Übungsrekapitulation

(1) a) $\sum \frac{2^n + \operatorname{arctag}(n^n) - \sin(n^2)}{n!}$

$\operatorname{arctag}(n^n) \leq \frac{\pi}{2} < 2$

$-\sin(n^2) \leq 1$

$\Rightarrow \left| \frac{2^n + \operatorname{arctg}(n^n) - \sin(n^2)}{n!} \right| \leq \frac{2^n + 3}{n!}$

Daher $\sum \left| \frac{2^n + \operatorname{arctg}(n^n) - \sin(n^2)}{n!} \right| \leq \sum \frac{2^n+3}{n!} < \infty$

denn $\frac{\frac{2^n+3}{n!}}{\frac{2^{n+1}+3}{(n+1)!}} = \frac{(n+1)(2^n+3)}{2^{n+1}+3} \longrightarrow \infty$ Quot. Krit.

$\Longrightarrow \sum \frac{2^n + \operatorname{arctg}(n^n) - \sin(n^2)}{n!}$ abs. konv $\Rightarrow$ konv.

b) $\sum \frac{2\cos(n^2\pi)\, n^2}{n^3+n} \underset{\substack{\uparrow \\ \text{WARUM?}}}{=} \sum \frac{2n^2(-1)^n}{n^3+n}$

$\sum \left| \frac{2n^2(-1)^n}{n^3+n} \right| = \sum \frac{2n^2}{n^3+n} \underset{\substack{\uparrow \\ \text{WARUM?}}}{\geq} \sum \frac{2n^2}{n^3+n^3} = \sum \frac{1}{n}$ div

$\Longrightarrow \sum \frac{2n^2(-1)^n}{n^3+n}$ ist NICHT abs. konv.

aber, sei

$a_n = \frac{2n^2(-1)^n}{n^3+n} \searrow 0$, denn $a_{n+1} < 0$ $\frac{2(n+1)^2}{(n+1)^3+(n+1)}$ ...

$\dots \Leftrightarrow 2n^2 + 2n - 2 > 0$ und $\frac{2n^2(-1)^n}{n^3(1+\frac{1}{n^2})} \longrightarrow 0$

$\Longrightarrow$ Leibnitz $\Longrightarrow \sum a_n$ Konv !

(7) $A = \begin{pmatrix} 1 & -K \\ 1 & -2 \\ 1 & \end{pmatrix}$

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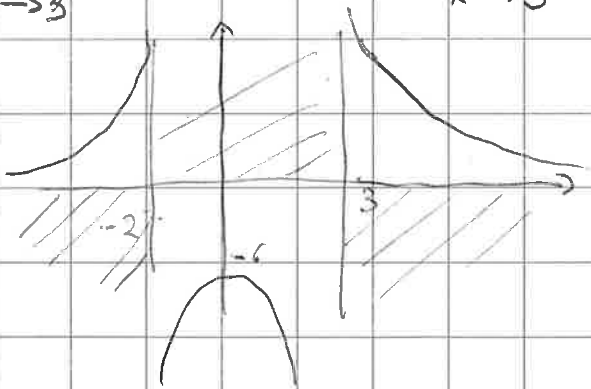
2 a) $f(x) = \dfrac{|x+1|}{x^2-x-6} = \begin{cases} \dfrac{x+1}{x^2-x-6} & x \geq -1 \\ \dfrac{-x-1}{x^2-x-6} & x < -1 \end{cases}$

- $D(f) = \{x \in \mathbb{R} : x^2-x-6 \neq 0\} = \mathbb{R} \setminus \{-2, 3\}$
- $f(x)$ stetig auf ihrem $D(f)$ (WARUM?)
- $f(x) > 0$ für $x < -2 \wedge x > 3$ (WARUM?)
- $\lim\limits_{x \to \pm\infty} f(x) = 0$ (Gr(Nenner) > Gr(Zähler))

$\lim\limits_{x \to 3^+} f(x) = +\infty$, $\lim\limits_{x \to 3^-} f(x) = -\infty$, $\lim\limits_{x \to -2^-} f(x) = +\infty$, $\lim\limits_{x \to -2^+} f(x) = -\infty$



$y = -2$, $x = 3$ vert Asym

$y = 0$ hor Asym.

b) $f(x) = e^{\frac{x}{|x-2|}} = \begin{cases} e^{\frac{x}{x-2}} & x \geq 2 \\ e^{\frac{x}{2-x}} & x < 2 \end{cases}$

$D(f) = \mathbb{R} \setminus \{2\}$, f stetig auf $D(f)$, $f(x) > 0 \ \forall x \in \mathbb{R}$ WARUM?

$\lim\limits_{x \to \pm\infty} f(x) = \begin{cases} e & x \to +\infty \\ e^{-1} & x \to -\infty \end{cases}$, $\lim\limits_{x \to 2^{\pm}} f(x) = \begin{cases} +\infty & x \to 2^+ \\ 0 & x \to 2^- \end{cases}$

$\hookrightarrow \lim\limits_{x \to 2^+} e^{\frac{x}{x-2}} = e^{\lim\limits_{x \to 2^+} \frac{x}{x-2}} = \text{"}e^{\infty}\text{"} = +\infty$, $\lim\limits_{x \to 2^-} f(x) = \text{"}e^{-\infty}\text{"} = 0$

($x \to 2$, $x - 2 \to 0^+$)

③ a) $\lim\limits_{x\to 1^-} e^{\frac{|x+1| \to 2}{x-1 \searrow 0^-}} = \lim\limits_{x\to 1^-} \text{"} e^{-\infty} \text{"} = 0$

b) $\lim\limits_{x\to -\infty}\left(1+\frac{\cos(x^2)}{\ln(|x|+1)}\right)^x$

Sei $f(x) = \frac{\cos(x^2)}{\ln(|x|+1)}$, $\lim\limits_{x\to-\infty} f(x) = 0 \Rightarrow \lim\limits_{x\to-\infty}(1+f(x))^{\frac{1}{f(x)}} = e$

$\Rightarrow \lim\limits_{x\to-\infty}\left(1+\frac{\cos(x^2)}{\ln(|x|+1)}\right)^{(x)} = \lim\limits_{x\to-\infty}\left(1+f(x)\right)^{\frac{1}{f(x)}\cdot f(x)\cdot (x)}$

$= \lim\limits_{x\to-\infty}\Big[\underbrace{(1+f(x))^{\frac{1}{f(x)}}}_{\to e}\Big]^{\frac{x\cos(x^2)}{\ln(1+|x|)}} \to \nexists$ WARUM? $= \nexists$

c) $\lim\limits_{x\to 0^-}\frac{\operatorname{tg}(\ln(1-x))}{\operatorname{sen}(x)} = \left[\lim\limits_{f(x)\to 0}\frac{\operatorname{tg}(f(x))}{f(x)} = \lim\limits_{f(x)\to 0}\frac{\ln(1+f(x))}{f(x)} = \; = \lim\limits_{f(x)\to 0}\frac{\operatorname{sen}(f(x))}{f(x)} = 1\right]$

$\lim\limits_{x\to 0^-} \underbrace{\frac{\operatorname{tn}(\ln(1-x))}{\underbrace{\ln(1-x)}_{f(x)}}}_{\searrow 1} \; \underbrace{\frac{\ln(1-x)}{\underbrace{-x}_{f(x)}}}_{\searrow 1} \; \frac{-x}{\operatorname{sen}(\underbrace{x}_{f(x)})} = -1$

④ a) $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 0 & 1 \\ -1 & 2 & 1 \end{pmatrix} = A \quad \det(A) = -1\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3+6 \neq 0$

$\Rightarrow$ Basis

b) $\begin{pmatrix} -1 & 2 & -3 \\ 3 & -4 & 8 \\ 1 & -3 & 2 \end{pmatrix} = A \quad \det(A) = -\begin{vmatrix} -4 & 8 \\ -3 & 2 \end{vmatrix} - 2\begin{vmatrix} 3 & 8 \\ 1 & 2 \end{vmatrix} + 3\begin{vmatrix} 3 & -4 \\ 1 & -3 \end{vmatrix}$

$= -16 + 4 + 45 \neq 0$

$\Rightarrow$ Basis

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⑥ a) $U_1 = \{(x_1, x_2, x_3, x_4) \mid x_1 - 2x_3 + 3x_4 = 0\}$

$x_1 = 2x_3 - 3x_4$

$$\begin{cases} x_1 = 2\mu - 3t \\ x_2 = \lambda \\ x_3 = \mu \\ x_4 = t \end{cases}$$

$\rightarrow U_1 = \{(2\mu - 3t, \lambda, \mu, t) \mid \lambda, \mu, t \in \mathbb{R}\} =$

$= \{\mu(2, 0, 1, 0) + \lambda(0, 1, 0, 0) + t(-3, 0, 0, 1)\}$
$\mu, \lambda, t \in \mathbb{R}$

$U_1 = \langle (2,0,1,0)\ (0,1,0,0)\ (-3,0,0,1) \rangle$ Basis

$\dim U_1 = 3$

b) $U_2 = \{(x_1, 0, 0, 0)\} = \langle (1,0,0,0) \rangle$ Basis

$\dim U_2 = 1$

7) $A = \begin{pmatrix} 1 & -k & -5 \\ 1 & -2 & -1 \\ k & 1 & -2 \end{pmatrix}$ $\det(A) = 5 - \begin{vmatrix} -k & -5 \\ 1 & -2 \end{vmatrix} + k \begin{vmatrix} -k & -5 \\ -2 & -1 \end{vmatrix}$

$= 5 - (2k+5) + k(k-10) = k^2 - 12k = k(k-12)$

- für $k \neq 0, 12$ Eindeutige Lösung $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$
(WARUM kann man A^{-1} finden?) Schreiben Sie die Lösung!
- für $k = 0$ $\begin{cases} x - 5z = 0 \\ x - 2y - z = 0 \\ y - 2z = 2 \end{cases}$ $\begin{cases} x = 5z \\ 5z - 4z - 4 - z = 0 \\ y = 2z + 2 \end{cases}$ $\lightning$ keine Lösung
- für $k = 12$ $\underbrace{rg(A) = 2 \quad rg(A \mid \begin{smallmatrix} 0 \\ 0 \\ 2 \end{smallmatrix}) = 3}_{\text{MIT GAUß.}} \Rightarrow$ keine Lösung

⑦ $A = \left(\begin{array}{cccc|c} 1 & -k & -5 & 1 & 1 \\ 1 & -2 & -1 & -1 & 0 \\ 1 & 0 & k & 0 & 1 \end{array}\right) \xrightarrow[III-I]{II-I} \left(\begin{array}{cccc|c} 1 & -k & -5 & 1 & 1 \\ 0 & -2+k & 4 & -2 & -1 \\ 0 & k & k+5 & -1 & 0 \end{array}\right) \xrightarrow{III+\frac{1}{2}II} \left(\begin{array}{cccc|c} 1 & -k & -5 & 1 & 1 \\ 0 & k-2 & 4 & -2 & -1 \\ 0 & \frac{k}{2}+1 & k-3 & 0 & \frac{1}{2} \end{array}\right)$

$rg(A) = 3 \quad \forall\, k \in \mathbb{R}$ WARUM?

$$\begin{cases} x = 1 - kz \\ 1 - kz - ky - 5z + w = 1 \\ 1 - kz - z - w = 0 \end{cases}$$

$\Rightarrow w - ky - 5z + w + z = 1 \qquad w = ky + 4z + 1$

$\Rightarrow 1 - kz - z - ky - 4z - 1 = 0 \qquad y = \frac{(k+5)z}{-k} \qquad k \neq 0$

für $k \neq 0$
$$\begin{cases} x = 1 - k\lambda \\ y = \frac{(k+5)\lambda}{-k} \\ z = \lambda \\ w = -k\lambda + 4\lambda + 1 \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -k \\ (k+5)/-k \\ 1 \\ 4-k \end{pmatrix}$$

für $k = 0$
$$\begin{cases} x = 1 \\ -5z + w = 0 \\ 1 - 2y - z - w = 0 \end{cases}$$

$$\begin{cases} x = 1 \\ w = 5z \\ 1 - 2y - 6z = 0 \end{cases} \rightarrow \begin{cases} x = 1 \\ y = (6\lambda - 1)/2 \\ z = \lambda \\ w = 5\lambda \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ -1/2 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ 1 \\ 5 \end{pmatrix}$$