KV 12 / 25.06.2018



**41** $\boxed{y'' - y = -2x^2 + 8xe^x}$ (*)

I Hom. Lösung : $P(\lambda)$ : $\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$

$\mu(\lambda) = 1$

$y_{hom}(x) = c_1 e^{1\cdot x} + c_2 e^{-1\cdot x}$

$\{e^x, e^{-x}\} \to$ Fundamentalsystem.

II Sp. Lösung : • $b(x) = \underbrace{-2x^2}+ 8xe^x = \underbrace{-2x^2 e^{0x}}_{b_1(x)} + \underbrace{8xe^x}_{b_2(x)}$

$b_1(x) = -2x^2 e^{0x}\cos 0x$

• $b_1(x) = -2x^2 e^{0x}$ ; $\mu(0) = 0$

• $y_{sp,1}(x) = (Ax^2 + Bx + C)$

$y'_{sp,1} = 2Ax + B$

$y''_{sp,1} = 2A$

in (*) einsetzen

$2A - \underline{A}x^2 - Bx - C = \underline{-2}x^2$

$\Rightarrow \boxed{A = 2}$ ; $B = 0$ ; $2A - C = 0 \Rightarrow \boxed{C = 4}$

$\Rightarrow \boxed{y_{sp,1}(x) = 2x^2 + 4}$

• $b_2(x) = 8xe^x$ ; $\mu(1) = 1$ ; $\boxed{b_2(x) = 8xe^x \cos 0x \quad \alpha + i\beta = 1}$

$y_{sp,2} = x(Dx + E)e^x = x^2e^x + Exe^x$

$y'_{sp,2}(x) = D(2xe^x + x^2e^x) + E(e^x + xe^x) =$

$= e^x(Dx^2 + (2D+E)x + E)$

$y''_{sp,2}(x) = e^x(Dx^2 + (2D+E)x + E) + e^x(2Dx + 2D + E)$

$= e^x(Dx^2 + (4D+E)x + (2D+2E))$

Einsetzen : $e^x(Dx^2 + (4D+E)x + (2D+2E)) - e^x(Dx^2 + Ex)$

$= 8xe^x \Rightarrow 4Dx + (2D+2E) = 8x \Rightarrow$

$4D = 8 \Rightarrow \boxed{D = 2}$

$2D + 2E = 0 \Rightarrow \boxed{E = -2}$

$\boxed{y_{sp,2}(x) = (2x^2 - 2x)e^x}$

Gesamt lösung:

$$\boxed{y(x) = 2x^2 + 4 + (2x^2 - 2x)e^{x} + c_1 e^{x} + c_2 e^{-x}}$$

**42** $y'' + 4y' + 4y = -8e^{-2x} + 8\sin 2x$

I Hom Lösung

$P(\lambda): \lambda^2 + 4\lambda + 4 = 0 \Rightarrow \lambda_{1;2} = \frac{-4 \pm \sqrt{16-16}}{2} = -2$

$(\lambda+2)^2 = 0 \Rightarrow \lambda = -2 \; ; \mu(\lambda) = 2$

$$\boxed{y_{hom} = c_1 e^{-2x} + c_2 x e^{-2x}}$$

Fundamentalsystem: $\{e^{-2x}, x e^{-2x}\}$

II Sp. Lösung:

$b(x) = -8e^{-2x} + 8\sin 2x$

$$\begin{cases} b(x) = p(x)e^{\lambda x}\cos x \\ \text{oder} \\ b(x) = p(x)e^{\lambda x}\sin x \end{cases}$$

$b(x) = \underbrace{-8e^{-2x}\cos 0x}_{b_1(x)} + \underbrace{8e^{0x}\sin 2x}_{b_2(x)}$

$b_1(x) = -8e^{-2x}\cos 0x$

$\mu(-2) = 2 \Rightarrow y_{sp,1}(x) = Ax^2 e^{-2x}$

Einsetzen in: $\boxed{y''_{sp,1} + 4y'_{sp,1} + 4y_{sp,1} = b_1(x)}$

$y'_{sp,1}(x) = 2Axe^{-2x} + Ax^2\cdot(-2)e^{-2x}$

$y''_{sp,1}(x) = 2Ae^{-2x} + 2Axe^{-2x}(-2) - 2A\left(2xe^{-2x} - 2x^2e^{-2x}\right)$

③

$$2Ae^{-2x} - \cancel{4Axe^{-2x}} - \cancel{4Axe^{-2x}} + \cancel{4Ax^2e^{-2x}} +$$
$$\cancel{8Axe^{-2x}} - \cancel{8Ax^2}e^{-2x} + \cancel{4}$$
$$+ 4A\cancel{x^2}e^{-2x} = -8e^{-2x}$$

$$\Rightarrow \quad 2Ae^{-2x} = -8e^{-2x} \Rightarrow \boxed{A=-4}$$

$$\boxed{y_{sp,1}(x) = -4x^2e^{-2x}}$$

$$b_2(x) = 8e^{0x}\sin 2x$$

$$\mu(0 \pm 2i) = 0$$

$$\boxed{y_{sp,2}(x) = A\sin 2x + B\cos 2x}$$

$$y'_{sp,2}(x) = 2A\cos 2x - 2B\sin 2x \quad ; \quad y''_{sp,2} = -4A\sin 2x - 4B\cos 2x$$

Einsetzen: $-4A\cancel{\sin 2x} - 4B\cancel{\cos 2x} + 8A\cos 2x - 8B\sin 2x +$
$4A\cancel{\sin 2x} + 4B\cancel{\cos 2x} = 8\sin 2x$

$$\boxed{y''_{sp,2} + 4y'_{sp,2} + 4y_{sp,2} = 8\sin 2x}$$

$$\Rightarrow 8A\cos 2x - 8B\sin 2x = 8\sin 2x$$

$$\Rightarrow \boxed{B=1} \quad \boxed{A=0}$$

$$\boxed{y_{sp,2}(x) = \cos 2x}$$

$$\boxed{y(x) = \underbrace{C_1e^{-2x} + C_2xe^{-2x}}_{y_{hom}(x)} + \underbrace{-4x^2e^{-2x}}_{y_{sp,1}(x)} + \underbrace{\cos 2x}_{y_{sp,2}(x)}}$$

43 $y'' + 9y = 2\sin x$

$P(\lambda)$ : $\lambda^2 + 9 = 0 \Rightarrow \lambda_{1} = 3i$ ; $\mu(3i) = 1$

$\lambda_2 = -3i$ ; $\mu(3i) = 1$

$$y_{hom} = c_1 e^{0x} \cos 3x + c_2 e^{0x} \sin 3x$$

$$\boxed{y_{hom}(x) = c_1 \cos 3x + c_2 \sin 3x}$$

Sp. Lösung : $b(x) = 2e^{0x} \sin x$

$\alpha + i\beta = 0 + 1i$

$\mu(0 + 1i) = 0$

$\Rightarrow y_{sp} = A \sin x + B \cos x$ ; $A, B = ?$

$y'_{sp} = A \cos x - B \sin x$ | Einsetzen

$y''_{sp} = -A \sin x - B \cos x$ | $-A \sin x - B \cos x + 9A \sin x + 9B \cos x = 2\sin x$

$\Rightarrow 8A \sin x + 8B \sin x = 2\sin x \Rightarrow$ $\begin{cases} 8B = 2 \\ 8A = 0 \end{cases} \Rightarrow \begin{matrix} B = \frac{1}{4} \\ A = 0 \end{matrix}$

$$\Rightarrow \boxed{y_{sp}(x) = \frac{1}{4} \cos x}$$

$$\boxed{y(x) = c_1 \cos 3x + c_2 \sin 3x + \frac{1}{4} \cos x}$$