Graz University of Technology

Department of Mathematical Structure Theory

Diskrete Stochastik und Informationstheorie – 14 May 2014

Exercise 23. Give an example to show that in general, a function of a Markov chain is not necessarily again a Markov chain (i.e., find a Markov chain $(X_n)_{n\geq 0}$ and a function f such that $(Y_n)_{n\geq 0}$ with $Y_n = f(X_n)$ is not a Markov chain).

Exercise 24. Show that for a Markov chain $(X_n)_{n\geq 0}$,

$$H(X_0 \mid X_n) \geq H(X_0 \mid X_{n-1}).$$

Exercise 25. Consider the Markov chain $(X_n)_{n\geq 0}$ on $\mathcal{X} = \{1, \ldots, 7\}$ with transition matrix

$$\mathbf{P} = \begin{pmatrix} 1/3 & 0 & 0 & 1/3 & 0 & 0 & 1/3 \\ 1/3 & 0 & 0 & 1/3 & 0 & 0 & 1/3 \\ 1/2 & 0 & 0 & 1/2 & 0 & 0 & 0 \\ 1/3 & 0 & 0 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0 & 1/2 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1/3 & 0 & 0 & 1/3 & 0 & 0 & 1/3 \end{pmatrix}$$

(a) Draw the transition graph. Is (X_n) irreducible?

(b) Calculate $\mathbb{P}(X_n = i \mid X_0 = 6)$ for $i \in \mathcal{X}$ and $n \in \{1, 2, 3\}$.

(c) Give a stationary distribution for X_n. Is it unique?

(d) Calculate $\lim_{n\to\infty} \mathbb{P}(X_n = i \mid X_0 = 6)$ for all $i \in \mathcal{X}$.

Exercise 26. Consider all simple random walks on undirected graphs of 4 vertices (transition to all connected neighbours with equal probability, with a uniform initial distribution).

(a) Which graph has the highest, which the lowest asymptotic entropy?

(b) What changes if you consider only connected graphs?

(c) What changes if you allow loops (transitions from a node to itself)?