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Mathematical foundations of information theory & Discrete Stochastics and Information Theory
SS 2018
Exercise Sheet 10

30^{th} of May 2018

Exercise 40 (1 Points)**.** Let $X \in \{1,2,3,4,5,6\}$ be a random variable with distribution

$$\mathbb{P}(X=1) = \frac{10}{16} \quad \mathbb{P}(X=2) = \frac{2}{16} \quad \mathbb{P}(X=3) = \mathbb{P}(X=4) = \mathbb{P}(X=5) = \mathbb{P}(X=6) = \frac{1}{16}\,.$$

Show that one can get two non-isomorphic binary trees by applying the Huffman-Algorithm.

Exercise 41 (2 Points)**.** Let X be the input and Y be the output of a binary symetric channel with transmission probabilities $p(y|x) = \mathbb{P}(Y = y \mid X = x)$ given as

$$Q = \begin{bmatrix} p(0|0) & p(1|0) \\ p(0|1) & p(1|1) \end{bmatrix} = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}.$$

Show that $I(X,Y) = H(p(1-2a)+a) - H(p)$, where $a = \mathbb{P}(X=0)$ is the distribution of the input.

Exercise 42 (2 Points)**.** A channel has a binary input and output alphabets and transition probabilities given by the following matrix:

$$Q = \begin{bmatrix} p(0|0) & p(1|0) \\ p(0|1) & p(1|1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/2 & 1/2 \end{bmatrix}.$$

Find the maximizing input probability of $I(X,Y)$ and the channel capacity.

Exercise 43 (2 Points)**.** Let the random variables X and Y be the input and output, respectively, of a channel. Let $X, Y \in \{0,1,\ldots,n-1\} = \mathcal{X} = \mathcal{Y}$, and let the transition matrix be given as

$$P(Y \mid X) = \begin{bmatrix} p(0|0) & \cdots & p(n-1|0) \\ \vdots & & \vdots \\ p(0|n-1) & \cdots & p(n-1|n-1) \end{bmatrix} = \begin{bmatrix} \frac{1}{n} & \cdots & \frac{1}{n} \\ \vdots & & \vdots \\ \frac{1}{n} & \cdots & \frac{1}{n} \end{bmatrix}.$$

Compute the channel capacity.