$\boxed{36}$ $y' - \frac{y}{x} = 3x \Rightarrow$

$$y' = \underbrace{\frac{1}{x}}_{a_1(x)} y + \underbrace{3x}_{a_0(x)} \quad ; \quad y(x) = y_{hom}(x) + y_{sp}(x)$$

I $\boxed{\text{Hom}}$ : $y' = \frac{1}{x} y$ $\rightarrow$ Trennung der Variablen

$$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx \Rightarrow \ln|y| = \ln|x| + C \Rightarrow$$

$$|y| = c\,|x| \Rightarrow \boxed{y_{hom} = \pm c x}$$

II Sp. Lösung.

Ansatz: $c \rightarrow c(x) \Rightarrow y_{sp} = c(x)\,x$ ; $c(x) = ?$

$$y'_{sp} = c'(x)\cdot x + c(x)$$

$$\Rightarrow \text{DGL.} \Leftrightarrow c'(x)\cdot x + \cancel{c(x)} - \frac{\cancel{c(x)}\,\cancel{x}}{\cancel{x}} = 3x \Rightarrow$$

$$\Rightarrow c'(x)\cdot x = 3x \Rightarrow c'(x) = 3 \Rightarrow \boxed{c(x) = 3x}$$

$$\Rightarrow y_{sp}(x) = 3x^2 \Rightarrow$$

$$\boxed{y(x) = cx + 3x^2}$$

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$\boxed{37}$ $\underbrace{(2xy - 1)}_{A(x,y)} + \underbrace{(x^2+1)}_{B(x,y)}\, y' = 0$

$$\left.\begin{aligned} A_y &= \frac{\partial A(x,y)}{\partial y} = 2x \\ B_x &= \frac{\partial B(x,y)}{\partial x} = 2x \end{aligned}\right\} \Rightarrow A_y = B_x \Rightarrow \text{Exakte DGL}$$

$\Rightarrow$ Gibt Fkt $F(x,y)$ mit $\nabla F(x,y) = (\underset{F_x}{A}, \underset{F_y}{B}) = (2xy-1, x^2+1)$

$$F_x = 2xy - 1 \Rightarrow F(x,y) = \int (2xy-1)\,dx = x^2 y - x + \boxed{\varphi(y)}$$

$\varphi = ?$

$F_y = B = x^2+1$

<u>Vorher</u> : $F(x,y) = x^2y - x + f(y)$ $\Big\} \Rightarrow$ $x^2 + f'(y) = x^2+1$

$f'(y) = 1 \Rightarrow f(y) = y$

$$\Rightarrow \boxed{F(x,y) = x^2y - x + y}$$

Lösung der DGL : $\boxed{\begin{array}{l} F(x,y) = C \\ x^2y - x + y = C \end{array}}$

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$\boxed{38}$ $\boxed{2x + (x^2 - e^{-y})y' = 0}$ ⊛

$A(x,y) = 2x$

$B(x,y) = x^2 - e^{-y}$

<u>Integrabilitätsbedingung</u> : $A_y = B_x$ ?

$\left.\begin{array}{l} A_y = 0 \\ B_x = 2x \end{array}\right\} A_y \neq B_x \Rightarrow$ DGL nicht exakt

<u>Integrierenden Faktor berechnen</u>

$$\frac{A_y - B_x}{B} \quad \text{oder} \quad \frac{A_y - B_x}{A} = ?$$

$$\frac{A_y - B_x}{B} = \frac{0 - 2x}{x^2 - e^{-y}} \rightarrow \text{nicht Fkt von } x$$

$$\frac{A_y - B_x}{A} = \frac{0 - 2x}{2x} = -1 \Rightarrow \text{eine Fkt von } y$$

Konstante

$$\mu(y) = \exp\left(\int 1\, dy\right) = e^y$$

Gleichung ⊛ wird mit $\mu(y) = e^y$ multipliziert

$$\Rightarrow 2xe^y + (x^2 - e^{-y})e^y \cdot y' = 0 \iff$$

$$\boxed{2xe^y + (x^2e^y - 1)y' = 0}$$ DGL ist exakt

$\Rightarrow \exists F(x,y)$ mit $\operatorname{grad} F = \nabla F = (2xe^y, x^2e^y - 1)$

$F_x = 2xe^y \Rightarrow F(x,y) = \int 2xe^y \, dx \Rightarrow$

$$\boxed{F(x,y) = x^2e^y + \varphi(y)} \quad \varphi(y) = ?$$

$$\frac{\partial F}{\partial y} = x^2e^y - 1$$

$\partial 1$

$$\frac{\partial F}{\partial y} = x^2e^y + \varphi'(y)$$

$\Rightarrow \varphi'(y) = -1 \Rightarrow$

$$\boxed{\varphi(y) = -y}$$

$$\Rightarrow F(x,y) = x^2e^y - y$$

Lösungen: Niveaulinien von $F$ : $F(x,y) = c$

$$\iff \boxed{x^2e^y - y = c}$$

**40** $y'' - 6y' + 9 = 0$

$P(\lambda): \lambda^2 - 6\lambda^1 + 9 = 0 \Rightarrow (\lambda-3)^2 = 0 \Rightarrow \lambda = 3$

$\mu(\lambda) = 2$

$y_{hom}(x) = c_1 e^{3x} + c_2 x e^{3x}$

$y(0) = 1 \Rightarrow \boxed{c_1 = 1}$

$y'(0) = -1 \; ; \; y'(x) = 3c_1 e^{3x} + c_2(e^{3x} + 3x e^{3x})$

$y'(0) = 3c_1 + c_2 = -1 \Rightarrow \boxed{c_2 = -1 - 3 = -4}$

$\Rightarrow \boxed{y_{hom}(x) = e^{3x} - 4x e^{3x}}$

**41** $y'' - 4y' + 5y = 5e^x \cos x$

Hom: $\lambda^2 - 4\lambda + 5 = 0 \Rightarrow \lambda_{1;2} = 2 \pm i$

$\boxed{y_{hom}(x) = c_1 e^{2x} \cos x + c_2 e^{2x} \sin x}$

Sp: $b(x) = \underbrace{5}_{p(x)} e^x \cos x$ $\quad \alpha + i\beta = 1 + i$ keine Lösung von $P(\lambda) \Rightarrow$

$\boxed{y_{sp}(x) = c_1 e^x \cos x + c_2 e^x \sin x}$

$c_1, c_2 = ?$

Einsetzen in DGL u:

$y'_{sp}(x) = (c_1 + c_2) e^x \cos x + (-c_1 + c_2) e^x \sin x$

$y''_{sp}(x) = 2c_2 e^x \cos x - 2c_1 e^x \sin x$

Einsetzen $\Rightarrow 2c_2 e^x \cos x - 2c_1 e^x \sin x -$

$$-4\left((c_1+c_2)e^x\cos x + (-c_1+c_2)e^x\sin x\right) +$$

$$5\left(c_1 e^x\cos x + c_2 e^x \sin x\right) = 5e^x\cos x$$

<u>linke Seite</u>: $= (c_1-2c_2)\,e^x\cos x + (2c_1+c_2)\,e^x\sin x$

<u>rechte Seite</u>: $5\,e^x\cos x + 0\cdot e^x\cdot\sin x$

$$\Rightarrow \begin{cases} c_1 - 2c_2 = 5 \\ 2c_1 + c_2 = 0 \end{cases} \Rightarrow \begin{matrix} c_1 = 1 \\ c_2 = -2 \end{matrix}$$

$$\Rightarrow \boxed{y_{sp} = e^x\cos x - 2e^x\sin x}$$