Graz University of Technology

Exercises in **Advanced Probability Theory**

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27.) [3 points] Compute all moments of the standard normal distribution $N(0,1)$ and its characteristic function.

28.) [2 points] *a)* Let X be a real random variable. Express the characteristic function of $a\,X + b$ in terms of the characteristic function of X.

b) Compute the characteristic function of the normal distribution $N(\mu, \sigma^2)$.

29.) [3 points] *a)* Let X, Y be two i.i.d. random variables with characteristic function φ_X $(= \varphi_Y)$. Compute φ_{-X} and φ_{X-Y} in terms of φ_X .

b) Decide if $e^{2(\cos t - 1)}$ is the characteristic function of some random variable.

c) Decide if $e^{-|t|^3}$ is the characteristic function of some random variable.

30.) [3 points] Let $\varphi(t)$ be the characteristic function of the distribution function (in the extended sense) $F(x)$. Show that for any $x \in \mathbb{R}$,

$$F(x) - F(x-) = \lim_{c\to\infty} \frac{1}{2c} \int_{-c}^{c} e^{-\mathrm{i}\,tx}\, \varphi(t)\, dt\,.$$

Hint: the continuous extension of the function $\sin t/t$ has value 1 at $t = 0$.