Abstracts of the talks of the first workshop

Marcelo Aguiar (Texas A&M University)

Infinitesimal Bialgebras

We will review basic notions pertaining to infinitesimal bialgebras, as well as some new points of view motivated mainly by work of Voiculescu (Free Analysis Questions I and II). These will include:

- A discussion of the interplay between infinitesimal and Frobenius bialgebras.
- An algebraic perspective on the algebras of fully matricial functions of Voiculescu.

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Michael Anshelevich (Texas A&M University)

Convolution semigroups with linear Jacobi parameters

For a convolution semigroup of measures, the dependence of the Jacobi parameters of the measure on the convolution parameter is typically quite complicated. However for some examples, such as the heat semigroup, the dependence is linear. I will show that the dependence is polynomial if and only if the measures lie in the Meixner class. The proof is simple but indirect. For the corresponding question for free convolution, there is a more explicit proof, based on non-crossing partitions machinery developed by Wojciech Mlotkowski (with whom this work is joint). Time permitting, I will also mention the corresponding result for the two-state free convolution.

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Octavio Arizmendi Echegaray (Universität des Saarlandes)

k-divisible Non-Crossing Partitions and Free Probability

In this talk we will give some results involving the combinatorics of k-divisible non-crossing partitions and explain consequences on Free Probability.

More specifically, let NC and NC_k be the sets of non crossing partitions and k-divisible non-crossing partitions. If we can look at NC_k as a sublattice of NC, it turns out that the (combinatorial) convolution with the zeta function in NC_k can be calculated by looking at the k-fold convolution of the convolution with the zeta function in NC_k. Using this result we can recover many counting results involving k-divisible partitions, k-equal partitions, k-multichains in NC and l-multichains in NC_k.

After this we will define naturally the notion of a k-divisible element x and derive a formula the free cumulants of x^s in terms of the free cumulants of x.

Finally we will explain some consequences on free multiplicative convolution with symmetric measures and free infinite divisibility.

Teodor Banica (Université de Cergy-Pontoise)

Probabilistic aspectsof free quantum groups

Hari Bercovici (Bloomington)

On sums and products in finite factors

We discuss the solution of the analogue of the Horn problem in finite factors This is the problem of determining the allowed spectral behavior of a sum of two self-adjoint elements with given eigenvalues. We also discuss the multiplicative version of this problem about which much less is known.

Marek Bożejko (Wrocław University)

New characterisation of free Meixner processes

We will present generalized stochastic processes with freely (classically) independent values.

They have representation as

$$P(t) = a^*(t) + a(t) + \lambda(t)a^*(t)a(t) + \eta(t)a^{2*}(t)a^2(t),$$

here $a(t)$ and $a^*(t)$ are free (classical) annihilation and creation distribution in the Hida (distribution) sense and λ and η are continuous functions on a non-atomic locally compact measure space (T, dx).

If that functions are constant we get the free (classical) representation of exactly Brownian motion, Poisson, gamma-case when $\eta = 0$ and Pascal and Meixner processes, when $\eta > 0$.

We will present the free version of results of E. Lytvynov and we get a new characterization of that class of processes.

Some relations with the papers with Demni on Meixner families will be also done.

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M.Bozejko, W.Bryc, On a class of free Levy laws related to a regression problem, J. Funct. Anal. 236(2006), 59-77.

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E.Lytvynov, Polynomials of Meixner's type in infinite dimensions-Jacobi fields and orthogonality measures, J. Funct. Analysisi 200(2003), 118-149.

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Michael Brannan (Queen's Univeristy, Kingston)

Approximation properties for free orthogonal and free unitary quantum groups

The free orthogonal and free unitary quantum groups, denoted by O_N^+ and U_N^+ ($N \geq 2$), were introduced by Shuzhou Wang in 1993. In recent years it has become increasingly apparent that these quantum groups share many deep connections with free probability theory. In particular, the associated reduced von Neumann algebras, $L^\infty(O_N^+)$ and $L^\infty(U_N^+)$, turn out to share many structural properties with the free group factors $L(\mathbb{F}_N)$.

In this talk, we will pursue this connection with $L(\mathbb{F}_N)$ further, and show that $L^\infty(O_N^+)$ and $L^\infty(U_N^+)$ always have the Haagerup approximation property. Using this result together with some Haagerup-type inequalities obtained by Roland Vergnioux (J. Operator Theory, 2007), we also show that the reduced C*-algebras $C(O_N^+)$ and $C(U_N^+)$ have the metric approximation property.

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Christian Brouder (Laboratory of Mineralogy and Crystallography (Paris))

Noncommutative Feynman graphs and Hopf algebra cohomology

Feynman graphs in quantum field theory can be generated as a convolution exponential over a free commutative algebra. We show that, similarly, any linear map f from the tensor algebra $T(V)$ to the scalars (with $f(1) = 1$), can be written as a convolution exponential and can be described with generalized non-commutative Feynman graphs. (joint work with Damien Manuel and Frederic Patras)

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Thierry Cabanal-Duvillard (Paris 5)

A generalization of a result of Marchenko & Pastur, providing a family of Bercovici-Pata bijections

In 1967, Marchenko and Pastur have determined the limit of the spectral law of sums of weighted rank one projectors. These limit distributions have been identified afterwards as free compound Poisson laws. In 2005, the authors have extended Marchenko-Pastur approach to any infinitely divisible law, giving a matricial realization of the Bercovici-Pata bijection. In this talk, another generalization will be presented, providing the limit distribution when the rank-one projectors are chosen with much fewer assumptions than in Marchenko-Pastur paper.

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Gennadii Chistyakov (Universität Bielefeld)

Infinitely divisible approximations of n-fold free convolutions

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Maciej Dołęga (Wrocław University)

Colorings of bipartite graphs and polynomial functions on the set of Young diagrams

Polynomial functions (in the sense of Kerov and Olshanski) on the set of Young diagrams are functions which have a prominent role in the asymptotic representation theory of a permutation groups. We will discuss some examples concerning normalized characters and free cumulants. We will also show how to construct a function on the set of Young diagrams from a given bipartite graph and when these kind of functions are polynomial functions. We will give a connection of our result with Jack symmetric functions and some conjectures of Lassalle. Our method involves a differential calculus on the set of Young diagrams and combinatorics of bipartite graphs which will be also discussed. This is a joint wotk with Piotr Śniady.

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Ken Dykema (Texas A&M University)

An application of asymptotic freeness to soficity of groups

After recalling some results about asymptotic freeness of random matrices and showing how they can be used to give lower bounds on free entropy dimension in amalgamated free products, we will examine an analogous discrete situation, and show how these can be used to prove soficity of certain amalgamated free products of groups.

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David Evans (Cardiff University)

Modular invariants, subfactors and twisted equivariant K-theory

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Valentin Feray (LABRI, Bordeaux)

Characters of symmetric groups, free cumulants and a combinatorial Hopf algebra

Representation theory of symmetric groups is a research field connected to free probability. Indeed, P. Biane and S. Kerov have shown that irreducible character values (which are central quantities in representation theory) can be expressed nicely in terms of the free cumulants of some natural measure. In this talk, we present a combinatorial Hopf algebra containing these objects.

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Terry Gannon (University of Alberta)

The search for the exotic

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Friedrich Götze (Universität Bielefeld)

Asymptotic Expansions in the Free Central Limit Theorem

We show asymptotic approximations of first and second order in the Central Limit Theorem of Free Probability. For the n-fold free convolution we establish error bounds of order $o(n^{-1/2})$ and $o(n^{-1})$ depending e.g. on the existence of three or four moments. The expansion results obtained are valid under minimal moment assumptions when compared to the classical case.

This is joint work with G. Chistyakov.

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Mikhail Gordin (POMI, Saint Petersburg)

Formal and Analytic Groups Related to Free Probability

At the formal level, with every probability distribution on the line a formal group law over reals can be associated. There exists also an analytic analogue of this correspondence. The concepts of the monotone and the free convolutions will be considered in the context of such group laws. Relation of the free convolution formalism to Lazard's universal formal group law will be discussed.

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Takahiro Hasebe (RIMS Kyoto)

On Cauchy distributions in non-commutative probability

Four independences are known in a non-commutative probability space, i.e., tensor, free, monotone and Boolean ones. For each independence, strictly stable distributions can be defined as analogues of the usual ones in probability theory. In general, strictly stable distributions depend on a choice of independence. However, strictly stable distributions with index one are Cauchy distributions in all the four independences. We will understand this coincidence by introducing generalized concepts of moments and cumulants.

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Claus Koestler (Aberystwyth University)

Noncommutative independence in the infinite braid and symmetric group

We introduce in an elementary setting to the recent merge of a noncommutative de Finetti type result with representations of the infinite braid and symmetric group which allows to derive factorization properties from symmetries. We explain some of the main ideas of this approach and discuss a constructive procedure to use in applications. Finally we illustrate the method by applying it to the theory of group characters.

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Christian Krattenthaler (Universität Wien)

Generalized non-crossing partitions for reflection groups and cyclic sieving

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Ilona Krolak (Wrocław University)

General commutation relations - properties of associated algebras and Ornstein-Uhlenbeck semigroup

We study a certain class of von Neumann algebras generated by selfadjoint elements $\omega_i = a_i + a_i^+$, for a_i , a_i^+ satisfying the general commutation relations:

$$a_i a_j^+ = \sum_{r,s} t_{js}^{ir} \, a_r^+ a_s + \delta_{ij} Id.$$

We assume that operator T for which the constants t_{js}^{ir} are matrix coefficients satisfies the braid relation. The choice of the relations was made since several examples of such structures are investigated in the literature (CAR, q-CCR, twisted commutation relations studied by Pusz and Woronowicz and their modifications which are type III factors). We concentrate on bounds for hypercontractivity of Ornstein-Uhlenbeck semigroup acting on these non-commutative algebras.

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Franz Lehner (TU Graz)

The normal law, free probability, and a Hopf algebra of rooted binary trees

In joint work with S. Belinschi, M. Bożejko and R. Speicher we proved some time ago the somewhat strange fact that the classical normal distribution is freely infinitely divisible. This fact is equivalent to the positive definiteness of the sequence of free cumulants of the normal distribution. In this talk we review combinatorial aspects of this sequence, which counts the number of connected pair partitions and has been studied by Touchard, Riordan and others previously. We present other combinatorial interpretations, from computer science to Hopf algebras of rooted binary trees which appear in the context of renormalization theory. In spite of the seeming simplicity of the sequence, there still is no direct combinatorial proof of its positive definiteness.

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Romuald Lenczewski (Wrocław University of Technology)

Matricial R-transform

We show that addition of strongly matricially free random variables leads to the 'matricial R-transform' related to the associated convolution. It is a linear combination of Voiculescu's R-transforms in free probability with coefficients given by internal units of the considered array of subalgebras. This allows us to view the associated linearization formula as the 'matricial linearization property' of the R-transform. Since strong matricial freeness unifies the main types of noncommutative independence, the matricial R-transform plays the role of a unified noncommutative analog of the logarithm of the Fourier transform for the main types of noncommutative independence.

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Jean-Louis Loday (CNRS, Strasbourg)

Generalized Hopf algebras and operads

A classical result of Hopf and Borel says that a Hopf algebra which is commutative, cocommutative and conilpotent is necessarily free as a commutative algebra. There is a similar theorem for associative algebras, but one has to modify the Hopf compatibility relation by the so-called "unital infinitesimal compatibility relation". We will show how to generalize these structure theorems to other settings involving different kinds of types of algebras, that is of operads.

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Mitja Mastnak (Saint Mary's University, Halifax)

Bialgebras and free multiplicative convolution

The talk is based on the joint paper with A. Nica entitled "Hopf algebras and the logarithm of the S-transform in free probability". There we discuss a bialgebra based on noncrossing partitions that encodes some of the combinatorics of the free multiplicative convolution of k-tuples of distributions in a non-commutative probability space. The emphasis of the talk will be on bialgebra aspects of the work in question. Some related work in progress will also be mentioned.

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Christian Mazza (University of Fribourg)

B-Series, Schwinger-Dyson Equations And Wigner Processes

We consider series indexed by rooted trees, which are relevant for some models in quantum field the established a link between the combinatorics of renormalization and the so-called Butcher's group. This group is composed of B-series, which are in fact numerical solutions of ordinary differential equations associated to methods. The Butcher's group is also the character group of Kreimer's algebra of rooted trees. We will focus on some combinatorial aspects of B-series, and derive associated Schwinger-Dyson equations in some special cases. We will also exhibit a special family of B-series which are related to traces of products of semi-circular elements.

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James Mingo (Queen's University)

Second Order Freeness and Wigner Ensembles

Twenty years ago Voiculescu showed that self-adjoint Gaussian random matrices and deterministic matrices were asymptotically free. This was later extended to Wigner random matrices.

Roland Speicher and I introduced a theory of second order freeness to do for fluctuation moments what freeness does for moments. Self-adjoint Gaussian random matrices and deterministic random matrices are asymptotically free of second order. With Wigner matrices this fails, but an interesting formula involving conditional expectations can still be proved. This is joint work with Roland Speicher.

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Naofumi Muraki (Iwate Prefectural University)

A certain q-interpolation between tensor and free independence.

I will construct a certain q-interpolation between tensor and free independence as a universal product for C*-probability spaces. This is a kind of 'q-independence' in the sense that this product produces the Bozejko-Speicher q-Brownian motion in the functional central limit. We remark that this is a kind of approximate notion for true q-independence that cannot exists as a universal product.

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Alexandru Nica (University of Waterloo)

On the C*-algebra of the Fock space representation for the q-commutation relations

For $q \in (-1, 1)$, the q-commutation relations have a natural representation on a deformed Fock space $\mathcal{F}_q$; this was introduced by Bozejko and Speicher in 1991, and was studied by many other authors after that. Let $\mathcal{C}_q \subseteq B(\mathcal{F}_q)$ be the C^*-algebra generated by this representation. For $q = 0$ one has that $\mathcal{C}_0$ is the full Fock space representation of the extended Cuntz algebra. It is widely believed that $\mathcal{C}_q$ is unitarily equivalent to $\mathcal{C}_0$ for all $q \in (-1, 1)$, but at present this is proved only for small values of $|q|$.

In a paper by Dykema and Nica in 1993 it was shown how to construct a unitary $U : \mathcal{F}_q \to \mathcal{F}$ such that $U\mathcal{C}_qU^* \supseteq \mathcal{C}_0$ for all $q \in (-1, 1)$, with equality holding when $|q| < 0.44$.

In this talk I will present a recent joint work with Matthew Kennedy, where we introduce a unitary $U_{opp} : \mathcal{F}_q \to \mathcal{F}$ (related to the U from the Dykema-Nica paper) which achieves the opposite inclusion:

$$U_{opp}\, \mathcal{C}_q\, U_{opp}^* \subseteq \mathcal{C}_0, \quad \forall\, q \in (-1, 1).$$

As a consequence, it follows that $\mathcal{C}_q$ is an exact C^*-algebra for all q.

In order to obtain the embedding into $\mathcal{C}_0$ stated above, we prove a "bicommutant type" result (with commutations considered modulo a suitable ideal of compact operators) which gives a sufficient condition for an operator $T \in B(\mathcal{F})$ to belong to $\mathcal{C}_0$. We then prove this sufficient condition to be satisfied by the generators of $U_{opp}\, \mathcal{C}_q\, U_{opp}^*$.

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Leonid Pastur (Institute of Low Temperature Physics, Kharkov)

Laws of Fluctuations for Spectral Statistics of Random Matrices

We present a review of recent results on the limiting laws for fluctuations of several classes of spectral statistics of random matrices as their size tends to infinity. We pay special attention to random matrices whose randomness is due Haar distributed random matrices of classical groups.

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Frederic Patras (Université de Nice Sophia-Antipolis)

Noncommutative Spitzer identities

Spitzer identities first appeared in fluctuation theory, allowing to understand algebraically the characteristic functions associated to the extrema of certain sequences of random variables. We derive here functional identities for noncommutative Rota–Baxter algebras. Our results generalize the seminal Cartier–Rota theory of classical Spitzer-type identities for commutative Rota–Baxter algebras. In the classical, commutative, case, these identities can be understood as deriving from the theory of symmetric functions. Here, we show that an analogous property holds for noncommutative Rota–Baxter algebras. That is, we show that functional identities in the noncommutative setting can be derived from the theory of noncommutative symmetric functions. Lie idempotents, and particularly the Dynkin idempotent play a crucial role in the process. Based on joint works with K Ebrahimi-Fard, J Gracia-Bondia and D Manchon.

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Mihai Popa (Ben Gurion University)

Non-commutative functions and some of their applications in free probability

Given two vector spaces, V and W over the complex numbers, a non-commutative function is, briefly, a mapping from a certain class of subsets of the matrix space over V to the matrix space over W satisfying some compatibility conditions: it has to respect direct sums and simultaneous similarities, or equivalently, simultaneous intertwinings. Noncommutative functions have very strong regularity properties and they admit a very nice differential calculus, closely related to some QD-bialgebras arising in free probabilities. Such objects were considered before by J. L. Taylor in his groundbreaking work on the noncommutative spectral theory, and more recently independently by D.-V. Voiculescu in free probability.

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Christian Sattlecker (TU Graz)

Free log-normal distribution and confluent hypergeometric series

Bercovici and Voiculescu (1992) introduced measures supported on the positive, real line $\mathbb{R}^+$ and the torus $\mathbb{T}$, which are free analogues of the log-normal distribution and the rolled-up normal distribution, respectively. These measures appear in the freen multiplicative central limit theorem, the free multiplicative brownian motion (Biane, 1997), and as limit distribution of some unitary random matrices (Biane, 1997).

We compute the free cumulants of these measures. This leads to some nice formulas for Kummer's confluent hypergeometric series ${}_1F_1(a;b;z)$. Connections between hypergeometric series ${}_1F_1(1 \pm n;2;z)$ and noncrossing partitions $NC(n)$ are obtained. We also discuss the density function and the support of these measures on the torus $\mathbb{T}$.

Piotr Śniady (Wrocław University)

Free cumulants in representation theory

Characters of the symmetric groups can be calculated by a combinatorial formula which involves summation over some graphs drawn on two-dimensional surfaces. The leading term of the character is related to the summands which are planar (graphs drawn on a sphere); this leading term turns out to be a free cumulant of the representation. Surprisingly, the sub-leading terms can be expressed as a polynomial function of the free cumulants (called Kerov polynomial). Similar combinatorial objects show up in the calculation of some matrix integrals; is it possible to extend Kerov polynomials to the domain of the random matrix theory?

Carlos Vargas (Universität Saarbrücken)

Different sized Haar-unitaries arising from random matrix models

Based on the well known results by Voiculescu on the asymptotic freeness of Haar distributed unitary random matrices, we study the asymptotic joint distribution of these matrices when their sizes grow at different rates. Freeness no longer holds between them, but interesting things can be said when amalgamating with a suitable algebra.

The study of such a setting was inspired by a random matrix model, solved recently by Hoydis, Coulliet and Debbah.

Jiun-Chau Wang (University of Saskatchewan)

A new approach to the monotone central limit theorem

Abstract: We will discuss the monotone central limit theorem for identical, unbounded summands. We show that the monotone central limit theorem holds under the same conditions of the classical central limit theorem. Our approach is based on the theory of Abel functional equations, rather than the usual combinatorial approach in the literature.