Technische Universität Graz

15th Austrian Numerical Analysis Day

Graz, May 9–10, 2019

Olaf Steinbach

Berichte aus dem
Institut für Angewandte Mathematik

Book of Abstracts 2019/1

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Time Schedule

Thursday, May 9, 2019

12:00		Registration
12:30	Michael Feischl	Multi-Index Finite Element Method
12:50	Joscha Gedicke	Robust adaptive *hp* discontinuous Galerkin finite element methods for the Helmholtz equation
13:10	Gerhard Unger	Boundary element methods for the modal approximation of plasmonic transmission problems
13:30	Stefan Takacs	Multigrid solvers for isogeometric multi-patch discretizations coupled by discontinuous Galerkin approaches
13:50	Maximilian Bernkopf	Optimal convergence rates in L^2 for a first order system least squares finite element method
14:10	Marco Zank	Realisation of a Galerkin-Bubnov FEM for the Heat Equation
14:30		Coffee Break
15:00	Walter Zulehner	A unified approach for mixed formulations of elliptic problems with application to models in structural mechanics
15:20	Lukas Kogler	An Algebraic Multigrid Method for Linear Elasticity
15:40	Patrick Bammer	Error Estimates for *hp*-FEM in Elastoplasticity
16:00	Douglas Pacheco	Pressure Poisson equation for the Stokes system: An ultra-weak formulation
16:20	Jana Fuchsberger	Simulating a Heart Valve using a Varying Permeability Approach
16:40		Coffee Break
17:00	Mirko Residori	A new splitting approach for dispersive problems on unbounded domains
17:20	Eric Setterqvist	Taut Strings and Total Variation Denoising on Graphs
17:40	Carl-Martin Pfeiler	Dörfler marking with minimal cardinality is a linear complexity problem
18:00	Jarle Sogn	Initial source recovery of the wave equation given internal boundary measurements
18:20	Manfred Liebmann	Reinforcement Learning for Hybrid Electric Vehicle Drivetrain System Controllers
19:00		Dinner at Gösser Bräu (Neutorgasse 48)

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8:30	Matthias Taus	Fast solvers for time-harmonic wave propagation in a parallel computational environment
8:50	Paul Stocker	Tent pitching and a Trefftz-DG method for the acoustic wave equation
9:10	Sjoerd Geevers	Doubling the convergence rate by pre- and post-processing the finite element approximation for linear wave problems
9:30	Michael Neunteufel	The Hellan–Herrmann–Johnson Method for Nonlinear Shells
9:50	Alexander Pichler	A virtual element method for the miscible displacement problem
10:10	Christof Haubner	A Priori Error Estimates for the Optimal Control of a Simplified Signorini Problem
10:30		Coffee Break
11:00	Julia I.M. Hauser	Variational Formulations for Maxwell's Equations
11:20	Victor A. Kovtunenko	Numerical Identification of Inhomogeneities by Inverse Scattering Using Topological Optimality Condition
11:40	Raphael Watschinger	Fast Directional Matrix-Vector Multiplications - Algorithm and Parameter Study
12:00	Tobias Danczul	A Reduced Basis Method for Fractional Diffusion Operators
12:20	Markus Faustmann	Optimal adaptivity and preconditioning for the fractional Laplacian
12:40	Clemens Hofreither	Fast Solvers for Isogeometric Analysis based on Alternating Linear Schemes for Tucker Tensors
13:00		Closing

Abstracts in Chronological Order

Multi-Index Finite Element Method

Michael Feischl[1], Josef Dick[2] and Christoph Schwab[3]

[1] TU Wien
[2] UNSW Sydney
[3] ETH Zürich

We show rigorous error and cost estimates for a multi-index MC method which includes the finite-element approximation, the approximation of the random field, and the Monte-Carlo quadrature error. This improves the cost estimates compared to multi-level algorithms for similar problems and mathematically underpins the outstanding practical performance of multi-index algorithms for partial differential equations with random coefficients.

Robust adaptive *hp* discontinuous Galerkin finite element methods for the Helmholtz equation

Joscha Gedicke[1], Scott Congreve[2], Ilaria Perugia[1]

[1] Faculty of Mathematics, University of Vienna, Vienna, Austria
[2] Faculty of Mathematics and Physics, Charles University, Praha, Czech Republic

This talk presents an *hp* a posteriori error analysis for the 2D Helmholtz equation that is robust in the polynomial degree p and the wave number k. For the discretization, we consider a discontinuous Galerkin formulation that is unconditionally well posed. The a posteriori error analysis is based on the technique of equilibrated fluxes applied to a shifted Poisson problem, with the error due to the nonconformity of the discretization controlled by a potential reconstruction. We prove that the error estimator is both reliable and efficient, under the condition that the initial mesh size and polynomial degree is chosen such that the discontinuous Galerkin formulation converges, i.e., it is out of the regime of pollution. We confirm the efficiency of an *hp*-adaptive refinement strategy based on the presented robust a posteriori error estimator via several numerical examples.

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[1] S. Congreve, J. Gedicke, and I. Perugia. Robust adaptive hp discontinuous Galerkin finite element methods for the Helmholtz equation. *SIAM J. Sci. Comput.*, accepted, 2019.

Boundary element methods for the modal approximation of plasmonic transmission problems

Gerhard Unger[1], Ulrich Hohenester[2], Andreas Trügler[2]

[1] TU Graz
[2] University of Graz

Light causes on the surface of metallic nanoparticles coherent charge oscillations, so-called surface plasmons. These are responsible that metallic nanoparticles have special optical properties which can hardly be achieved by other optical materials. The interaction of light with metallic nanoparticles is modeled as a transmission problem for Maxwell's equations. The concept of resonances and modes for the description of surface plasmons has recently received great interest, both in the context of efficient simulations as well as for an intuitive interpretation in physical terms. While resonance modes have been successfully employed for geometries whose optical response is governed by a few modes only, the resonance mode description exhibits considerable difficulties for larger nanoparticles with their richer mode spectra. In this talk we analyze the problem using a boundary element method approach and identify the fixed link between the electric and magnetic components in the modal expansion of the optical response as the main source for this shortcoming. We suggest a novel modal approximation scheme that allows in principle to overcome this problem by proposing separate coefficients of the the electric and magnetic components of the modal expansion.

References

[1] U. Hohenester, A. Trügler, G. Unger: Novel Modal Approximation Scheme for Plasmonic Transmission Problems. Phys. Rev. Lett. 121 (2018), 246802.

Multigrid solvers for isogeometric multi-patch discretizations coupled by discontinuous Galerkin approaches

Stefan Takacs

Johann Radon Institute for Computational and Applied Mathematics (RICAM),
Austrian Academy of Sciences, Austria

Isogeometric Analysis is a spline-based discretization for partial differential equations (PDEs) which allows to archive convergence rates of a high-order discretization for costs (in terms of the number of unknowns) of a low-order method. The spline-based discretization is first constructed on some simple parameter domain, like the unit square and then mapped to the domain of interest using a global geometry parameterization. More complicated domains are typically decomposed into subdomains, usually called patches, where each of them is parameterized separately. The patches can be coupled in a strong sense or using discontinuous Galerkin approaches. The latter allows non-matching discretizations on the interfaces and extends easier to high-order PDEs. The focus of the talk is set on the solution of the resulting systems with multigrid solvers. In principle, standard Jacobi or Gauss-Seidel smoothers can be applied also in the framework of isogeometric analysis, however their convergence behavior deteriorates significantly if the spline degree is increased. So, we introduce alternative approaches and compare them with the standard Jacobi or Gauss-Seidel smoothers and discuss their advantages and disadvantages. We will see numerous numerical experiments and discuss what can be explained by the convergence theory.

Optimal convergence rates in L^2 for a first order system least squares finite element method

Maximilian Bernkopf, Jens Markus Melenk

TU Wien

We consider a Poisson-like second order model problem written as a system of first order equations. For the discretization an $\boldsymbol{H}(\Omega, \mathrm{div}) \times H^1(\Omega)$-conforming least squares formulation is employed. A least squares formulation has the major advantage that regardless of the original formulation the linear system resulting from a least squares type discretization is always positive semi-definite, making it easier to solve. Even though our model problem in its standard $H^1(\Omega)$ formulation is coercive our methods and lines of proof can most certainly be applied to other problems as well, see [2, 3] for an application to the Helmholtz equation. A major drawback of a least squares formulation is that the energy norm is somewhat intractable. Deducing error estimates in other norms, e.g., the $L^2(\Omega)$ norm of the scalar variable, is more difficult. Numerical examples in our previous work [2] suggested convergence rates previous results did not cover. Closing this gap in the literature will be the main focus of the talk. To that end we showcase a duality argument in order to derive L^2 error estimates of the scalar variable, which was the best available estimate in the literature. We then perform a more detailed analysis of the corresponding error terms. This analysis then leads to improved convergence rates of the method. The above procedure can then be applied to more complicated boundary conditions, for which an analogous result is a nontrivial task. As a tool, which is of independent interest, we develop $\boldsymbol{H}(\Omega, \mathrm{div})$-conforming approximation operators satisfying certain orthogonality relations. For the analysis, a crucial tool are recently developed projection based commuting diagram operators, see [4].

References

[1] M. Bernkopf and J.M. Melenk, Optimal convergence rates in L^2 for a first order system least squares finite element method, in preparation

[2] M. Bernkopf and J.M. Melenk, Analysis of the *hp*-version of a first order system least squares method for the Helmholtz equation, arXiv:1808.07825

[3] H. Chen and W. Qiu A first order system least squares method for the Helmholtz equation, *Journal of Computational and Applied Mathematics* **309** (2017), pp. 145–162.

[4] J.M. Melenk and C. Rojik, On commuting p-version projection-based interpolation on tetrahedra, arXiv:1802.00197

Realisation of a Galerkin-Bubnov FEM for the Heat Equation

Marco Zank

University of Vienna

For the discretisation of time-dependent partial differential equations, the standard approaches are explicit or implicit time stepping schemes together with finite element methods in space. An alternative approach is the usage of space-time methods, where the space-time domain is discretised and the resulting global linear system is solved at once. In this talk, the model problem is the heat equation. First, a space-time variational formulation in anisotropic Sobolev spaces for the heat equation is discussed, where a linear isometry $\mathcal{H}_T$ is used such that ansatz and test spaces are equal, see [1, 2] for details. A conforming discretisation of this space-time variational formulation in anisotropic Sobolev spaces leads to a Galerkin-Bubnov finite element method, which is unconditionally stable, i.e. no CFL condition is required. However, for the implementation of this method, the realisation of the linear isometry $\mathcal{H}_T$ is crucial. The main part of this talk investigates possible ways of doing this realisation for piecewise linear, globally continuous ansatz and test functions. In the last part of the talk, numerical examples are shown and discussed.

References

[1] O. Steinbach and M. Zank: Coercive space-time finite element methods for initial boundary value problems, *Submitted*, 2018.

[2] M. Zank: Inf-Sup Stable Space-Time Methods for Time-Dependent Partial Differential Equations, PhD thesis, TU Graz, 2019.

A unified approach for mixed formulations of elliptic problems with application to models in structural mechanics

Walter Zulehner[1], Dirk Pauly[2]

[1] Johannes Kepler University Linz
[2] University Duisburg-Essen

A general and rather flexible approach will be presented how to derive mixed variational formulations of elliptic problems. The approach is based on the concept of densely defined linear operators and their adjoints, rather than on the well-known technique of integration by parts, which is typically used for the construction of mixed variational formulations otherwise. The construction of the mixed formulation starts with the primal variational formulation rather than the strong (or classical) form of the problem. This allows to address the relation between the primal and the mixed problem quite thoroughly.

As a particular application of the approach, the Reissner-Mindlin plate bending model is discussed, for which a decomposition of the problem into three simpler second-order problems is shown.

An Algebraic Multigrid Method for Linear Elasticity

Lukas Kogler, Joachim Schöberl

TU Wien

In this talk we present a new variation of Smoothed Aggregation AMG for linearized elasticity. The method directly incorporates rigid body modes into the coarse spaces, which feature both translational and rotational degrees of freedom. Coarsening and smoothing are both based on a replacement matrix which induces an equivalent norm to the energy norm. It is, however, of a simpler structure than the system matrix, as it only consists of edge contributions which are computed from element matrices during the assembly loop and can be kept unassembled. This gives us a simple way to smooth out coarse basis functions while preserving rigid body modes.

Error Estimates for *hp*-FEM in Elastoplasticity

Patrick Bammer, Lothar Banz, Andreas Schröder

University of Salzburg

In this talk we consider a variational inequality formulation as well as a equivalent mixed variational formulation for a model problem in elastoplasticity with linear kinematic hardening and present *hp*-discretizations for both formulations. We prove that the discrete variational inequality formulation is equivalent to the discrete mixed formulation and show the unique existence of discrete solutions. The main focus is on the derivation of a-priori error estimates and reliable and efficient a-posteriori error estimates.

Pressure Poisson equation for the Stokes system: An ultra-weak formulation

Douglas R. Q. Pacheco, Olaf Steinbach

Institute of Applied Mathematics, Graz University of Technology

For the standard variational formulation of the Stokes system, we consider the pressure Poisson equation by choosing gradient fields as appropriate test functions. This results in an ultra-weak variational problem to find the pressure in L^2, which allows the use of a piece-wise constant approximation for the pressure. In this case, the test functions must be of second order to guarantee conformity and stability. We derive the ultra-weak formulation and present several applications of this approach which can be used for pressure retrieval from measured velocities, stabilization of lowest-order finite element approximations for the Stokes system, and the analysis and simulation of Stokes eigenvalue problems.

Simulating a Heart Valve using a Varying Permeability Approach

Jana Fuchsberger[1], Gundolf Haase[1], Elias Karabelas[2], Gernot Plank[2]

[1] University Graz
[2] Medical University Graz

Models of total heart function include computational fluid dynamics models of blood flow. The effect of heart valves upon flow patterns can be taken into account by a fictitious domain approach in combination with the Navier-Stokes-Brinkman equations.

The motion of the valve is represented by means of a spatio-temporal varying permeability function while the underlying mesh remains unchanged.

We present first proof-of-concept simulations of blood flow in the left ventricle and aorta to demonstrate feasibility.

A new splitting approach for dispersive problems on unbounded domains

Mirko Residori, L. Einkemmer, A. Ostermann

University of Innsbruck

We propose a new numerical method for solving the linearized Korteweg–de Vries (KdV) equation

$$u_t + g(x)u_x + u_{xxx} = 0, \quad (t, x) \in [0, T] \times \mathbb{R} \tag{1}$$

with initial data $u(0, x) = u_0(x)$. In order to perform numerical simulations, we truncate the unbounded domain $\mathbb{R}$ to a finite computational domain $[a, b]$, $a < b$. Boundary conditions must be imposed on a, b. We employ the so called transparent boundary conditions. These conditions allow us to compute numerical solutions that approximate the solution over the unbounded domain $\mathbb{R}$ restricted to the interval $[a, b]$. We follow a splitting strategy in order to solve separately the advective and the dispersive part of equation (1). For this purpose, we employ a Lie–Trotter splitting. We then take advantage of the employed Lie–Trotter splitting by carrying out the spatial discretization by a pseudo-spectral approach, as it has been done for the pure dispersive equation in [1]. This approach gives us a very accurate spatial numerical solution using a modest number of grid points.

References

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Taut Strings and Total Variation Denoising on Graphs

Eric Setterqvist[1], Clemens Kirisits[1], Otmar Scherzer[1,2]

[1] Faculty of Mathematics, University of Vienna, Austria
[2] Johann Radon Institute for Computational and Applied Mathematics (RICAM), Austrian Academy of Sciences, Linz, Austria

For one-dimensional discrete signals, it is known that total variation flow, total variation regularization and the taut string algorithm are equivalent filters. A notable feature of the filtered signal, inherited from the taut string algorithm, is that it simultaneously minimizes a large number of convex functionals in a certain neighbourhood of the data. In this talk we address the question to what extent this situation can be carried over to a more general setting. We consider data given on the vertices of a finite oriented graph and total variation defined as $J(f) = \sum_{i,j} |f(v_i) - f(v_j)|$ for adjacent vertices v_i and v_j. It turns out that the minimizer of the corresponding Rudin-Osher-Fatemi (ROF) model on the graph may be viewed as the outcome of a generalized taut string algorithm. In particular, the minimizer has the same universal minimality property as in the one-dimensional setting. However, this property is lost if J is replaced by discrete isotropic total variation. Next, we show that, in contrast to the one-dimensional setting, the minimizer of the ROF model and the solution to the gradient flow for J do not coincide in general.

Dörfler marking with minimal cardinality is a linear complexity problem

Carl-Martin Pfeiler and Dirk Praetorius

TU Wien

Adaptive finite element methods (AFEM) iterate the procedure *Solve-Estimate-Mark-Refine* to generate a sequence of locally refined meshes $(\mathcal{T}_\ell)_{\ell\in\mathbb{N}_0}$, where the degrees of freedom are chosen more carefully than for uniform mesh refinement: First, the discrete solution is computed on the given mesh $\mathcal{T}_\ell$. Then, local refinement indicators $(\eta_\ell(T))_{T\in\mathcal{T}_\ell}$ are computed. Based on these indicators a subset $\mathcal{M}_\ell \subseteq \mathcal{T}_\ell$ is marked for refinement. Finally, (at least) the marked elements are refined to obtain an improved mesh $\mathcal{T}_{\ell+1}$.

In his seminal work [1], Dörfler proposes a marking criterion, which allows to prove linear convergence of the energy error for each iteration of the AFEM algorithm. This marking criterion is commonly known as *Dörfler marking*: Given $\eta_\ell(T)$ for all $T \in \mathcal{T}_\ell$ and a marking parameter $0 < \theta \leq 1$, construct a set $\mathcal{M}_\ell \subseteq \mathcal{T}_\ell$ such that

$$\theta \sum_{T\in\mathcal{T}_\ell} \eta_\ell(T)^2 \leq \sum_{T\in\mathcal{M}_\ell} \eta_\ell(T)^2 \,.$$

Later it was shown in [2] that the Dörfler marking criterion is not only sufficient to prove linear convergence, but even in some sense necessary.

Clearly, one aims for a subset $\mathcal{M}_\ell \subseteq \mathcal{T}_\ell$ containing as few elements as possible, which satisfies the Dörfler marking criterion. In the best case, the set $\mathcal{M}_\ell \subseteq \mathcal{T}_\ell$ has minimal cardinality, i.e.,

$$\#\mathcal{M}_\ell = \min\{\#\mathcal{N} \colon \mathcal{N} \subseteq \mathcal{T}_\ell \text{ satisfies the Dörfler marking criterion}\} \,.$$

Dörfler [1] notes that sorting the refinement indicators would be sufficient to find such a set of minimal cardinality. Since sorting an array of length N requires $\mathcal{O}(N \log N)$ operations, while *Solve*, *Estimate* and *Refine* are (in principle) of linear cost, he, however, notes that sorting should be avoided. Stevenson [2] proposes a linear complexity algorithm to find a set of minimal cardinality up to some absolute factor 2, which satisfies the marking criterion.

In our talk, we propose a new algorithm for finding a set with minimal cardinality satisfying the Dörfler marking criterion. We show that this new algorithm terminates after at most $\mathcal{O}(N)$ operations. In particular, Dörfler marking with minimal cardinality is a linear complexity problem.

References

[1] W. Dörfler: A convergent adaptive algorithm for Poisson's equation. SIAM *Journal on Numerical Analysis*, 33(3) (1996), 1106-1429.

[2] R. Stevenson: Optimality of a standard adaptive finite element method. Foundations of Computational Mathematics, 7(2) (2007), 245-269.

Initial source recovery of the wave equation given internal boundary measurements

Jarle Sogn[1], Alexander Beigl[2], Otmar Scherzer[2], Walter Zulehner[1]

[1] Johannes Kepler University Linz
[2] University of Vienna

We consider an optimization problem in function spaces with a hyperbolic partial differential equation as constraint with application Photoacoustic Tomography. Our goal is to identify the state variable at the initial time given some data on an internal boundary. We derive the KKT-system and show that the problem is well-posed, moreover, the involved constants are independent of the regularization parameter. This leads to robust preconditioning. A stable discretization technique is introduced, however, at the price of introducing a stabilization parameter. Finally, numerical result are shown

Reinforcement Learning for Hybrid Electric Vehicle Drivetrain System Controllers

Manfred Liebmann, Huan Chen

Institute for Mathematics and Scientific Computing
University of Graz

The aim of the research project is to use state of the art machine learning techniques, specifically *reinforcement learning* [1], to implement system controllers for hybrid electric vehicle (HEV) drivetrains.

Reinforcement learning offers a very flexible framework to solve optimal control problems, where the system controller is identified as the *agent* and the hybrid electric vehicle drivetrain as the *environment*. The research is geared towards the identification of suitable and efficient reinforcement learning algorithms to handle the hybrid electric vehicle drivetrain based on industry standard simulation software tools.

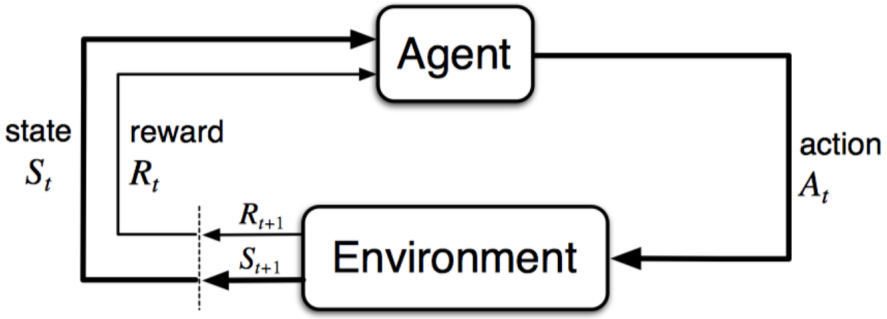

Figure 1: The agent-environment feedback loop.

Reinforcement learning algorithms also allow very flexible definitions of *reward functions* to specify the goal of the optimization and enable different profiles for example for fuel economy, performance, emissions, or drivability for the system controller to optimize.

References

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Fast solvers for time-harmonic wave propagation in a parallel computational environment

Matthias Taus[1,4], Leonardo Zepeda-Núñez[2], Russell J. Hewett[3], Laurent Demanet[4]

[1] Technische Universität Wien, Austria
[2] Lawrence Berkley National Laboratory, USA
[3] Virginia Polytechnic Institute and State University, USA
[4] Massachusetts Institute of Technology, USA

In many science and engineering applications, solving time-harmonic high-frequency wave propagation problems quickly and accurately is of paramount importance. For example, in geophysics, particularly in oil exploration, such problems can be the forward problem in an iterative process for solving the inverse problem of subsurface inversion. It is important to solve these wave propagation problems accurately in order to efficiently obtain meaningful solutions of the inverse problems: low order forward modeling can hinder convergence. Additionally, due to the volume of data and the iterative nature of most optimization algorithms, the forward problem must be solved many times. Therefore, a fast solver is necessary to make solving the inverse problem feasible. For time-harmonic high-frequency wave propagation, obtaining both speed and accuracy is historically challenging.

Recently, there have been many advances in the development of fast solvers for such problems, including methods which have linear complexity with respect to the number of degrees of freedom. While most methods scale optimally only in the context of low-order discretizations and smooth wave speed distributions, the method of polarized traces has been shown to retain optimal scaling for high-order discretizations, such as hybridizable discontinuous Galerkin methods and for highly heterogeneous (and even discontinuous) wave speeds. The resulting fast and accurate solver is consequently highly attractive for geophysical applications. To date, this method relies on a layered domain decomposition together with a preconditioner applied in a sweeping fashion, which has limited straightforward parallelization.

In this work, we introduce a new solution strategy based on a checkerboard domain decomposition. This allows us to exploit more parallel structure than previous fast solvers while preserving all of the advantages. We introduce the strategy and provide numerical examples that show that independently of the frequency ω the Helmholtz equation can be solved in $O((N/p)\log N)$ complexity where N is the number of degrees of freedom and p the number of processors used in the parallel computational environment. Our numerical examples include several different wave speed distributions, mainly motivated by geophysical applications.

Tent pitching and a Trefftz-DG method for the acoustic wave equation

Ilaria Perugia[1], Joachim Schöberl[2], Paul Stocker[1], Christoph Wintersteiger[2]

[1] Faculty of Mathematics, University of Vienna, Vienna, Austria
[2] Institute for Analysis and Scientific Computing, Vienna University of Technology, Vienna, Austria

We present a space-time Trefftz discontinuous Galerkin (DG) method for the approximation of the acoustic wave equation on space-time tent-pitched meshes. Tent-pitching is a front-advancing mesh technique that allows to completely localize the solution of the discrete system. Trefftz basis functions are local solutions to the wave equation, that allow to simply advance the solution from the bottom to the top of each tent-element. The method has been implemented in NGSolve, solving the space-time elements in parallel. Insights into the implementation details are given, including the case of propagation in heterogeneous media.

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Doubling the convergence rate by pre- and post-processing the finite element approximation for linear wave problems

Sjoerd Geevers

University of Vienna

A novel pre- and post-processing algorithm is presented that can double the convergence rate of finite element approximations for linear wave problems. In particular, it can be shown that a q-step pre- and post-processing algorithm can improve the convergence rate of the finite element approximation from order $p+1$ to order $p+1+q$ in the L^2-norm and from order p to order $p+q$ in the energy norm, in both cases up to a maximum of order $2p$, with p the polynomial degree of the finite element space. The q-step pre- and post-processing algorithms only need to be applied *once* and require solving at most q linear systems of equations.

The biggest advantage of the proposed method compared to other post-processing methods is that it does not suffer from convergence rate loss when using unstructured meshes. Other advantages are that this new pre- and post-processing method is straightforward to implement, incorporates boundary conditions naturally, and does not lose accuracy near boundaries or strong inhomogeneities in the domain. Numerical examples illustrate the improved accuracy and higher convergence rates when using this method. In particular, they confirm that $2p$-order convergence rates in the energy norm are obtained, even when using unstructured meshes or when solving problems involving heterogeneous domains and curved boundaries.

The Hellan–Herrmann–Johnson Method for Nonlinear Shells

Michael Neunteufel, Joachim Schöberl

TU Vienna

Finding appropriate discretizations for nonlinear shells is still a challenging problem. For Kirchhoff plates the Hellan–Herrmann–Johnson method introduces a moment tensor for computing the fourth order equation as a mixed method [1].

In this talk we present a generalization of these methods to nonlinear shells, where we allow large strains and rotations. We may assume the Kirchhoff–Love hypothesis to neglect shearing terms and focus on the bending energy, which is defined as the difference between the curvatures of the deformed and undeformed configuration of the shell. Therefore, we introduce the moment tensor $\sigma \in \mathrm{H(divdiv)}$ [2] as the difference between these curvatures. With these elements, also non-smooth surfaces with kinks can be handled directly without rewriting terms.

The method is implemented in NGS-Py, which is based on the finite element library Netgen/NGSolve (www.ngsolve.org). Finally, we present numerical results.

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A virtual element method for the miscible displacement problem

Lourenço Beirão da Veiga[1], Alexander Pichler[2], Giuseppe Vacca[1]

[1] Università di Milano-Bicocca
[2] University of Vienna

The miscible displacement of one incompressible flow by another in a porous medium can be described by a nonlinear coupling of an elliptic equation for the pressure with a parabolic one for the concentration of one of the fluids. Since the pressure appears in the concentration equation only through its velocity field, a mixed method can be chosen to approximate both pressure and velocity simultaneously, whereas a standard Galerkin method can be applied for the concentration, see e.g. [1, 2] for finite element methods.

In this talk, we discuss an extension of this approach to the recently introduced virtual element framework [3]. By an implicit definition of the local basis functions and suitable projectors onto polynomial spaces, this methodology allows the use of general polygonal meshes, leading to a series of advantages for diffusion problems. Among them are an easier handling of complex domain and geometry data as in reservoir simulations (including the presence of cracks), an automatic use of nonconforming grids, more efficient and easier adaptivity, a higher robustness to mesh deformation, etc.

Both theoretical and numerical results are presented.

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A Priori Error Estimates for the Optimal Control of a Simplified Signorini Problem

Christof Haubner

Universität der Bundeswehr München

In the context of distributed control we consider a simplified Signorini problem, an elliptic variational inequality of first kind with unilateral constraints on the boundary. The state is discretized using linear finite elements while a variational discretization is applied to the control. We derive a priori error estimates for the control and state based on strong stationarity and a quadratic growth condition. The convergence rates depend on H^1 and L^2 error estimates of the simplified Signorini problem. Furthermore, we discuss under what conditions we can expect quadratic growth. Numerical experiments are presented that confirm our results.

Variational Formulations for Maxwell's Equations

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Maxwell's equations are the groundwork for electromagnetism. Because of their importance there are several approaches to solve them. For most people the time dependence and especially the time derivative pose the biggest problem. There are some, who eliminate the time derivative by using finite differences. However this method will be stretched to it's limits when it comes to time dependent domains. Another approach is inspired by physics and uses the fact that electromagnetic fields behave like waves. In these methods the time dependencies of the solution is more or less known and only the space dependencies are computed. In contrast to these works we will consider Maxwell's equations as a whole. We want to extent the known theory of space-time methods for the wave equation to Maxwell's equations. For that we will look at the differential equations in a 4D space-time setting. Although it is not obvious why one should use such space-time methods since they usually are involve with more computational effort, we want to quickly give an idea why space-time methods are interesting before we explain them in more detail.

Numerical Identification of Inhomogeneities by Inverse Scattering Using Topological Optimality Condition

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The inverse scattering problem modeled by the Helmholtz equation is considered within the topology optimization framework. Varying the complex-valued refractive index we derive a zero-order necessary optimality condition in minimizing the least square misfit cost functional. The topology asymptotic expansion of the optimality condition leads to an imaging operator used to identify the center of unknown inhomogeneity.

Our algorithm realizes the Vainikko fast solver for formulation of the scattering problem in the form of a weakly singular integral equation of Lippmann–Schwinger. The numerical tests show high precision and stability in the reconstruction both in two and three dimensions. We present numerical examples indicating what happens in cases when our theory does not apply, e.g. in the case of multiple objects or absorption.

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Fast Directional Matrix-Vector Multiplications - Algorithm and Parameter Study

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We consider a matrix $A \in \mathbb{C}^{N \times N}$ with entries

$$A[j,k] = f(x_j, y_k) = \frac{\exp(i\kappa|x_j - y_k|)}{4\pi|x_j - y_k|}, \quad j,k \in \{1,...,N\},$$

where $\{x_j\}_{j=1}^N, \{y_k\}_{k=1}^N \subset \mathbb{R}^3$, f is the 3D Helmholtz kernel and $\kappa > 0$ the wave number. Corresponding matrix-vector multiplications have a complexity of order $\mathcal{O}(N^2)$ and are therefore prohibitive for large N. Standard matrix approximation schemes can be used to overcome this problem in low frequency regimes, but are inefficient in high frequency regimes.

We consider an approximation of the matrix A based on a clustering strategy and a directional approximation of the Helmholtz kernel. This approximation allows to construct an algorithm for fast matrix-vector multiplications, which has a complexity of order $\mathcal{O}(N \log(N))$ in all frequency regimes under suitable assumptions on N, κ and the distribution of points $\{x_j\}_{j=1}^N$ and $\{y_k\}_{k=1}^N$. The effective runtime and accuracy of the algorithm is influenced by the choice of two parameters. We conduct a parameter study to investigate this influence and summarize our observations in a parameter selection strategy.

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A Reduced Basis Method for Fractional Diffusion Operators

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Throughout the last two decades, there has been a growing body of literature that highlights the importance of fractional differential operators in modern science. The design of both efficient and accurate approximations is challenging. By means of a reduced basis method, we aim to address each of these difficulties. The desired operator is projected to a low dimensional space $\mathcal{V}_r$, where the fractional power can be directly evaluated via the small eigensystem. The projection relies on several independent standard Poisson problems, whose decoupled structure is amenable to parallel computations. The optimal choice of $\mathcal{V}_r$ is provided by the so-called Zolotarëv points, for which we prove exponential convergence rates. Numerical experiments evaluating the operator confirm the analysis and demonstrate the efficacy of our algorithm.

Optimal adaptivity and preconditioning for the fractional Laplacian

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We present novel inverse estimates for the integral fractional Laplacian. More precisely, we show that a weighted L^2-norm, where the weight is a power of the local mesh-width, of the fractional Laplacian can be bounded by the energy norm. Generalizing the arguments used in the boundary element method, [1], the non-local operator is split into a localized near-field and a smoother far-field part, which is treated using the so-called Caffarelli-Silvestre extension problem and interior regularity estimates.

Weighted L^2-norms appear naturally in the context of *a-posteriori* error estimation in adaptive methods. As the classical weighted residual error estimator is not well-defined for fractional powers larger than 3/4, we propose a different, reliable weighted error estimator to cover this open case. Using our inverse estimate, we prove optimal convergence rates of an adaptive algorithm steered by the classical and modified weighted residual error estimator with the axiomatic approach of [2].

As a second application of the inverse inequalities, we obtain that an additive Schwarz preconditioner of BPX-type for the fractional Laplacian on locally refined meshes leads to condition numbers that are uniformly bounded in the refinement level.

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Fast Solvers for Isogeometric Analysis based on Alternating Linear Schemes for Tucker Tensors

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In this talk, we consider the construction of fast and memory-efficient solvers for tensor product Isogeometric Analysis by means of low-rank approximation. In particular, we consider the approximation of the solution fields by means of Tucker tensors and then propose an iterative method for the approximation of the solution based on a so-called Alternating Linear Scheme. The idea is to reduce the nonlinear best tensor approximation problem in the energy norm to a series of linear approximation steps for the individual factors of the Tucker tensor. We study several numerical examples in 2D and 3D where the proposed solution method exhibits very robust performance in both the spline degree and the geometry mapping.

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