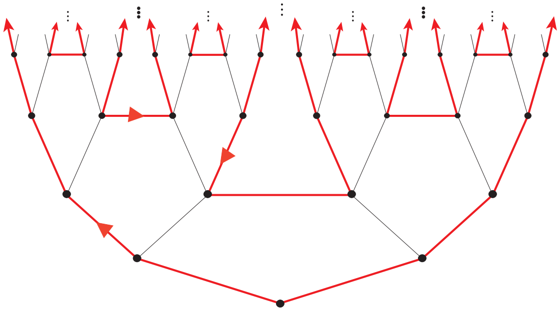

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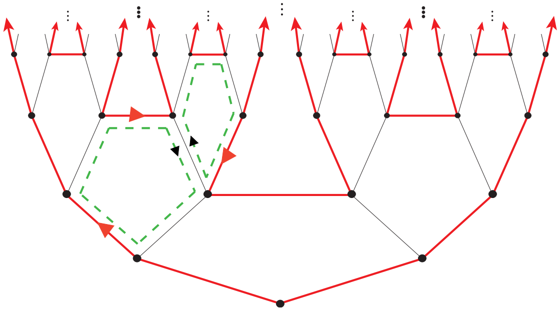

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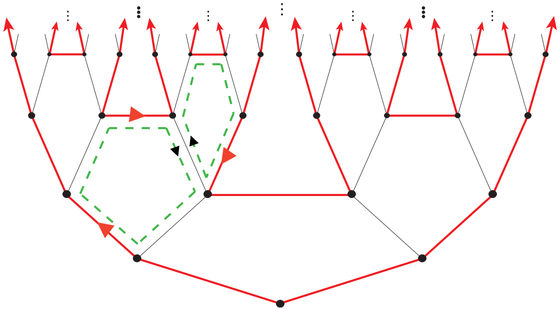

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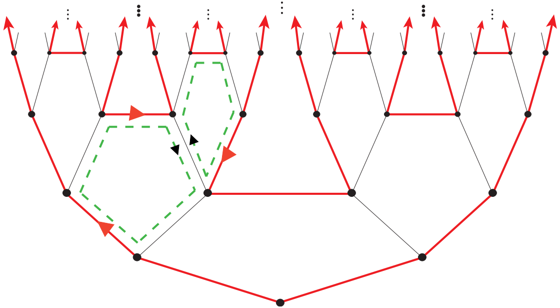

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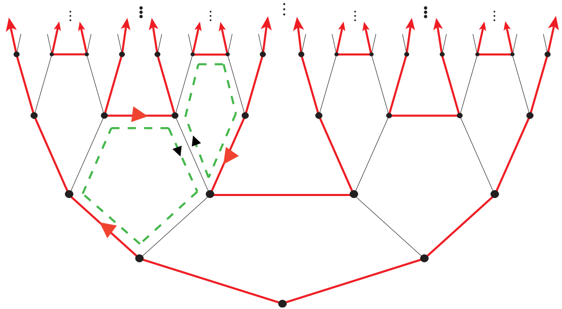

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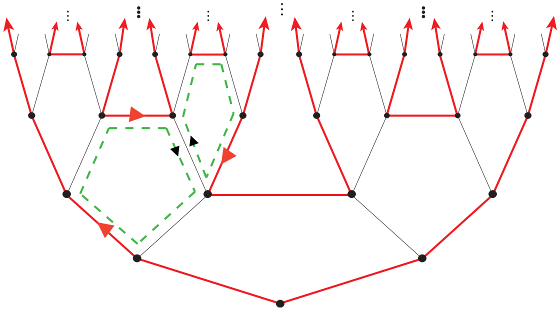

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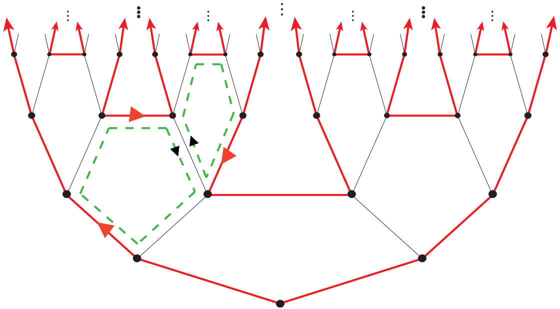

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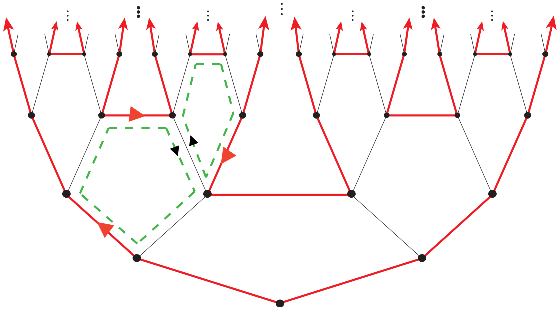

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OK if $\sum r(e) < \infty$

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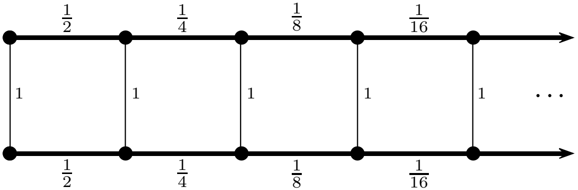

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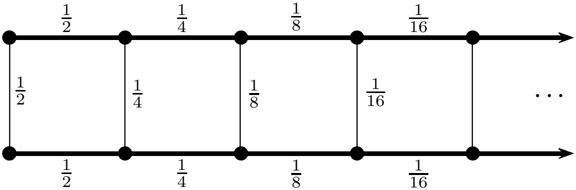

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Theorem (G '06 (easy))

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Theorem (G '06 (easy))

If $\sum_{e \in E} r(e) < \infty$ then $|G|_r \approx |G|$.

Kirchhoff's cycle law for wild circles

Theorem (Diestel & G)

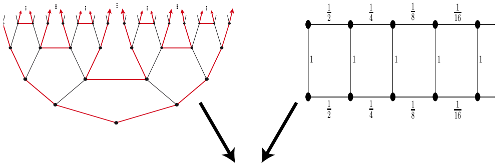
The circles of an electrical network N satisfy Kirchhoff's cycle law if the sum of the resistances in N is finite.

The Theorem

Theorem (G '08)

In a network with $\sum_{e \in E} r(e) < \infty$ there is a unique non-elusive flow with finite energy that satisfies Kirchhoff's cycle law.

The Theorem



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The Dirichlet Problem

Continuous version

Discrete version

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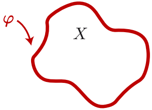

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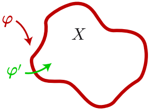

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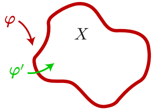

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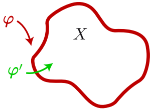

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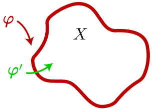

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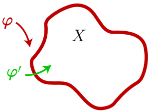

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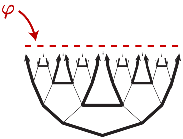

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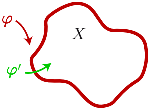

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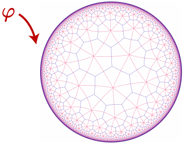

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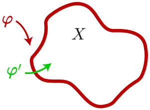

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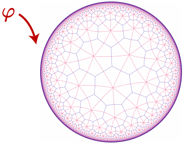

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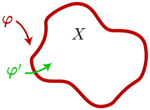

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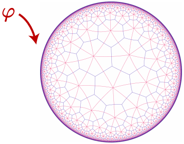

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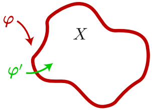

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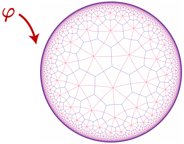

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Studied intensively (Woess, Kaimanovich, Benjamini & Schramm)

Problem

For evey assignment $r : E \to \mathbb{R}_+$ *(such that* $|G|_r$ *is compact) the Dirichlet problem is solvable for every continuous* $\phi : \partial|G|_r \to \mathbb{R}$.

The Dirichlet Problem

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Interesting because:

Theorem (Gromov '87 (indirect proof))

For every compact metric space X there is a locally finite graph G and $r : E \to \mathbb{R}_+$ such that $X = \partial|G|_r$.

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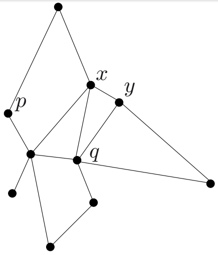
The converse works:

Theorem

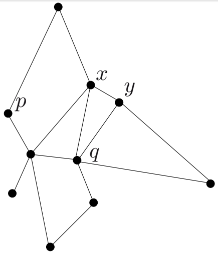
If $f : \vec{E} \to \mathbb{R}$ is a flow of finite energy in G satisfying Kirchhoff's cycle law then it is possible to extend the corresponding potentials continuously to $\partial|G|_r$.

Random Walks & Electrical networks



Random Walks & Electrical networks

Every edge e has a weight $c(e)$



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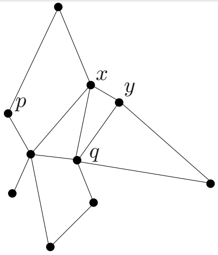

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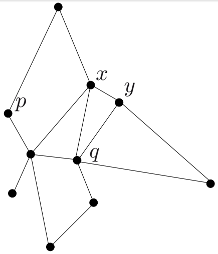

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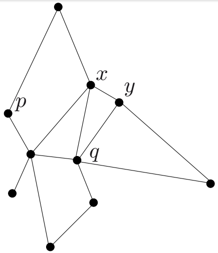

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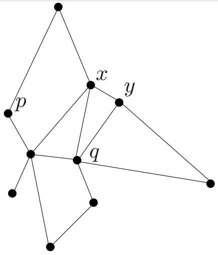

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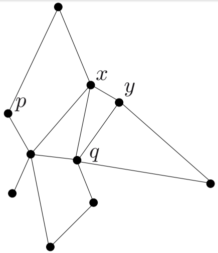

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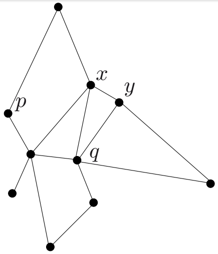

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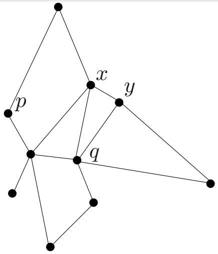

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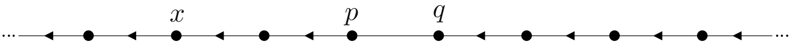

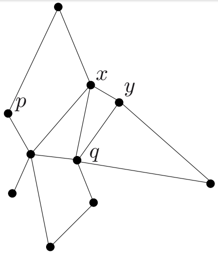

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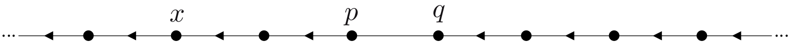

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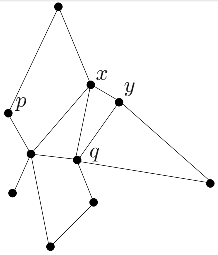

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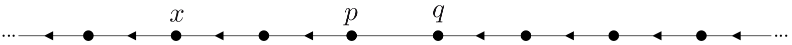
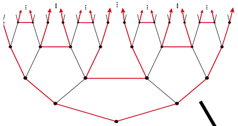
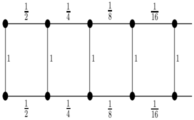


Problem

Define brownian motion on $|G|_\ell$

Summary





Theorem (G '08)

In a network with $\sum_{e \in E} r(e) < \infty$ there is a unique 'good' current

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Problem

Define brownian motion on $|G|_\ell$

Problem

Solve the Dirichlet Problem at the $|G|_\ell$ boundary