Technische Universität Graz

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9. Workshop on

Fast Boundary Element Methods in Industrial Applications

Söllerhaus, 29.9.–2.10.2011

U. Langer, O. Steinbach, W. L. Wendland (eds.)

Berichte aus dem Institut für Numerische Mathematik

Book of Abstracts 20011/4

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Program

Thursday, September 29, 2011	
15.00–16.20	Coffee
16.20–16.30	Opening
16.30–17.00	R. Hiptmair (ETH Zürich) Multilevel preconditioning for Edge BEM
17.00–17.30	S. Hardesty (Rice University) Adaptive Cross Approximation of BEM shape derivative tensors
17.30–18.00	C. Hofreither (JKU Linz) A BEM–based FEM for convection–diffusion equations
18.30	Dinner
Friday, September 30, 2011	
9.00–9.30	S. R. Arridge (University College London) BEM and BEM–FEM in Optical Tomography
9.30–10.00	G. Of (TU Graz) On the coupling of interior penalty Galerkin and boundary element methods
10.00–10.30	M. Feischl (TU Wien) Convergence of adaptive FEM–BEM coupling driven by residual–based error estimators
10.30–11.00	Coffee
11.00–11.30	A. Kimeswenger (TU Graz) Symmetric BEM–FEM coupling for fluid–structure interaction
11.30–12.00	G. Unger (TU Graz) Coupled FE–BE eigenvalue problems for fluid–structure interaction
12.00–12.30	J. Xin (Karlsruhe Institute of Technology) Boundary element approximation for Maxwell's eigenvalue problem
12.30	Lunch
15.00–15.30	Coffee
15.30–16.00	M. Bebendorf (U Bonn) Constructing nested bases approximation from the entries of non–local operators
16.00–16.30	D. Praetorius (TU Wien) Quasi–optimal convergence rate for an adaptive boundary element method
16.30–17.00	P. Urthaler (TU Graz) A convolution quadrature boundary element method for poroelastodynamics
17.00–17.30	Break
17.30–18.00	L. Kielhorn (ETH Zürich) An approach to a unified implementation of boundary element methods
18.00–18.30	T. Traub (TU Graz) HYENA–A modular and extensible C++ library for solving hyperbolic and elliptic PDEs
18.30	Dinner

Saturday, October 1, 2011	
9.00–9.30	E. P. Stephan (Leibniz Universität Hannover) Large boundary element computation: Molodensky problem and sound radiation of car tyres
9.30–10.00	X. Claeys (Universite de Toulouse) Multi–trace boundary integral formulation of the first kind of acoustic scattering by composite structures
10.00–10.30	T. Betcke (University College London) Spectral properties of boundary integral operators in acoustic scattering
10.30–11.00	Coffee
11.00–11.30	L. Banjai (MPI Leipzig) A modified TDBIE for computation of the first reflection of an impulse–like wave
11.30–12.00	M. Kachanovska (MPI Leipzig) A fast Runge–Kutta convolution quadrature for solution of the 3D wave equation in unbounded domains
12.00–12.30	D. Lukas (TU Ostrava) A parallel Galerkin ACA–BEM for the Helmholtz equation
12.30	Lunch
13.30–18.00	Hiking Tour
18.30	Dinner
Sunday, October 2, 2011	
9.00–9.30	Z. Andjelic (ABB Switzerland) Direct and indirect free–form optimization using BEM
9.30–10.00	M. Aurada (TU Wien) Mixed conforming elements for the large–body limit in micromagnetics – A FEM–BEM approach
10.00–10.30	Coffee
10.30–11.00	M. Kolmbauer (JKU Linz) The multi–harmonic FEM–BEM coupling method for simulation and control of eddy current problems
11.00–11.30	O. Steinbach (TU Graz) Boundary integral equations for linear elasticity problems
11.30	Closing

Direct and Indirect Free-form Optimisation using BEM

Z. Andjelic
ABB Corporate Research, Baden, Switzerland

Here we present several approaches for automatic free-form optimization based on BEM. We elaborate to different optimization approaches: a direct and an indirect approach. In the direct approach we try to minimize the maximal electrical field at the interface between different media by changing the form of the surface in normal direction. The objective function is typically the Maxwell stress or the total or tangential field strength.
In the indirect approach we introduce a novel approach for simple sensitivity calculation. The analogy with the sensitivity calculation in signal processing is used. The typical optimization task is to find a form of the interface that should provide a required field/stress/force distribution in the separate space of interest. In other words, by changing the form of the interface we indirectly change the value of the objective function in the space of interest.
For both approaches we use BEM as a solution engine. We elaborate briefly the mathematical background of the BEM for non-linear magnetostatics.
Finally, we demonstrate the above procedures on some academic, as well as some industrial problems.

BEM and BEM-FEM in Optical Tomography

S. R. Arridge, J. Elisee, A. Zacharopoulos, O. Dorn, M. Bonnet
University College London, UK

Diffuse Optical Tomography (DOT) is a medical imaging modality in which light is detected after transmission through a highly scattering medium which is described by a diffusion type forward model. Since the light is detected on the boundary this methodology is usually formulated as an inverse boundary value problem and is non-linear and exponetially ill-posed.

In some applications, boundary element methods are useful to deal with multiple thin layers of tissue, such as in the head. In this talk I will describe recent methods for the forward model in DOT using BEM and BEM-FEM. Inverse problems using these models can be developled for shape based reconstruction, for localised reconstruction in a FEM region surrounded by BEM layers, and for cortical mapping, in which the image is assumed to be constrained to a surface.

Mixed Conforming Elements for the Large-Body Limit in Micromagnetics – A FEM-BEM Approach

M. Aurada, J. M. Melenk, D. Praetorius
TU Wien, Austria

We consider the large-body limit of the stationary Landau-Lifshitz minimization problem introduced by DeSimone in 1993, [1]: Find a minimizer $\mathbf{m} : \Omega \to \mathbb{R}^d$ with $|\mathbf{m}| \leq 1$ a.e. of the bulk energy

$$E(\mathbf{m}) = \frac{1}{2} \int_{\mathbb{R}^d} |\nabla u|^2 \, \mathrm{d}x + \int_{\Omega} \varphi^{**}(\mathbf{m}) \, \mathrm{d}x - \int_{\Omega} \mathbf{f} \cdot \mathbf{m} \, \mathrm{d}x. \tag{1}$$

Here, $\Omega \subset \mathbb{R}^d$, for $d = 2, 3$, is the spatial domain of the ferromagnetic material, φ^{**} is the (convexified) anisotropy density, and $\mathbf{f} : \Omega \to \mathbb{R}^d$ is an applied exterior field. The magnetic potential $u : \mathbb{R}^d \to \mathbb{R}$ is related to the magnetization $\mathbf{m}$ by the magnetostatic Maxwell equation, which reads in distributional form

$$\mathrm{div}(-\nabla u + \mathbf{m}) = 0 \quad \text{in } \mathcal{D}'(\mathbb{R}^d). \tag{2}$$

For the numerical treatment of this minimization problem we have to deal with the following issues:

a) The side constraint $|\mathbf{m}| \leq 1$: We enforce this by using a penalization strategy, which means that we add an appropriate penalization term to (1).

b) The side constraint given by Maxwell's equation (2): We append (2) to the energy functional by a Lagrange multiplier. To ensure stability of the discrete saddle point problem, we add a consistent stabilization term.

c) The full space problems involved in the first integral in (1) and in Maxwell's equation (2): The energy contribution

$$\int_{\mathbb{R}^d \setminus \Omega} |\nabla u|^2 \, \mathrm{d}x$$

in (1) is realized by a boundary integral term. For the Maxwell equation (2) we use a similar idea. The numerical realization of the boundary integral terms then results in a FEM-BEM coupling.

In this talk, we discuss the well-posedness of the discrete problem and present an *a priori* error analysis, where we show optimal convergence rates under sufficient regularity assumptions. We illustrate our analysis with numerical examples.

References

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A modified TDBIE for computation of the first reflection of an impulse-like wave

L. Banjai

Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany

We consider the numerical computation of time-domain scattering of acoustic waves by a bounded obstacle in an infinite homogeneous medium. As the incident wavewe take a highly peaked Gaussian plane wave, which excites a large frequency bandwidth. In order to speed up computation, we form a modified time-domain boundary integral equation which includes the knowledge of the incident wave. Further, we analyse the numerical solution of this equation by convolution quadrature in time and Galerkin boundary integral method in space. If only a single reflection is to be computed, the solution of the integral equation is very cheap. If the scatterer is convex a single reflection is indeed all that needs to be computed. We will compare such a method with recently developed asymptotic methods for frequency domain computations of scaterring by convex obstacles. Numerical results for 2D scatterers will also be presented.

This is a joint work with Fatih Ecevit of Bogazici University, Turkey.

Constructing nested bases approximation from the entries of non–local operators

M. Bebendorf
Universität Bonn, Germany

The adaptive cross approximation (ACA) [1, 2] is a method for constructing data–sparse approximations to large–scale fully populated discretizations of integral operators with logarithmic linear cost. A characteristic property of this method is that the approximation is based on few of the original matrix entries. The ease of use of ACA, however, usually comes with a higher memory consumption compared with the fast multipole method (FMM) [3, 4]. A new method will be presented which combines the desired properties of ACA with the improved storage requirement of FMM.

References

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Spectral properties of boundary integral operators in acoustic scattering

T. Betcke
University College London, UK

Spectral properties of boundary integral operators in acoustic scattering are essential to understand properties such as coercivity or convergence of iterative methods. Yet, very little is known apart from the unit disk case, where the Green's function has a simple decomposition into Fourier modes. For more general domains it is not even known whether boundary integral operators are normal.
In this talk we first extend known results about eigenvalues on the circle to the case of an ellipse, where a decomposition of the acoustic Green's function in elliptic coordinates is possible. Based on this it is shown that scaled versions of the standard boundary integral operators are normal in a modified L^2 inner product on the ellipse. For more general domains we consider pseudospectra to visualise spectral behaviour. In particular, we are interested in pseudospectra of operators around resonances of trapping domains.

Multi–trace boundary integral formulation of the first kind for acoustic scattering by composite structures

X. Claeys

Université de Toulouse, ISAE, France

We study the scattering of acoustic waves by an object composed of several adjacent parts with different material properties. Starting from an already well known single trace integral formulation of the first kind for this problem, we derive a new boundary integral formulation of the first kind where all unknowns are doubled on each interface. This formulation does not suffer from any spurious mode phenomenon, and it satisfies a stability property that ensures quasi-optimal convergence of conforming boundary element methods. Besides, for certain cases, this formulation satisfies a relation very similar to a Calderón formula.

Convergence of adaptive FEM-BEM coupling driven by residual-based error estimators

M. Aurada, M. Feischl, T. Führer, M. Karkulik, M. Melenk, D. Praetorius
TU Wien, Austria

Model problem & discretization. For the ease of presentation, we consider a linear interface problem with the 2D Laplacian which is equivalently reformulated by means of the Johnson-Nédélec FEM-BEM coupling: Find $(u, \phi) \in \mathcal{H} := H^1(\Omega) \times H^{-1/2}(\Gamma)$ such that

$$\begin{aligned} \langle \nabla u, \nabla v \rangle_\Omega - \langle \phi, v \rangle_\Gamma &= \langle f, v \rangle_\Omega + \langle \phi_0, v \rangle_\Gamma, \\ \langle \psi, (1/2 - K)u + V\phi \rangle_\Gamma &= \langle \psi, (1/2 - K)u_0 \rangle_\Gamma, \end{aligned} \tag{1}$$

for all $(v, \psi) \in \mathcal{H}$. Here, $\Omega \subset \mathbb{R}^2$ is a bounded Lipschitz domain with polygonal boundary, and $f \in L^2(\Omega)$, $u_0 \in H^{1/2}(\Gamma)$, and $\phi_0 \in H^{-1/2}(\Gamma)$ are given data, which satisfy the compatibility condition $\langle \phi_0, 1 \rangle_\Gamma + \langle f, 1 \rangle_\Omega = 0$. With a regular triangulation $\mathcal{T}_\ell$ of Ω, we consider the lowest-order Galerkin discretization, where $u \approx u_\ell \in \mathcal{S}^1(\mathcal{T}_\ell)$ is approximated by piecewise affine and globally continuous functions and $\phi \approx \phi_\ell \in \mathcal{P}^0(\mathcal{E}_\ell^\Gamma)$ by piecewise constants on the boundary. The set $\mathcal{E}_\ell^\Gamma$ denotes the set of boundary edges of $\mathcal{T}_\ell$. It is well-known that the continuous and discrete formulations admit unique solutions.

Residual error estimator. The Galerkin error is controlled by some residual error estimator from [2]

$$\|u - u_\ell\|_{H^1(\Omega)}^2 + \|\phi - \phi_0\|_{H^{-1/2}(\Gamma)}^2 \lesssim \varrho_\ell^2 := \sum_{E \in \mathcal{E}_\ell^\Omega} \varrho_\ell(E)^2 + \sum_{E \in \mathcal{E}_\ell^\Gamma} \varrho_\ell(E)^2 \tag{2}$$

with $\mathcal{E}_\ell^\Omega$ being the set of edges of $\mathcal{T}_\ell$ inside Ω. For these $E \in \mathcal{E}_\ell^\Omega$, the local contributions read

$$\varrho_\ell(E)^2 = |\omega_{\ell,E}| \, \|f - f_{\ell,E}\|_{L^2(\omega_{\ell,E})}^2 + \mathrm{diam}(E) \, \|[\partial_n u_\ell]\|_{L^2(E)}^2,$$

while for boundary edges $E \in \mathcal{E}_\ell^\Gamma$

$$\varrho_\ell(E)^2 = \mathrm{diam}(E) \left(\|\phi_0 + \phi_\ell - \partial_n u_\ell\|_{L^2(E)}^2 + \|\big((1/2 - K)(u_0 - u_\ell) - V\phi_\ell\big)'\|_{L^2(E)}^2 \right).$$

Here, $\omega_{\ell,E} = T_+ \cup T_-$ with $T_\pm \in \mathcal{T}_\ell$ is the edge patch of an interior edge $E = T_+ \cap T_- \in \mathcal{E}_\ell^\Omega$, and $f_{\ell,E}$ is the corresponding integral mean of f. Moreover, $(\cdot)'$ denotes the arclength derivative.

Convergence of adaptive algorithm. We follow the concept of estimator reduction from [3] which is applied for $(h - h/2)$-based adaptivity in [1]: If Dörfler marking is used to mark edges for refinement,

$$\varrho_{\ell+1}^2 \le \kappa \, \varrho_\ell^2 + C \left(\|u_{\ell+1} - u_\ell\|_{H^1(\Omega)}^2 + \|\phi_{\ell+1} - \phi_\ell\|_{H^{-1/2}(\Gamma)}^2 \right) \tag{3}$$

with ℓ-independent constants $0 < \kappa < 1$ and $C > 0$. From the best-approximation property of Galerkin schemes (Céa lemma) and nestedness of discrete spaces, one obtains a priori convergence $u_\ell \to u_\infty$ in $H^1(\Omega)$ and $\phi_\ell \to \phi_\infty$ in $H^{-1/2}(\Gamma)$ with

certain (unknown) limits. Elementary calculus and (3) then predict $\varrho_\ell \to 0$ as $\ell \to \infty$. Therefore, (2) guarantees convergence of the adaptive algorithm.
The main ingredient to prove (3) for residual error estimators are novel inverse-type estimates for the integral operators involved. We stress that the proposed approach also works for other coupling strategies in 2D and 3D and even certain nonlinearities inside of Ω.

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Adaptive Cross Approximation of BEM Shape Derivative Tensors

S. Hardesty
Rice University, Houston, USA

This work presents a new algorithm based on Heirarchical Matrices and the Adaptive Cross Algorithm for the fast approximation of shape derivative tensors appearing in the context of shape optimization problems constrained by the solution of a system of boundary integral equations. The approach is natural for problems with multiphysics coupling, and is contrasted with that of extraction techniques, which use commutators of derivatives with the boundary integral operators, and require the solution of additional linear systems.

Multilevel Preconditioning for Edge BEM

R. Hiptmair
ETH Zürich, Switzerland

We establish the stability of nodal multilevel decompositions of lowest-order conforming boundary element subspaces of the trace space $H^{-1/2}(\mathrm{div}_\Gamma, \Gamma)$ of $H(\mathrm{curl}, \Omega)$ on boundaries of triangulated Lipschitz polyhedra. The decompositions are based on nested triangular meshes created by regular refinement and the stability bounds are uniform in the number of refinement levels.
The main tool is the general theory of [P. Oswald, Interface preconditioners and multilevel extension operators, in Proc. 11th Intern. Conf. on Domain Decomposition Methods, London 1998, pp. 96–103] that teaches, when stability of decompositions of boundary element spaces with respect to trace norms can be inferred from corresponding stability results for finite element spaces. $H(\mathrm{curl}, \Omega)$-stable discrete extension operators are instrumental in this.
Stable multilevel decompositions immediately spawn subspace correction preconditioners whose performance will not degrade on very fine surface meshes. Thus, the results of this article demonstrate how to construct optimal iterative solvers for the linear systems of equations arising from the Galerkin edge element discretization of boundary integral equations for eddy current problems.

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A BEM-based FEM for convection-diffusion equations

C. Hofreither, U. Langer, C. Pechstein
Johannes Kepler Universität Linz, Austria

We present a nonstandard finite element method for elliptic partial differential equations with piecewise constant coefficients which is based on element-local boundary integral operators. The method is able to treat general polyhedral meshes and employs locally PDE-harmonic trial functions.
In this talk, we apply the method to convection-diffusion-reaction problems. We compare the performance of the method for convection-dominated problems in three dimensions to a standard Finite Element Method and observe very promising stability properties.

A Fast Runge-Kutta Convolution Quadrature for Solution of the 3D Wave Equation in Unbounded Domains

L. Banjai, M. Kachanovska

Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany

We consider a Dirichlet problem for the three-dimensional wave equation in an unbounded domain with zero initial conditions. To solve the problem we use time-domain boundary element formulation and employ Runge-Kutta convolution quadrature approach for time discretization and Galerkin method for space discretization. We improve the method of the solution of the discretized system presented in [1] exploiting Huygens' principle, which allows for truncation of the series representing the discretized kernel of the wave equation at a finite number of terms.
The sparse matrices constructed for the approximation of the convolution weights are reused during different stages of the recursive matrix-vector multiplication. The algorithm presented allows reduction of the time and storage costs for both matrix construction and matrix-vector multiplication, retaining the original linear order of complexity.

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An approach to a unified implementation of Boundary Element Methods

L. Kielhorn
ETH Zürich, Switzerland

In the past decade there has been an explosion of available Open Source libraries dealing with Finite Element Methods. But unfortunately, there exist only very few approaches to develop Open Source libraries which are well-suited to tackle the discretization of Boundary Integral operators.
It is inherent to the method itself, that the development of Boundary Element codes is more challenging than it might be for Finite Element codes. Contrary, even today there are ongoing discussions on what might be the best Boundary Element formulation for some particular problem. From this point of view a Boundary Element library being well-suited for the discretization of Boundary Integral operators would lead to considerable shorter development times of new Boundary Element codes.
This talk is about some ideas and thoughts about a unified approach on the implementation of discrete Boundary Integral operators. In addition, a specific implementation with applications to electrostatic, magnetostatic, eddy-current, and optimization problems will be presented.

Symmetric BEM-FEM-coupling for Fluid-Structure-Interaction

A. Kimeswenger, O. Steinbach
TU Graz, Austria

In this talk the focus is on the acoustic-structure-interaction, where for a time harmonic ansatz the partial differential equations of linear elasticity and the wave equation are considered in the domain of the structure and in the fluid domain, respectively. In addition, two transmission conditions are used.

The standard variational formulation is used to handle the equations of linear elasticity. Due to the fact that the fluid domain is unbounded, the Helmholtz boundary integral equations are used. Because one cannot neglect the effect of acoustic pressure onto the structure, a strong coupling will be used. By using both boundary integral equations we obtain a (symmetric) coupled system with the unknown displacement and the unknown pressure. This system will be analyzed and afterwards a coupled finite-element boundary-element method is used to discretize the system. Eventually, numerical examples will be considered.

The advantage of the proposed symmetric coupled system is the stability for all wave numbers, independent of the regularity of the domain under consideration.

The multi–harmonic FEM–BEM coupling method for simulation and control of eddy current problems

M. Kolmbauer, U. Langer
Johannes Kepler Universitä Linz, Austria

This talk is devoted to the simulation and control of time-dependent eddy current problems. In order to discretize in time, we apply a multiharmonic approach. The resulting system of frequency domain equations is discretized in terms of a symmetrically coupled FEM-BEM method. For the resulting large system of linear equations, we construct block-diagonal preconditioners, used in a MinRes setting, that are robust with respect to the space and time discretization parameters and all additionally involved parameters (i.e. conductivity, reluctivity, regularization parameters).

A Parallel Galerkin ACA-BEM for the Helmholtz Equation

D. Lukas
VSV TU Ostrava, Czech Republic

We deal with a parallel implementation of a Galerkin BEM accelerated by the Adaptive Cross Approximation (ACA) technique. In particular, the method is employed to the Neumann problem of the Helmholtz equation with an application to acoustic noise analysis of a railway wheel. Our improvement relies on approximation of a part of the hypersingular operator by piecewise constant basis functions instead of the linear ones. Numerical results document a significant reduction of computational time.

On the coupling of interior penalty Galerkin and boundary element methods

G. Of[1], G. Rodin[2], O. Steinbach[1], M. Taus[2]

[1]TU Graz, Austria, [2]The University of Texas at Austin, USA

In this talk we provide three new formulations for the coupling of discontinuous Galerkin finite element and boundary element methods. The proposed approaches are different in the use of boundary integral equations, which also allow the use of either collocation or Galerkin discretizations. The different formulations are presented in a unified framework, which allows the application of standard stability and error estimates. Numerical results confirm the theoretical statements.

Quasi-optimal convergence rate for an adaptive boundary element method

M. Feischl, M. Karkulik, M. Melenk, D. Praetorius
TU Wien, Austria

Our prior works on convergence of adaptive BEM considered $(h - h/2)$-based error estimators. Reliability of these type of estimators is, however, equivalent to the so-called saturation assumption. Although this is widely believed to hold in practice, it still remains mathematically open. For this reason, these convergence results are not fully satisfactory.
In this year's talk, based on the recent work [3], we consider the weakly singular integral equation

$$V\phi = f \quad \text{on } \Gamma$$

and a weighted residual error estimator

$$\mu_\ell = \|h_\ell^{1/2}\nabla(f - V\Phi_\ell)\|_{L^2(\Gamma)}$$

from [1,2]. Here, $V \in L(\widetilde{H}^{-1/2}(\Gamma), H^{1/2}(\Gamma))$ is the simple-layer potential of the 2D or 3D Laplacian, $\Phi_\ell \in \mathcal{P}^0(\mathcal{E}_\ell)$ is the $\mathcal{E}_\ell$-piecewise constant Galerkin approximation of ϕ, and ∇ denotes the arclength derivative for 2D resp. the surface gradient for 3D. Throughout, the index ℓ denotes quantities associated with the ℓ-th step of the adaptive mesh-refining algorithm.
The weighted residual estimators enjoy the property to be reliable

$$|||\phi - \Phi_\ell||| \le C_{\rm rel}\, \mu_\ell,$$

where the constant $C_{\rm rel} > 0$ depends only on the shape of the elements in $\mathcal{E}_\ell$ and where $|||\cdot||| \simeq \|\cdot\|_{\widetilde{H}^{-1/2}(\Gamma)}$ denotes the energy norm induced by V.
We prove a certain (local) inverse-type estimate which allows us to conclude, for the usual h-adaptive algorithm, that μ_ℓ satisfies the estimator reduction

$$\mu_{\ell+1}^2 \le \widetilde{\kappa}\, \mu_\ell^2 + C\, |||\Phi_{\ell+1} - \Phi_\ell|||^2$$

with ℓ-independent constants $0 < \widetilde{\kappa} < 1$ and $C > 0$. Boots trapping this result by means of the Galerkin orthogonality, we then prove that a properly weighted sum of Galerkin error and weighted residual error estimator is contractive in each step of the adaptive algorithm, i.e.

$$\Delta_{\ell+1} \le \kappa\, \Delta_\ell \quad \text{with} \quad \Delta_\ell = |||\phi - \Phi_\ell|||^2 + \gamma\, \mu_\ell^2$$

with certain ℓ-independent constants $0 < \kappa, \gamma < 1$. This proves linear convergence of adaptive BEM.
Finally, we identify the right notion of quasi-optimal convergence for an adaptive BEM algorithm. The neccessary ingredients, that led to success in proving quasi-optimal convergence rates in adaptive FEM, are extended to weakly singular integral equations and associated weighted residual error estimators. Finally, we show that adaptive BEM algorithms choose substantially an optimal sequence of meshes.

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Boundary integral equations for linear elasticity problems

O. Steinbach
TU Graz, Austria

In this talk I will review several analytic properties of boundary integral operators which are related to linear elasticity problems. This covers ellipticity estimates for two–dimensional problems, almost incompressible materials, contraction estimates for the double layer potential, boundary value problems with inclusions, and the coupling of boundary element methods with finite elements.

Large Boundary Element Computation: Molodensky Problem and Sound Radiation of Car Tyres

E. P. Stephan, A. Costea, Z. Nezhi
Leibniz Universität Hannover, Germany

For given potential and gravity vector of the earth, we perform boundary element methods for the Molodensky problem. Our algorithm constructs the surface of the earth and the gravitational potential by combining iterative solutions of integral equations. We start from the unit sphere and determine in each interative step the correction to the surface. Crucial for our method is the computation of the gravity gradient for the updated surface. Next, we analyse the numerical solution of time dependent scattering phenomena in unbounded domains using retarded potential boundary integral equations also known as time domain boundary integral equations. We employ an unconditionally stable space-time boundary element scheme based on a Burton-Miller type formulation which uses various retarded potential. Its fully discrete formulation results in a marching-on-in-time (MOT) scheme through a history of sparse matrices and solution vectors.
The main focus of this work lies on the efficient computation of the matrix entries. We study the discrete retarded potentials evaluated on one element of a triangulation of the surface. Furthermore we discuses the use of hp-quadrature to compute the entries of the Galerkinmatrix.
We present numerical experiments for the Molodensky problem and for the sound radiation of car tyres.

HYENA - A modular and extensible C++ library for solving hyperbolic and elliptic PDEs

T. Traub, B. Kager, Ma. Messner, Mi. Messner, F. Rammerstorfer, P. Urthaler
TU Graz, Austria

In this presentation we outline the basic concepts and structure of the open source boundary element library HyENA (**Hy**perbolic and **e**lliptic **n**umerical **a**nalysis) which is jointly developed by the Institute of Applied Mechanics and the Institute of Computational Mathematics at the Graz University of Technology.
Its primary goal is to provide a flexible and extensible framework to implement and test new algorithms. The focus of the library lies on the solution of static and time dependent problems in the area of applied mechanics.
Our aim is to increase productivity through the implementation of reusable modules, which results in a library structure and allow the design of specific high level solvers. This is achieved by employing a generic programming approach. Consequently, different programs, which lack unnecessary control instructions, can be build in order to solve specific problems.
Starting from essential implementation strategies and the chosen program structure we continue with an overview of modularity, functionality and usability. Features like, Convolution quadrature methods for time discretization and fast methods, are described briefly. Since the modularity of the library offers a variety of interfaces for further development another main objective is to demonstrate this extensibility. Finally, we end up with some examples and an outlook of HyENA's future.

Coupled FE-BE eigenvalue problems for fluid-structure interaction

A. Kimeswenger, O. Steinbach, G. Unger
TU Graz, Austria

In this talk we present a coupled finite and boundary element eigenvalue problem formulation for the simulation of the vibro-acoustic behavior of bodies in fluids as submarines. The proposed eigenvalue problem is nonlinear in the frequency parameter and it is analyzed in the framework of eigenvalue problems for holomorphic Fredholm operator–valued functions. For the solution of the eigenvalue problem we use the contour integral method which reduces the algebraic nonlinear eigenvalue problem to a linear one. The method is based on a contour integral representation of the resolvent operator and it is suitable for the extraction of all eigenvalues which are enclosed by a given contour. The dimension of the resulting linear eigenvalue problem corresponds to the number of eigenvalues inside the contour. The main computational effort consists in the evaluation of the resolvent operator for the contour integral which requires the solution of several linear systems involving finite and boundary element matrices.

A convolution quadrature boundary element method for poroelastodynamics

O. Steinbach, P. Urthaler
TU Graz, Austria

We consider wave propagation in porous media. We formulate a linear system of coupled partial differential equations in the Laplace domain based on Biot's theory with the solid displacements and the pore pressure as the primary unknowns. To solve this system of coupled partial differential equations in a semi-infinite homogeneous domain the Boundary Element Method and the Convolution Quadrature Method are applied. The boundary integral operators are analyzed, i.e., unique solvability of the analytic and of the discrete system is shown. Suitable error estimates are given and discussed with the help of numerical examples.

Boundary Element Approximation for Maxwell's Eigenvalue Problems

C. Wieners, Jiping Xin
Karlsruhe Institute of Technology, Germany

We present the Galerkin boundary element methods to solve the eigenvalue problem for Maxwell's equations. This uses the method of Steinbach/Unger for the Laplace eigenvalue problem. The main idea is to apply Newton method to find a nontrivial solution of the equation curlcurl$u + k^2 u = 0$ together with a normalization for u. This approach is compared with the contour integral method recently introduced by W. Beyn and applied to the Laplace eigenvalue problem by G. Unger.
Next, we use the interface conditions to couple the Calderón projections for different domains with piecewise constant material parameters. Finally, we discuss periodic and quasi-periodic boundary conditions. This is applied to the band gap computation of photonic crystals. Our results are also compared with finite element computations using standard Nédélec elements.
The presentation summarizes results from my PhD work (Karlsruhe 2011).

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Erschienene Preprints ab Nummer 2010/1

2010/1	G. Of, T. X. Phan, O. Steinbach: Boundary element methods for Dirichlet boundary control problems.
2010/2	P. D. Ledger, S. Zaglmayr: hp–Finite element simulation of three–dimensional eddy current problems on multiply connected domains.
2010/4	S. Engleder, O. Steinbach: Boundary integral formulations for the forward problem in magnetic induction tomography.
2010/5	Z. Andjelic, G. Of, O. Steinbach, P. Urthaler: Direct and indirect boundary element methods for magnetostatic field problems.
2010/8	A. Klawonn, U. Langer, L. F. Pavarino, O. Steinbach, O. B. Widlund (eds.): Workshop on Domain Decomposition Solvers for Heterogeneous Field Problems, Book of Abstracts.
2010/9	O. Steinbach, G. Unger: Convergence analysis of a Galerkin boundary element method for the Dirichlet Laplacian eigenvalue problem.
2010/10	O. Steinbach (ed.): Workshop on Computational Electromagnetics, Book of Abstracts.
2010/11	S. Beuchler, V. Pillwein, S. Zaglmayr: Sparsity optimized high order finite element functions for H(div) on simplices.
2010/12	U. Langer, O. Steinbach, W. L. Wendland (eds.): 8th Workshop on Fast Boundary Element Methods in Industrial Applications, Book of Abstracts.
2011/1	O. Steinbach, G. Unger: Convergence orders of iterative methods for nonlinear eigenvalue problems.
2011/2	M. Neumüller, O. Steinbach: A flexible space–time discontinuous Galerkin method for parabolic initial boundary value problems.
2011/3	G. Of, G. J. Rodin, O. Steinbach, M. Taus: Coupling methods for interior penalty discontinuous Galerkin finite element methods and boundary element methods.