(46) X = Anzahl 6 ; $X \sim B(15000, \frac{1}{6})$

Approx De Moivre-Laplace:

Wenn $n \min\{p, \frac{1}{1-p}\} = 15.000 \cdot \frac{1}{6} > 5$ ✓

$$\mathbb{P}[l \le X \le k] = \Phi\left(\frac{k+\frac{1}{2}-np}{\sqrt{np(1-p)}}\right) - \Phi\left(\frac{l-\frac{1}{2}-np}{\sqrt{np(1-p)}}\right) \Rightarrow$$

$$\mathbb{P}[2380 \le X \le 2550] = \Phi\left(\frac{2550+\frac{1}{2}-2500}{\sqrt{2083,33}}\right) - \Phi\left(\frac{2380-\frac{1}{2}-2500}{\sqrt{2083,33}}\right)$$

$$= \Phi\left(\frac{50,5}{45,64}\right) - \phi\left(\frac{-120,5}{45,64}\right) = \Phi(1,10) - \Phi(-2,64) = 0,8643 -$$

$$(1-\Phi(2,64)) = 0,8643 - 1 + 0,9959 = \underline{0,8602}$$

(51) $\mathbb{P}[0 \le X \le c] = 1$ z.z. $\boxed{Var(X) \le \frac{c^2}{4}}$

$0 \le X \le c \Rightarrow \mathbb{P}[0 \le X^2 \le cX] = 1$

$\mathbb{E}[X^2] \le \mathbb{E}[cX] = c\,\mathbb{E}[X]$

$Var(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$

$\mathbb{E}[X^2] \le c\mathbb{E}[X]$

$$Var(X) \le c\mathbb{E}[X] - (\mathbb{E}[X])^2 = \cancel{\mathbb{E}[X](c-\mathbb{E}[X])} =$$

$$= c^2\left[\frac{\mathbb{E}[X]}{c} - \left(\frac{\mathbb{E}[X]}{c}\right)^2\right] = c^2\,\alpha(1-\alpha)$$

mit $\alpha = \frac{\mathbb{E}[X]}{c}$

Z. zeigen $\boxed{\alpha \le \frac{1}{4}}$

$0 \le X \le c \Rightarrow 0 \le \mathbb{E}[X] \le c \Rightarrow$

$0 \le \frac{\mathbb{E}[X]}{c} \le 1 \Rightarrow \boxed{0 \le \alpha \le 1}$

Ist $\boxed{\alpha(1-\alpha) \le \frac{1}{4} \text{ für } \alpha \in [0,1]^c ?}$

Mathe A: $\frac{\partial}{\partial \alpha}\alpha(1-\alpha) = 1 - 2\alpha = 0 \Rightarrow \alpha = \frac{1}{2}$; $f(\frac{1}{2}) = \frac{1}{2}(1-\frac{1}{2}) = \frac{1}{4}$

max ist in $\frac{1}{4}$ angenommen!

(47) $X \sim \mathcal{N}(100, 2)$; $Z = \frac{X-100}{2} \sim \mathcal{N}(0,1)$

a) $\mathbb{P}[X \geq 97] = 1 - \mathbb{P}[X \leq 97] = 1 - \mathbb{P}\left[\frac{X-100}{2} \leq \frac{97-100}{2}\right] =$

$1 - \mathbb{P}[Z \leq -1{,}5] = 1 - \Phi(-1{,}5) = 1 - (1 - \Phi(1{,}5)) =$

$= \Phi(1{,}5) = \underline{\underline{0{,}9332}}$

b) $c = ?$ $\mathbb{P}[100 - c \leq X \leq 100 + c] = \frac{95}{100}$ $\Leftrightarrow$

$\mathbb{P}\left[\frac{100-c-100}{2} \leq \underbrace{\frac{X-100}{2}}_{Z} \leq \frac{100+c-100}{2}\right] = \mathbb{P}\left[-\frac{c}{2} \leq Z \leq \frac{c}{2}\right] = \frac{95}{100}$

$\Rightarrow \Phi\left(\frac{c}{2}\right) - \Phi\left(-\frac{c}{2}\right) = \Phi\left(\frac{c}{2}\right) - \left(1 - \Phi\left(\frac{c}{2}\right)\right) = 2\Phi\left(\frac{c}{2}\right) - 1$

$= \frac{95}{100} \Rightarrow 2\Phi\left(\frac{c}{2}\right) = \frac{195}{100} \Rightarrow \Phi\left(\frac{c}{2}\right) = \underbrace{\frac{195}{200}}_{0{,}975} \Rightarrow$

$\Rightarrow \frac{c}{2} = 1{,}96 \Rightarrow \boxed{c = 3{,}92}$

c) $p = \mathbb{P}[X \leq 97] = 1 - \mathbb{P}[X \geq 97] = 1 - 0{,}9332 = 0{,}0668$.

$Y \sim \mathcal{B}(20; 0{,}0668)$

$\mathbb{P}[Y \leq 3] = \sum_{k=0}^{3} \binom{20}{k} (0{,}0668)^k (0{,}9332)^{20-k} = 0{,}9593$

d) $\sigma = ?$

$\mathbb{P}[X \geq 97] = \mathbb{P}\left[\frac{X-100}{\sigma} \geq \frac{97-100}{\sigma}\right] = 1 - \mathbb{P}\left[\frac{X-100}{\sigma} \leq \frac{-3}{\sigma}\right] =$

$1 - \Phi\left(\frac{-3}{\sigma}\right) = \frac{97}{100} \Rightarrow \Phi\left(\frac{3}{\sigma}\right) = 0{,}97 \Rightarrow \frac{3}{\sigma} = 2{,}06 \Rightarrow$

Tabelle

$\sigma = \frac{3}{2{,}06} = 1{,}456$

48) $X \sim \mathcal{N}(5,2)$

a) $\mathbb{P}[X < c] = \frac{95}{100} \Leftrightarrow \mathbb{P}[\underbrace{\frac{X-5}{2}}_{N(0,1)} < \frac{c-5}{2}] = 0,95 \Rightarrow$

$\Phi\left(\frac{c-5}{2}\right) = 0,95 \Rightarrow \frac{c-5}{2} = 1,65 \Rightarrow \boxed{c = 8,3}$

b) $\mathbb{P}[5-d < X < 5+d] = 98\% \Leftrightarrow \mathbb{P}[-\frac{d}{2} < \underbrace{\frac{X-5}{2}}_{N(0,1)} < \frac{d}{2}] =$

$= \Phi\left(\frac{d}{2}\right) - \Phi\left(\frac{-d}{2}\right) = 2\Phi\left(\frac{d}{2}\right) - 1 = 0,98$

$\Rightarrow \Phi\left(\frac{d}{2}\right) = \frac{1,98}{2} \Rightarrow \Phi\left(\frac{d}{2}\right) = 0,99 \Rightarrow \frac{d}{2} = 2,33$

$\Rightarrow \boxed{d = 4,66}$

49) $\mathbb{P}[X > 9] = \mathbb{P}[\frac{X-5}{\sigma} > \frac{9-5}{\sigma}] = 1 - \mathbb{P}[\frac{X-5}{\sigma} \leq \frac{4}{\sigma}] = 0,2 \Rightarrow$

$X \sim \mathcal{N}(5,\sigma)$

$\Rightarrow 1 - \Phi\left(\frac{4}{\sigma}\right) = 0,2 \Rightarrow \Phi\left(\frac{4}{\sigma}\right) = 0,8 \Rightarrow \frac{4}{\sigma} = 0,85 \Rightarrow$

$\sigma = 4,70$

$\sigma = \sqrt{Var(X)} = 4,70 \Rightarrow \boxed{Var(X) = 22,9}$

50) $X \sim Exp(1) \Rightarrow f_X(x) = \begin{cases} e^{-x} & , x > 0 \\ 0 & , x \leq 0 \end{cases} \Rightarrow F_X(x) = \begin{cases} 1 - e^{-x} & , x > 0 \\ 0 & , x \leq 0 \end{cases}$

$Y = \log X$

$F_Y(y) = \mathbb{P}[Y \leq y] = \mathbb{P}[\log X \leq y] = \mathbb{P}[X \leq e^y] = F_X(e^y) =$

$= 1 - e^{-e^y}, \forall y \Rightarrow f_Y(y) = \frac{\partial}{\partial y} F_Y(y) = e^y \cdot e^{-e^y}$

$f_Y(y) = e^y \cdot e^{-e^y}$