Graz University of Technology

Department of Mathematical Structure Theory

Diskrete Stochastik und Informationstheorie – 26 Mar 2014

Exercise 6. Let X, Y be two independent, dice-valued (i.e., uniformly distributed in $\{1, \ldots, 6\}$) random variables.

(a) Calculate $\mathbb{E}[2X + Y^2]$.

(b) Calculate $\mathrm{Cov}(X, Y)$.

(c) Are $X + Y$ and $X - Y$ independent?

(d) Calculate the covariance of $(X + Y)$ and $(X - Y)$.

Exercise 7. Let X, Y be two independent, dice-valued (i.e., uniformly distributed in $\{1, \ldots, 6\}$) random variables. Calculate

(a) The distribution and expectation of X given the event $Y = 3$,

(b) The distribution of $X + Y$ given the event $Y \in \{2, 4\}$,

(c) The distribution and expectation of Y given the event $X > Y$.

Exercise 8. Let X be a discrete random variable taking values in $\mathbb{N}$, with a finite expected value

$$\mathbb{E}(X) = \sum_{n \geq 0} n \cdot \mathbb{P}[X = n] < \infty.$$

Let $a > 0$. Prove Markov's inequality:

$$\mathbb{P}\big[X \geq a\,\mathbb{E}(X)\big] \leq \frac{1}{a}.$$

Exercise 9. A roulette gambler always bets \$ 10 on a color (red or black). Thus, for 18 of the 37 possible outcomes, he wins his stakes back plus a prize of the same amount; for the other 19 outcomes, he loses his bet. Use the strong law of large numbers to argue that the gambler will go bankrupt in the long run.

Exercise 10. Let X be a geometrically distributed random variable on $\mathbb{N}$, i.e., $\mathbb{P}(X = k) = (1 - p)^{k-1} p$ for some $p \in [0, 1]$. Define the sequence of random variables $(X_n)_{n \geq 0}$ by

$$X_n := \begin{cases} (1-p)^{1-n} & \text{if } X = n \\ 0 & \text{otherwise.} \end{cases}$$

(a) Does (X_n) converge in probability?

(b) What can you say about the almost sure limit of X_n?

(c) Can you interchange limit and expectation?