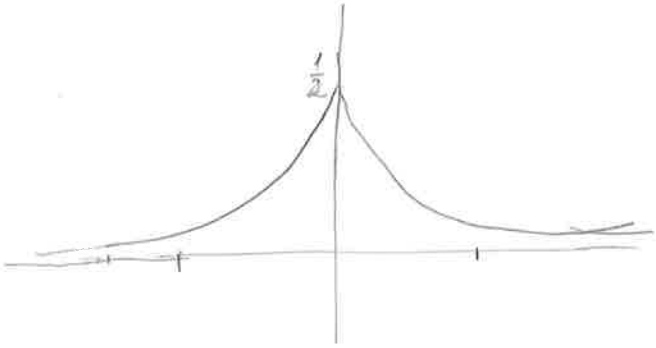
7. Übungsblatt ; 29/11/2016

Ü34 Siehe Seite 3, 6. Übungsblatt.

Ü35

$$f_X(x) = \begin{cases} \frac{1}{2} e^{x} & , x \leq 0 \\ \frac{1}{2} e^{-x} & , x > 0 \end{cases}$$



$x<0$: $F_X(x) = \int_{-\infty}^{x} \frac{1}{2} e^{t} dt = \frac{1}{2} e^{t} \Big|_{-\infty}^{x} = \frac{1}{2} e^{x}$

$x \geq 0$: $F_X(x) = \int_{-\infty}^{0} \frac{1}{2} e^{t} dt + \int_{0}^{x} \frac{1}{2} e^{-t} dt = \frac{1}{2} + \frac{1}{2}(-e^{-t}) \Big|_{0}^{x} = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} e^{-x}$

$$\Rightarrow F_X(x) = \begin{cases} \frac{1}{2} e^{x} & , x < 0 \\ 1 - \frac{1}{2} e^{-x} & , x \geq 0 \end{cases}$$

$$\mathbb{E}[X] = \int_{-\infty}^{+\infty} x f(x) dx = \frac{1}{2} \int_{-\infty}^{0} x e^{x} dx + \frac{1}{2} \underbrace{\int_{0}^{+\infty} x e^{-x} dx}_{-x = y} = \frac{1}{2} \int_{-\infty}^{0} x e^{x} dx - \frac{1}{2} \int_{-\infty}^{0} y e^{y} dy = 0$$

$$Var(X) = \mathbb{E}[X^2] - \underbrace{\mathbb{E}[X]^2}_{0} = \frac{1}{2} \underbrace{\int_{-\infty}^{0} x^2 e^{x} dx}_{A} + \frac{1}{2} \underbrace{\int_{0}^{+\infty} x^2 e^{-x} dx}_{B} =$$

$$A = \int_{-\infty}^{0} x^2 (e^{x})' dx = \underbrace{x^2 e^{x} \Big|_{-\infty}^{0}}_{0} - \int_{-\infty}^{0} 2x e^{x} dx = -2 \int_{-\infty}^{0} x (e^{x})' dx =$$

$$= -2 \left(x e^{x} \Big|_{-\infty}^{0} - \int_{-\infty}^{0} e^{x} dx \right) = +2 \int_{-\infty}^{0} e^{x} dx = 2 e^{x} \Big|_{-\infty}^{0} = 2$$

$$B = \int_{0}^{+\infty} x^2 e^{-x} dx = \int_{-\infty}^{0} y^2 e^{-y} dy = 2 \rightarrow Var(X) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 = 1$$

Ü36 X = # defekten Chips ; $N = 300$; $M = ?$; $n = 20$

$\underbrace{X \sim H(300, 6, 20)}$; $\boxed{M = \frac{2}{100} \cdot 300 = 6}$

a) $\mathbb{E}[X] = n \frac{M}{N} = 20 \frac{6}{300} = 0{,}4$

$Var(X) = 20 \cdot \frac{6}{300} \left(1 - \frac{6}{300}\right) \frac{300 - 20}{300 - 1} = 0{,}367$

2

i) $\mathbb{P}[X<2] = \mathbb{P}[X=0] + \mathbb{P}[X=1] = \frac{\binom{6}{0}\binom{294}{20}}{\binom{300}{20}} + \frac{\binom{6}{1}\binom{295}{19}}{\binom{300}{20}} =$

$\mathbb{P}[X \geq 2] = 1 - \mathbb{P}[X<2] = 1 - \mathbb{P}[X=0] - \mathbb{P}[X=1] = \underline{0{,}054}$

ii) Faustregel: $N = 300 \geq 30 \ \checkmark$

$\frac{1}{15} = \frac{20}{300} = \frac{n}{N} \leq \frac{1}{10} \ \checkmark \quad \Big\} \Rightarrow$

$X \sim H(N, M, n) \approx \underbrace{B\left(n, \frac{M}{N}\right)}_{\sim Y}$

$\frac{M}{N} = \frac{6}{300} = \frac{1}{50}$

$\mathbb{P}[X \geq 2] = 1 - \mathbb{P}[X=0] - \mathbb{P}[X=1] \approx 1 - \mathbb{P}[Y=0] - \mathbb{P}[Y=1] =$

$= 1 - \binom{20}{0}\left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{20} - \binom{20}{1}\left(\frac{1}{50}\right)^1 \left(\frac{49}{50}\right)^{19} = \underline{0{,}06}$

Ü37 $\quad p$ = Erfolgsw'keit $= \frac{12}{37}$

X = # Gewinne ; $\quad X \sim B\left(n, \frac{12}{37}\right)$

$n = ?$: $\mathbb{P}[X \geq 1] \geq \frac{98}{100} \Leftrightarrow 1 - \mathbb{P}[X=0] \geq \frac{98}{100} \Rightarrow$

$1 - \underbrace{\binom{n}{0}}_{1}\underbrace{\left(\frac{12}{37}\right)^0}_{1} \cdot \left(\frac{25}{37}\right)^n \geq \frac{98}{100} \Rightarrow \frac{2}{100} \geq \left(\frac{25}{37}\right)^n \Rightarrow$

$\Rightarrow n \log \frac{25}{37} \leq \log(0{,}02) \Rightarrow n \geq 9{,}98 \Rightarrow \boxed{n_{min} = 10}$

Ü38 $\quad n = 15 \quad p =$ Verspätungswahrscheinlichkeit $= \frac{1}{4}$

Dann $X \sim B(15, \frac{1}{4})$

a) $P[X \leq 2] = 1 - P[X=0] - P[X=1] - P[X=2] =$

$$= 1 - \binom{15}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{15} - \binom{15}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^{14} - \binom{15}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^{13} =$$

$$= \underline{\underline{0{,}236}}$$

b) Poisson-approx: $\left.\begin{array}{l} n = 15 \overset{?}{\geq} 30 \\ \frac{1}{4} = p \leq \frac{1}{10} \end{array}\right\}$ nicht erfüllt

$\Rightarrow$ Poissonvtlg. approximiert nicht gut $X \sim B(15, \frac{1}{4})$

$Y \sim P(\frac{15}{4})$

Falls $P[X \leq 2] \approx P[Y \leq 2] = \underline{\underline{0{,}277}}$

$Y \sim P(\frac{15}{4})$

Ü39 $\quad L =$ Lebensdauer ; $L \sim Exp(\lambda)$

$E[L] = 3 = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{3} \Rightarrow \boxed{L \sim Exp(\frac{1}{3})}$

$$f_L(x) = \begin{cases} \frac{1}{3} e^{-\frac{1}{3}x} & , x > 0 \\ 0 & , x \leq 0 \end{cases} \quad ; \quad F_L(x) = \begin{cases} 1 - e^{-\frac{1}{3}x} & , x > 0 \\ 0 & , x \leq 0 \end{cases}$$

a) $p = P[\text{Erfolgs}] = P[\text{Päckchen verdorben nach 4 Monaten}] =$

$= P[L \leq 4] = F_L(4) = 1 - e^{-\frac{1}{3}\cdot 4} = 0{,}73$

Sei $Y =$ Anzahl verdorbenen Päckchen aus 50 Stück

Dann $Y \sim B(50, \underbrace{0{,}73}_{p})$

$$P[Y \leq 5] = ? = \sum_{k=0}^{5} \binom{50}{k} (0{,}73)^k (1 - 0{,}73)^{50-k} = \text{---}$$

(b) Sei G = Einnahme

$$G = \begin{cases} 10\cdot(50-k), & \text{falls } Y = 50-k \\ -15\cdot k, & \text{falls } Y = k \end{cases}$$

$\Leftrightarrow$

$$G = 10\cdot(50-Y) - 15\cdot Y$$

$Y \sim B(50, \underbrace{0{,}73}_{p})$; $\mathbb{E}[Y] = 50\cdot 0{,}73 = 36{,}82$

$$\mathbb{E}[G] = \mathbb{E}[10(50-Y) - 15\cdot Y] = 500 - 10\cdot\mathbb{E}[Y] - 15\,\mathbb{E}[Y] =$$

$$= 500 - 25\,\mathbb{E}[Y] = 500 - 920{,}5 = -420{,}5$$

$\Rightarrow$ $\boxed{\mathbb{E}[G] = -420{,}5}$