Diskrete Stochastik und Informationstheorie – 19 Mar 2014

Exercise 1. A 6-sided die with values $\Omega = \{1, 2, 3, 4, 5, 6\}$ is thrown and the upside facet is read.

(a) What is the minimal σ-algebra that contains the information if the value of the die is even or odd?

(b) What is the minimal σ-algebra that contains the information if the value of the die is either in $\{1, 2\}$ or in $\{3, 4\}$ or in $\{5, 6\}$?

(c) What is the σ-algebra that allows you to determine the value of the die?

Exercise 2. Is $\{\emptyset, \{b\}, \{a, c\}, \{d, b\}, \{a, b, c, d\}\}$ a σ-algebra over $\Omega = \{a, b, c, d\}$? If not, which elements are missing? What is the "information" described by this σ-algebra?

Exercise 3. How many different σ-algebras can be constructed from the basic events $\Omega = \{\text{red}, \text{blue}, \text{green}\}$? How many of them are truly structurally different (non-isomorphic)?

Exercise 4. Karl has two kids. Let $\mathcal{A}$ be the minimal σ-algebra that describes whether each of the two kids is a boy or a girl.

(a) Describe the corresponding probability space $(\Omega, \mathcal{A}, \mathbb{P})$ and its probability distribution.

(b) Assume Karl's firstborn child is a girl. What is the chance that the second is a boy?

(c) Assume at least one of the two kids is a girl. What is the chance that the other is a girl as well?

Exercise 5. A fair die is rolled first. It gives a random number k from $\{1, 2, 3, 4, 5, 6\}$. Next, a fair coin is tossed k times.

(a) Assume we know that all coins show heads, what is the probability that the score of the die was equal to 4?

(b) Are the events "k is even" and "all coins show heads" independent?