Abstracts of the talks of the second workshop

Nikita Alekseev (St.-Peterburg State University)

Genus expansion for some ensembles of random matrices

There exists the famous genus expansion of the moments of the eigenvalue distribution of GUE. Namely, we consider a random square $N \times N$ Hermitian Gaussian matrix H with complex entries. It turns out that the k^{th} moment of the eigenvalue distribution of the matrix H, which equals to $\frac{1}{N}\,\mathbf{E}\,\mathrm{Tr}\,H^{2k}$, has a nice topological combinatorial interpretation

$$\mathbf{E}\,\mathrm{Tr}\,H^{2k} = N^{k+1}\sum_{g=0}^{[k/2]} T(k,g)\frac{1}{N^{2g}},$$

where $T(k,g)$ is the number of ways to glue pairwise all the edges of a $2k$-gon so as to produce a surface of a given genus g (see Haagerup and Thorbjornsen, "'Random Matrices with Complex Gaussian Entries"'). Also in this paper the authors consider the complex Wishart case and obtain the asymptotic of $\mathrm{Tr}(XX^*)^k$. In the talk we discuss these theorems and consider generalizations for the real Wishart case and for the case of the singular value distribution of the product of two (or more) independent matrices.

Michael Anshelevich (Texas A&M University)

Two-state free Brownian motions.

A familiar phenomenon in free probability is that many purely algebraic constructions and notions extend to the von Neumann algebra context. This is already the case for the notion of free independence itself. I will show that such behavior need not hold in the two-state free probability theory. Specifically, I will construct a large family of processes which, in the algebraic setting, deserve to be called two-state free Brownian motions. However, in the von Neumann algebra setting, among all these processes, only a one-parameter family exists.

Serban T. Belinschi (Saskatoon)

Convolution semigroups for operator valued distributions

A well-known result of Nica and Speicher states that, unlike in classical probability, any probability measure μ on the real line embeds in a partial FREE convolution semigroup of probability measures $\{\mu^{\boxplus t} : t \geq 1\}$, starting at time t=1. In this talk we shall discuss an extension of this result to operator-valued distributions. The result will be viewed in the context of free infinite divisibility of operator-valued distributions. (This is joint work with M. Popa and V. Vinnikov and with M. Anshelevich, M. Fevrier and A. Nica.)

Florent Benaych-Georges (Université Paris 6)

Finite rank perturbations of random matrices and free probability theory

Let us consider a random Hermitian matrix X which empirical eigenvalue distribution tends to a limit distribution as the dimension tends to infinity and such that the extreme eigenvalues tend to the bounds of the support of the limit distribution (it is for example the case when the X is a Wigner matrix: in this case, the limit distribution is the semi-circle law). We shall add a perturbation to X, and thus consider X +P, under the hypothesis is that the rank of P stays bounded as the dimension tends to infinity and that the eigenspaces of X and P are in generic position one to each other (it is for example the case when X is distributed according to the GUE).

Then, a natural question arises: how are the eigenvalues and the eigenvectors of X perturbed by the addition of P ?

This question had first been asked, for a quite close model, by Johnstone, and been solved, in a several particular cases, by Baik, Ben Arous, Péché, Féral, Capitaine and Donati-Martin.

We shall give a general answer, uncovering a remarkable phase transition phenomenon: the limit of the extreme eigenvalues of the perturbed matrix differs from the original matrix if and only if the eigenvalues of the perturbing matrix are above a certain critical threshold. We also examine the consequences of this eigenvalue phase transition on the associated eigenvectors and generalize our results to examine the case of multiplicative perturbations or of additive perturbations for the singular values of rectangular matrices.

Philippe Biane (Université de Marne la Vallée)

Brownian motion on matrices

The motion of the eigenvalues of a matrix performing Brownian motion is a very interesting object. We describe it using a multidimensional generalization of Pitman's theorem, to provide a probabilistic interpretation of some convexity properties of the Duistermaat-Heckmann measure, whose Fourier transform is the HCIZ integral.

Natasha Blitvic (MIT)

Chords, Norms, and q-Commutation Relations

The q-commutation relations, represented on the q-Fock space of Bo.zjeko and Speicher, interpolate between the classical commutation relations and the classical anti-commutation relations. In this setting, one can construct the q-semicircular and q-circular operators, acting as deformations of the classical Gaussian and complex Gaussian random variables, respectively.

Considering the moments of the q-semicircular and q-circular, we contrast the combinatorial structure of the two operators and provide some new characterizations of their moments. As a surprising consequence, the $2n$-norms of the q-circular turn out to be significantly less well behaved (in a certain analytic sense) than those of q-semicircular. In addition, connecting these moments to several essential combinatorial objects appearing in the classical work of Touchard and Riordan and the recent work of Corteel and Williams provides new indication of the structural depth of the of the q-commutative framework.

Marek Bożejko (Wrocław University)

Deformed Fock spaces, Hecke operators and non-commutative Levy processes for generalized "anyonic" statistics.

(with E.Lytvynov and J.Wysoczanski) We will present the following topics:

1. Fock spaces of Yang-Baxter type.
2. Hecke operators
3. Woronowicz-Pusz CAR operators $T(\mu)$ and connections with Muraki monotone Fock space.
4. "Anyonic" Fock space and Q-CCR relations

$$a(s)a^*(t) - q(s,t)a^*(t)a(s) = \delta(s,t),$$

where $Q = q(s,t)$, $|q(s,t)| = 1$, and s, t are in a non-atomic measure space (T, σ), and a construction of Q-Wick product and Q-Levy processes.

5. Applications to Haagerup approximation property.

Mireille Capitaine (Toulouse)

Free subordination property and deformed matricial models

We will show how the subordination function related to the free additive resp. multiplicative convolution allows to describe the eigenstructure of large additive resp. multiplicative spiked deformations of classical matricial models.

Hayat Cheballah (Université Paris 13)

Gog, Magog and Schützenberger Involution

We describe an approach to finding a bijection between Alternating Sign Matrices (ASM) and Totally Symmetric Self-Complementary Plane Partitions (TSSCPP) which is based on the Schützenberger Involution.

Benoit Collins (University of Ottawa)

Random matrices, representations of $GL(n)$ and free probability of higher order.

We study random matrices whose entries are in the enveloping Lie algebra of $GL(n)$, and show that under suitable conditions, their moments and their fluctuations have the same behaviour as unitarily invariant random matrices. As an application, we generalize previous results about the asymptotic behaviour of representations of $GL(n)$ and obtain new results about their fluctuations. This is joint work with Piotr Sniady.

Stephen Curran (UCLA)

On the symmetric enveloping algebra of planar algebra subfactors

In a recent paper, Guionnet, Jones and Shlyakhtenko gave a diagrammatic method for constructing a subfactor, starting from a planar algebra. In this talk we will give a graphical description of Popa's symmetric enveloping algebras of these subfactors. As an application, we compute a free entropy dimension type quantity associated to these factors. This is based on joint work with D. Shlyakhtenko.

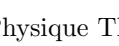
Yoann Dabrowski (UCLA)

Applications of free SDEs to von Neumann algebras of q-gaussian variables.

Abstract : We will explain two kinds of applications of free stochastic differential equations to von Neumann algebras $M_{q,N}$ generated by N q-Gaussian variables (for small q). First, we can use them to compute microstate free entropy dimension of N q-Gaussian variables if $|q|N < 1$ and $|q|N^{1/2} < 0.13$. Its value is identically $\delta_0(X_1, ..., X_N) = N$. In fact we can show q-Gaussian variables have finite Fisher information in this range of q, even though the computation of microstate free entropy dimension is not a consequence and also involves various almost coassociative derivations. Second, we can show that for small q, $M_{q,N}$ have complete metric approximation property (and as a consequence of a result of Popa and Ozawa, they are thus strongly solid). We will discuss how a better understanding of our general dilation results for Markov semigroups (solving variants of free SDEs) could improve the range of q for this second kind of statements.

Nizar Demni (Université de Rennes)

Kanter random variable and positive free stable laws.



Philippe Di Francesco (Institut de Physique Theorique, CEA Saclay)

The Proof of the ASM-DPP Conjecture

We prove a 28-year old conjecture by Mills-Robbins-Rumsey (1983) relating some refined enumerations of Alternating Sign Matrices (ASM) and Descending Plane Partitions (DPP). These are performed by reformulating the enumeration problems in terms of statistical models, namely the 6Vertex model for ASMs and Rhombus tilings/Dimers or Lattice Paths for DPPs. The conjecture then boils down to a determinant identity, which is proved by use of generating function techniques. Remarkably, the main player is the transfer matrix for discrete 1+1-dimensional Lorentzian quantum gravity, which generates random Lorentzian triangulations of the two-dimensional space-time. (This is joint work with Roger Behrend and Paul Zinn-Justin).

Catherine Donati (Université Pierre et Marie Curie Paris)

Truncations of Haar distributed matrices and bivariate Brownian bridge

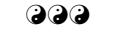
Let U be a Haar distributed matrix on the unitary group or the orthogonal group of size N. Let $p, q \leq N$ and $U_{p,q}$ the left upper corner of U. We prove that after centering, the sequence of two-parameter processes

$$W_{s,t}^{(N)} = Tr(U_{\lfloor Ns \rfloor, \lfloor Nt \rfloor} U_{\lfloor Ns \rfloor, \lfloor Nt \rfloor}^{*}),\ s,t \in [0,1]$$

converges in distribution to the bivariate tied-down Brownian bridge.

Uwe Franz (Université de Franche-Comté)

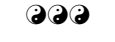
Symmetries of Levy processes on compact quantum groups



Marius Junge (Urbana-Champaign)

Martingales with continuous time and application to brownian motion and dilation

I will briefly discuss a general characterization of Levy concerning brownian motion and how minimal knowledge of stochastic integration (in the noncommutative context) is used towards a classification of certain classes. Another related appearance is the construction of brownian motions from martingales with continuous time parameter following an idea of Doob in the classical case. This construction is useful in construction a free brownian motion driving force behind a semigroup of completely positive unital selfadjoint maps on an arbitrary von Neumann algebra.



Claus Koestler (Aberystwyth University)

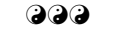
Noncommutative independence from characters of the infinite symmetric group

Recently we have found a new operator algebraic proof of Thoma's theorem which characterizes the extremal characters of the infinite symmetric group. We give an outline of the underlying ideas of our approach and address in particular spectral properties of certain mean ergodic averages of cycles.

Hans Maassen (Nijmegen)

Entanglement of Werner states: greatest cross norm and immanant inequalities

We discuss the greatest cross norm on multiple tensor products of state spaces as a measure of entanglement of quantum states. In particular the completely symmetric (or "Werner") states on $B(H^{\otimes k})$ are expressed in terms of Littlewood's immanants of Gram matrices. Immanant inequalities such as those of Schur and Lieb provide bounds on these Werner states.



Camille Male (ENS Lyon)

The norm of polynomials in random and deterministic matrices

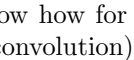
In this talk, I will present a strengthened version of the Asymptotic freeness of Gaussian and deterministic matrices. Let $\mathbf{X}_N = (X_1^{(N)}, \ldots, X_p^{(N)})$ be a family of $N \times N$ independent, normalized random matrices from the Gaussian Unitary Ensemble. We state sufficient conditions on matrices $\mathbf{Y}_N = (Y_1^{(N)}, \ldots, Y_q^{(N)})$, possibly random but independent of $\mathbf{X}_N$, for which the operator norm of $P(\mathbf{X}_N, \mathbf{Y}_N, \mathbf{Y}_N^*)$ converges almost surely for all polynomials P. The method of the proof is based on recent works of Haagerup and Thorbjornsen, where the case $\mathbf{Y}_N = 0$ has been studied.

Alexandru Nica (University of Waterloo)

Convolution powers in operator-valued framework

I will present a recent joint work with Michael Anshelevich, Serban Belinschi and Maxime Fevrier, concerning convolution powers in the framework of an operator-valued noncommutative probability space over a C^*-algebra B. We show how for a B-valued distribution μ one can define convolution powers $\mu^{\boxplus\eta}$ (for free additive convolution) and $\mu^{\uplus\eta}$ (for Boolean convolution) where the exponent η is a suitable positive map from B to B, instead of being just a non-negative real number. We show moreover how these two types of convolution powers can be combined into an "evolution" semigroup related to the Boolean Bercovici-Pata bijection, and we prove some basic properties for this semigroup.

The talk will focus on combinatorial aspects of the B-valued convolution powers, and will complement the talk given by Serban Belinschi (who will discuss some of their analytic aspects).



Maciej A. Nowak (Jagiellonian University Krakow)

Multiplication law and S-transform for non-hermitian random matrices

We derive the multiplication law for free non-hermitian matrices using the planar diagrammatic technique. We define the corresponding non-hermitian S transform being a natural generalization of the Voiculescu S transform. Using examples we show how to use this law to determine the complex eigenvalue density for matrices given as free products of nonhermitian ensembles.

Florin Radulescu (University Roma II)

A "quantum double" representation of the Hecke algebra given by matrix coefficients of discrete series representation of $PSL(2,R)$

Dimitri Shlyakhtenko (UCLA)

Planar algebras and free probability

Piotr Soltan (Warszawa University)

An application of property (T) for discrete quantum groups

A short introduction to property (T) for discrete quantum groups will be given. Using various equivalent descriptions of this property we will be able to solve some questions about existence of so called "exotic" completions of algebras of polynomials on compact quantum groups.

Steen Thorbjørnsen (Aarhus Universitet)

Asymptotic expansions for GUE and Wishart random matrices

For each n in $\mathbb{N}$ let X_n be a (suitably normalized) GUE random matrix, let $g\colon \mathbb{R} \to \mathbb{C}$ be a C^∞-function with all derivatives bounded and let tr_n denote the normalized trace on the $n \times n$ matrices. Reporting on joint work with Uffe Haagerup we describe an analytical approach to the derivation of an asymptotic expansion for the mean value $\mathbb{E}\{\mathrm{tr}_n(g(X_n))\}$, previously established by Ercolani and McLaughlin. Specifically we derive the expansion:

$$\mathbb{E}\big\{\mathrm{tr}_n(g(X_n))\big\} = \frac{1}{2\pi}\int_{-2}^{2} g(x)\sqrt{4-x^2}\,\mathrm{d}x + \sum_{j=1}^{k}\frac{\alpha_j(g)}{n^{2j}} + O(n^{-2k-2}),$$

where k is an arbitrary positive integer. Considered as mappings of g, we describe the coefficients $\alpha_j(g)$, $j \in \mathbb{N}$, as distributions (in the sense of L. Schwarts). We derive a similar asymptotic expansion for the covariance $\mathsf{Cov}\{\mathrm{Tr}_n[f(X_n)], \mathrm{Tr}_n[g(X_n)]\}$, where f is a function of the same kind as g, and $\mathrm{Tr}_n = n\mathrm{tr}_n$. Special focus is drawn to the case where $g(x) = \frac{1}{\lambda - x}$ and $f(x) = \frac{1}{\mu - x}$ for λ, μ in $\mathbb{C} \setminus \mathbb{R}$. We finally describe similar results for Wishart matrices.

Alexander Tikhomirov (Syktyvkar)

On the asymptotics of the spectrum of products and powers of large random matrices

Gabriel Tucci (Alcatel-Lucent)

Random Vandermonde Matrices and Covariance Estimates

The talk will consist of two parts. In the first part we will center on the limit eigenvalue distribution of random Vandermonde matrices with unit magnitude complex entries. The phases of the entries are chosen independently and identically distributed from the interval $[-\pi, \pi]$. Various types of distribution for the phase are considered and we establish the limit eigenvalue distribution in a wide range of cases. We also provide a combinatorial and analytic formula for the sequence of moments. The rate of growth of the maximum and minimum eigenvalue is examined.

In the second part, we will discuss a new approach to the estimation of covariance estimates. The estimation of a covariance matrix from insufficient data is one of the most common problems in multivariate statistics. More specifically, assume we have a set of n independent identically distributed measurements of an m dimensional random vector where $n < m$. The maximum likelihood estimate is the sample covariance matrix but in the case $n < m$ this estimate is singular, and therefore it is a fundamentally bad estimate. In this part we will discuss a new approach to this problem where we use random matrices techniques and free probability.

Yoshimichi Ueda (Kyushu University)

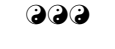
On free product von Neumann algebras

Abstract: I'll report my recent works (arXiv:1011.5017, arXiv:1101.4991) on arbitrary free product von Neumann algebras.

Roland Vergnioux (Université de Caen)

Path cocycles in quantum Cayley trees and L^2-cohomology.

I will report on my work on L^2-cohomology of universal discrete quantum groups. In particular I will present the notion of quantum Cayley graph, which is the geometrical tool of the study, and I will explain the strategy of the proof of the vanishing of the first L^2-Betti number in the orthogonal case.



John David Williams (Indiana University)

Decomposition and Tightness in Free Probability.

I will present some recent results on a 'prime' decomposition for free probability. Time permitting, I will speak about the tightness phenomenon for divisors of a given probability measure that underlie the proof of this Theorem.

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Stanisław Woronowicz (University of Warszawa)

Simplified $E(2)$ quantum group

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Paul Zinn-Justin (Université Pierre et Marie Curie, Paris)

Planar algebras and Potts model on random lattices

We discuss a recent proposal to use random matrix techniques in the context of planar algebras. We focus on a particular case of relevant matrix model, which turns to be equivalent to the Potts model on dynamical random lattices, and solve it. This is joint work with A. Guionnet, V. Jones and D. Shlyakhtenko.