Technische Universität Graz

21. Workshop on

Fast Boundary Element Methods in Industrial Applications

Söllerhaus, 1.–4.10.2023

U. Langer, M. Schanz, O. Steinbach, W. L. Wendland (eds.)

Berichte aus dem Institut für Angewandte Mathematik

Book of Abstracts 2023/7

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Program

Sunday, October 1, 2023	
15.00	Coffee
16.30–16.45	Opening
16.45–17.15	Heiko Gimperlein (Innsbruck) A posteriori error estimates and space-time mesh refinements for the boundary element method
17.15–17.45	Remo von Rickenbach (Basel) Anisotropic wavelet compression of boundary integral operator matrices
17.45–18.15	Ignacio Labarca (Zürich) Coupled boundary and volume integral equations for electromagnetics
18.30	Dinner
Monday, October 2, 2023	
8.00–9.00	Breakfast
9.00–9.30	Stefan Kurz (Jyväskylä) On the exponential stability of uniformly damped wave equations
9.30–10.00	M. Zank (Wien) A coercive boundary element method for the wave equation for flat objects
10.00–10.30	Daniel Hoonhout (Delft) Stable adaptive least-squares space-time BEM for the wave equation
10.30–11.00	Break
11.00–11.30	Matthias Ospel (Saint–Louis) Inverse estimation of hypersonic trajectories using the time reversal symmetry of the linearized acoustic wave equation
11.30–12.00	Richard Löscher (Graz) On some properties of the modified Hilbert transformation
12.00	Lunch
14.00–14.30	Ernst Stephan (Hannover) Numerical evaluation of retarded potentials
14.30–15.00	Dalibor Lukáš (Ostrava) On condition number of the Steklov–Poincaré operator discretized by FEM and BEM multiscreens: a two-level substructuring preconditioner
15.00–16.00	Coffee
16.00–16.30	Hieu Hoang (Innsbruck) Stabilized finite elements for unique continuation problems
16.30–17.00	Olaf Steinbach (Graz) Regularization and discretization error estimates for Dirichlet control problems
17.00–17.15	Break
17.15–17.45	Christian Köthe (Graz) Adaptive least-squares space-time finite element methods
17.45–18.15	Michael Reichelt (Graz) Space-time finite element methods for thermo-elastodynamics
18.30	Dinner

Tuesday, October 3, 2023	
8.00–9.00	Breakfast
9.00–9.30	Jörg Nick (Zürich) Numerical analysis for electromagnetic scattering with nonlinear boundary conditions
9.30–10.00	Van Chien Le (Ghent) A well-conditioned combined field integral equation for electromagnetic scattering in Lipschitz domains
10.00–10.30	Fernando Henriquez (Lausanne) Parametric shape holomorphy of boundary integral operators with applications
10.30–11.00	Break
11.00–11.30	Christina Schwarz (Bayreuth) Using a mixed approximation to solve the mixed boundary value problem of linear elasticity
11.30–12.00	Anouk Wisse (Delft) First-kind Galerkin BEM for the Hodge–Helmholtz equation
12.00	Lunch
13.00–18.00	Hiking Tour
18.30	Dinner
Wednesday, October 4, 2023	
8.00–9.00	Breakfast
9.00–9.30	G. Of (Graz) A space-time fast boundary element method for the heat equation with temporal nearfield compression
9.30–10.00	P. Panchal (Zürich) Magnetostatic force computation using boundary element method
10.00–10.30	Ulrich Langer (Linz) Matrix-free monolithic multigrid methpods for Stokes and generalized Stokes problems
10.30	Closing

22. Söllerhaus Workshop on

Fast Boundary Element Methods and Space-Time Discretization Methods

26.9.–29.9.2024

Boundary element methods for the hypersingular integral equation on multiscreens: a two-level substructuring preconditioner

Martin Averseng

France

We introduce a preconditioning method for the linear systems arising from the boundary element discretization of the Laplace hypersingular equation on a 2-dimensional triangulated surface Γ in $\mathbb{R}^3$. We allow Γ to belong to a rather large class of geometries that we call polygonal multiscreens, which may not even be manifolds. After presenting a new, simple conforming Galerkin discretization (which reduces to the standard one when Γ is a screen or the boundary of a regular polyhedron), we analyze a preconditioner based on ideas from substructuring domain decomposition in the Finite Element Method. The surface Γ is subdivided into non-overlapping regions, and the application of the preconditioner is obtained via the solution of the hypersingular equation on each patch, plus a coarse subspace correction. We prove that the condition number of the preconditioned linear system grows poly-logarithmically with H/h, the ratio of the coarse mesh and fine mesh size, thereby allowing for significant speedups of iterative solvers, even when a large number of subdomains is used. Numerical experiments illustrate our analysis.

A posteriori estimates and space-time mesh refinements for the boundary element method

Heiko Gimperlein

Universität Innsbruck, Austria

We discuss recent and on-going work for time-domain boundary element methods for the wave equation, witha focus on locally refined meshes in space and/or time. First, a posteriori estimates for the approximation error are presented for the weakly singular and the hypersingular integral equations. The a posteriori estimates lead to an adaptive mesh refinement procedure. More generally, we discuss the numerical analysis, computational aspects and numerical experiments on locally refined meshes in space and/or time.

Parametric shape holomorphy of boundary integral operators with applications

Fernando Henriquez
EPFL, Lausanne, Switzerland

We consider a family of boundary integral operators supported on a collection of parametrically defined bounded Lipschitz boundaries. Thus, the boundary integral operators themselves also depend on the parametric variables, leading to a parameter-to-operator map. In this talk, we discuss the analytic or holomorphic dependence of said boundary integral operators upon the parametric variables, i.e., of the parameter-to-operator map. As a direct consequence we also establish holomorphic dependence of solutions to boundary integral equations, i.e., holomorphy of the parameter-to-solution map. To this end, we construct a holomorphic extension to complex-valued boundary deformations and investigate the complex Fréchet differentiability of boundary integral operators with respect to each parametric variable. The established parametric holomorphy results have been identified as a key property to derive best N-term approximation rates to overcome the so-called curse of dimensionality in the approximation of parametric maps with distributed, high-dimensional inputs.

To demonstrate the applicability of the derived results, we consider as a concrete example the sound-soft Helmholtz acoustic scattering problem and its frequency-robust boundary integral formulations. For this particular application, we explore the consequences of our results in reduced order modelling, Bayesian shape inversion, and the construction of efficient surrogates using artificial neural networks.

Joint work with Jürgen Dölz, Institute for Numerical Simulation, University of Bonn, Germany.

Stabilized finite elements for unique continuation problems

Hieu Hoang
Universität Innsbruck, Austria

We consider the finite element approximation of unique continuation problems for the Laplace equation: For given data in a subdomain $\omega \subset \Omega$, determine the solution in the larger domain Ω. Classical approaches to this ill-posed inverse problem include Tikhonov-stabilized methods. In this talk, we present certain stabilized finite element methods and discuss their numerical analysis. In particular, the unique continuation problem allows a natural interpretation as a Dirichlet boundary control problem, so that ideas from optimal control can be used for this inverse problem.

Stable adaptive least-squares space-time BEM for the wave equation

D. Hoonhout[1], R. Löscher[2], O. Steinbach[2], C. Urzúa–Torres[1]
[1]Delft University of Technology, Delft, Netherlands
[2]TU Graz, Graz, Austria

We consider space-time boundary element methods for the weakly singular operator V corresponding to transient wave problems. In particular, we restrict ourselves to the one-dimensional case and work with prescribed Dirichlet data and zero initial conditions. We begin by revisiting two approaches: energetic BEM [1] and the more recent formulation proposed in [2], for which the weakly singular operator is continuous and satisfies inf-sup conditions in the related spaces. However, numerical evidence suggests that it is unstable when using low-order Galerkin-Bubnov discretisations. As an alternative, it was shown in [3] that one obtains ellipticity -and thus stability- by composing V with the modified Hilbert transform [4].
In this talk, we reformulate these variational formulations as minimisation problems in L^2. For discretisation, the minimisation problem is restated as a mixed saddle point formulation. Unique solvability can be established by combining conforming nested boundary element spaces for the mixed formulation such that the first-kind variational formulation is discrete inf-sup stable. We will analyse under which conditions the discrete inf-sup stability is satisfied, and, moreover, we will show that the mixed formulation provides a simple error estimator, which can be used for adaptivity. The theory is complemented by several numerical examples.

References

[1] A. Aimi, M. Diligenti, C. Guardasoni, S. Panizzi: A space-time energetic formulation for wave propagation analysis by BEMs. Riv. Mat. Univ. Parma 8 (2008) 171–207.

[2] O. Steinbach, C. Urzúa–Torres: A new approach to space-time boundary integral equations for the wave equation. SIAM J. Math. Anal. 54 (2022) 1370-1392.

[3] O. Steinbach, C. Urzúa–Torres, M. Zank: Towards coercive boundary element methods for the wave equation. J. Integral Equations Appl. 34 (2022) 501–515.

[4] O. Steinbach, M. Zank: Coercive space-time finite element methods for initial boundary value problems. Electron. Trans. Numer. Anal. 52 (2020) 154–194.

Adaptive least-squares space-time finite element methods

C. Köthe, O. Steinbach
Institut für Angewandte Mathematik, TU Graz, Austria

For the numerical solution of an operator equation $Bu = f$ we consider a least-squares approach. We assume that $B : X \to Y^*$ is an isomorphism and $A : Y \to Y^*$ implies a norm, where X and Y are Hilbert spaces.
Firstly, we assume the differential operator B to be linear. The minimizer of the least-squares functional $\frac{1}{2}\|Bu - f\|_{A^{-1}}$ is then characterized by the gradient equation which involves an elliptic operator $S = B^* A^{-1} B : X \to X^*$. We introduce the adjoint $p = A^{-1}(f - Bu)$ and reformulate the first order optimality system as a saddle point system. Based on a discrete inf-sup condition we discuss related a priori error estimates and use the discrete adjoint p_h to drive an adaptive refinement scheme. Numerical examples will be presented which confirm our theoretical findings.
Secondly, we demonstrate how to apply the least-squares approach for the numerical solution of a non linear operator equation $B(u) = f$. We derive the related first order optimality system and discuss its solution via Newton's method. Numerical examples involving the semi-linear heat equation and the quasi-linear Poisson equation are presented in order to show the correctness of the proposed method.
Finally, we will conclude with some remarks on future work in this area which needs to be done.

On the exponential stability of uniformly damped wave equations

H. Egger[1], S. Kurz[2], R. Löscher[3]

[1]Institute of Numerical Mathematics, JKU Linz, Austria
[2]Faculty of Information Technology, University of Jyväskylä, Finland
[3]Institute of Applied Mathematics, TU Graz, Austria

We aim at providing exponential stability for a large class of wave propagation problems, including acoustics, the vibration of strings or membranes, electromagnetics, etc. To this end, we establish an abstract model in terms of a Hilbert complex, which provides the minimum structure required to properly model the considered problem class. The desired exponential stability follows straightforwardly from the model's structural properties, while avoiding excessive details that would occur on the level of specific problem instances.

Coupled boundary and volume integral equations for electromagnetics

Ignacio Labarca

Seminar für Angewandte Mathematik, ETH Zürich, Switzerland

We study frequency domain electromagnetic scattering at a bounded, penetrable, and inhomogeneous obstacle. By defining constant reference coefficients, a representation formula for the electric and magnetic fields is derived. The resulting expression contains volume operators related to the standard volume integral equations. Besides, it features integral operators defined on the boundary of the obstacle and closely related to boundary integral equations of the first-kind for transmission problems with piecewise constant coefficients. We show well-posedness of the continuous variational formulation and asymptotic convergence of Galerkin discretizations. Numerical experiments validate our expected convergence rates for different material properties.

Matrix-free monolithic multigrid methods for Stokes and generalized Stokes problems

D. Jodlbauer[1], U. Langer[1], T. Wick[2], and W. Zulehner[1]

[1]Institut für Numerische Mathematik, Johannes Kepler Universität Linz, Austria
[2]Institut für Angewandte Mathematik, Leibniz Universität Hannover, Germany

We consider the widely used continuous $\mathcal{Q}_k$-$\mathcal{Q}_{k-1}$ quadrilateral or hexahedral Taylor-Hood elements for the finite element discretization of the Stokes and generalized Stokes systems in two and three spatial dimensions. For the fast solution of the corresponding symmetric, but indefinite system of finite element equations, we propose and analyze matrix-free monolithic geometric multigrid solvers that are based on appropriately scaled Chebyshev-Jacobi smoothers. The analysis is based on results by Schöberl and Zulehner (2003). We present and discuss several numerical results for typical benchmark problems.

A well-conditioned combined field integral equation for electromagnetic scattering in Lipschitz domains

Van Chien Le, Kristof Cools

Department of Information Technology, Ghent University, Belgium

In this talk, we address two issues of integral equations for the scattering of time-harmonic electromagnetic waves by a perfect electric conductor with Lipschitz continuous boundary: resonant instability and dense discretization breakdown. The remedy to resonant instability is a combined field integral equation, and dense discretization breakdown is eliminated by means of operator preconditioning. The exterior traces of single and double layer potentials are complemented by their interior counterparts of a pure imaginary wave number. We derive the corresponding variational formulation in the natural trace space for electromagnetic fields and establish its well-posedness for all wave numbers. A Galerkin discretization scheme is employed using conforming edge boundary elements on dual meshes, which produces wellconditioned discrete linear systems of the variational formulation. Some numerical results are also provided to support the numerical analysis.

On some properties of the modified Hilbert transformation

Richard Löscher, Olaf Steinbach
Institut für Angewandte Mathematik, TU Graz, Austria

Space-time variational formulations of PDEs often lead to Petrov- Galerkin schemes. Thus, in the numerical treatment, stable pairs of ansatz and test spaces are of highest interest. The application of the modified Hilbert transformation to the test space, has proved itself very useful in this manner, leading to stable Galerkin-Bubnov schemes for both, parabolic and hyperbolic problems [1, 3]. Though, a rigorous analysis of the stability of this approach is still open.
In this talk we will first outline the open problems concerning stability, followed by deriving an alternative relation between the modified Hilbert transformation and the Hilbert transformation, see [2]. The link of these two operators will then enable us to give a partial result, concerning the stability.

[1] R. Löscher, O. Steinbach, M. Zank: Numerical results for an unconditionally stable space-time finite element method for the wave equation. In: Domain Decomposition Methods in Science and Engineering XXVI, pp. 625–632, 2022.

[2] O. Steinbach, A. Missoni: A note on a modified Hilbert transform. Appl. Anal. 102 (2023) 2583–2590.

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On condition number of the Steklov-Poincaré operator discretized by FEM and BEM

P. Vodstrčil, D. Lukáš, Z. Dostál, M. Sadowská, D. Horák, and J. Bouchala
TU VŠB Ostrava, Czech Republic

The finite element (FEM) and boundary element method (BEM) are two fundamental methods for the discretization of elliptic boundary value problems. Here we are interested in their application in the context of un-preconditioned FETI (finite element tearing and interconnecting) and BETI (boundary element) domain decomposition method for solving huge linear systems arising from the discretization of variational inequalities for elliptic partial differential equations. The reason for no preconditioning is that the duality transforms the general inequality constraints to bound constraints that can be solved very efficiently by the specialized algorithms, but the standard preconditioners transform the variables and do not preserve the bound constraints.

The local problems in FETI and BETI are defined by the Schur complements S_i^{FEM} and S_i^{BEM}. Since they both approximate the Steklov-Poincaré operator, it is natural to assume that that they are very similar to each other. Qualitatively, it is true, as the condition numbers of both matrices are proportional to H/h, where H and h denote the diameter of the subdomain and the discretization parameter, respectively. However, closer inspection of the conditioning of S_i^{BEM} is more favorable than that of S_i^{FEM}. In this talk we shall prove that for a 2D model scalar problem the regular condition number κS_i^{BEM} is less than half of that of κS_i^{FEM} for $h \to 0$.

Numerical analysis for electromagnetic scattering with nonlinear boundary conditions

Jörg Nick

Mathematisches Seminar, ETH Zürich, Switzerland

The talk covers a time-dependent electromagnetic scattering problem from obstacles whose interaction with the wave is fully determined by a nonlinear boundary condition. In particular, the boundary condition studied enforces a power law type relation between the electric and magnetic field along the boundary. Based on time-dependent jump conditions of classical boundary operators, we derive a nonlinear system of time-dependent boundary integral equations that determines the tangential traces of the scattered electric and magnetic fields.
Fully discrete schemes are obtained by discretising the nonlinear boundary integral equations with Runge–Kutta based convolution quadrature in time and Raviart–Thomas boundary elements in space. Error bounds with explicitly stated convergence rates are presented. Numerical experiments illustrate the use of the proposed method and provide empirical convergence rates.

A space-time fast boundary element method for the heat equation with temporal nearfield compression[1]

Günther Of, Raphael Watschinger

Institut für Angewandte Mathematik, TU Graz, Austria

We consider a space-time boundary element method for the solution of initial boundary value problems of the heat equation in three spatial dimensions. In particular we deal with tensor product meshes with adaptive decompositions of the considered time interval and adaptive spatial meshes. We apply a space-time fast multipole method as well as shared and distributed memory parallelization with respect to space and time.

We present a novel temporal nearfield compression technique which enables efficient computations for fine spatial mesh resolutions related to the considered adaptive tensor product meshes. In particular, we introduce a version of the adaptive cross approximation tailored to the nature of the considered heat kernel. Finally, we present numerical experiments that demonstrate the great benefits of the new method for tensor product meshes with spatially fine meshes and adaptive spatial meshes.

[1]The authors acknowledge support provided by the Austrian Science Fund (FWF) under the project I 4033-N32 in a joint project with the Czech Science Foundation (project 17-22615S).

Inverse estimation of hypersonic trajectories using the time reversal symmetry of the linearized acoustic wave equation

Matthias Ospel

Deutsch-Französisches Forschungsinstitut Saint-Louis, France

The presentation focuses on the inverse estimation of hypersonic trajectories by exploiting the time reversal symmetry of the linearized acoustic wave equation. The inverse estimation of hypersonic flight paths using pressure sensor arrays poses unique challenges and accurate estimation of the trajectory and velocity of hypersonic objects plays a crucial role in various applications such as aerospace engineering and security. We present an inverse trajectory estimation technique based on the advanced Green's function combined with optimization methods. We begin by briefly introducing the concept of time reversal symmetry in the context of acoustics and the fundamentals of the forward modeling of sonic booms. Then, results of numerical simulations are shown, demonstrating the efficacy of the proposed technique in reconstructing the flight path of hypersonic projectiles. Furthermore, we showcase calculations based on measurement data obtained from a field experiment in which a conical object was launched at hypersonic velocity.

Magnetostatic force computation using boundary element method

Piyush Panchal
Seminar für Angewandte Mathematik, ETH Zürich, Switzerland

We derive novel expressions for calculating local/global forces and torques on a piece of linear magnetic material, applying the virtual work principle via the shape derivative of the magnetic field energy. We use both scalar potential (H field) and vector potential (B field) based field descriptions, along with a volume based and a BIE based variational formulation to describe the solutions of the model boundary value problem. Different variational constraints lead to different expressions for the shape derivative, recovering classical expressions and yielding new ones. The expression arising from BIE based formulation of the vector potential description produces the best convergence rate which we demonstrate through numerical experiments computing total force, torque and the dual norm.

Space-time finite element methods for thermo-elastodynamics

M. Reichelt, O. Steinbach
Institut für Angewandte Mathematik, TU Graz, Austria

We present a novel approach for the thermo-elastic coupling using space-time finite elements. This method enables the simultaneous simulation of heat transfer and structural mechanics in a fully coupled manner. We will discuss the mathematical formulation of this approach, highlight its advantages, and present numerical examples that demonstrate its accuracy and computational efficiency in analyzing multiphysics systems. Further we will comment on preconditioning for linear systems arising from a least squares approach for the heat equation.

Anisotropic wavelet compression of boundary integral operator matrices

Helmut Harbrecht, Remo von Rickenbach
Universität Basel, Switzerland

On a domain $\Omega \subset \mathbb{R}^3$ with a Lipschitz boundary $\Gamma = \partial\Omega$, we consider a boundary integral equation $\mathcal{A}u = g$, where the kernel of the operator $\mathcal{A}$ is asymptotically smooth of the order $2q$. The discretisation of the boundary integral equation with standard, isotropic wavelet functions, leads to a quasi-sparse Galerkin matrix. In particular, all entries except $\mathcal{O}(N)$ in the Galerkin matrix can be dropped, while the solution with respect to the compressed matrix still converges at discretisation error accuracy.

Moreover, there exist respective adaptive wavelet methods, which can approximate the unknown solution at the rate of the best N-term approximation with linear complexity. However, in order to optimally resolve anisotropic singularities, such as edge singularities for example, anisotropic bases are required. In this talk, we present a compression scheme taylored to non-adaptive, anisotropic tensor-product wavelet bases, for which the same properties as for the isotropic bases hold. Investigating the adaptive method is work in progress.

Using a mixed approximation to solve the mixed boundary value problem of linear elasticity

Christina Schwarz
Chair of Scientific Computing, University of Bayreuth, Germany

We consider the mixed boundary value problem of linear elasticity (Lamé equation). Using the boundary element method to solve this problem, we only consider the unknowns on the boundary, resulting in a reduction of spatial dimension, but also in a non-sparse operator. Thus, in order to reduce storage, the involved integral operators need to be approximated.
The symmetric formulation of the boundary integral equations can be written as

$$\begin{pmatrix} V & -K \\ K^\top & D \end{pmatrix} \begin{pmatrix} \widetilde{t} \\ \widetilde{u} \end{pmatrix} = \begin{pmatrix} -V & \frac{1}{2}I + K \\ \frac{1}{2}I - K^\top & -D \end{pmatrix} \begin{pmatrix} \widetilde{g}_N \\ \widetilde{g}_D \end{pmatrix},$$

with the extensions of the solutions $\widetilde{u}$, $\widetilde{t}$, the Dirichlet and Neumann data $\widetilde{g}_D$, $\widetilde{g}_N$, and with the boundary integral operators

- single layer potential operator V,
- double layer potential operator K,
- hypersingular integral operator D.

In order to avoid the time-consuming computation of the hypersingular integral operator D, we can use an alternative approach, a mixed approximation, which was introduced by Olaf Steinbach for the Laplace equation [1].
With the help of an approximated Steklov-Poincaré operator a coupled saddle point problem can be derived

$$\begin{pmatrix} V & -\frac{1}{2}I - K \\ I^\top & \end{pmatrix} \begin{pmatrix} w \\ \hat{u} \end{pmatrix} = \begin{pmatrix} 0 \\ g_N - S\widetilde{g}_D \end{pmatrix},$$

which only involves single and double layer potential operators.
Our aim is now the application of this mixed formulation to the Lamé equation from linear elasticity. Therefore, we expand the library AHMED for solving elliptic differential equations, which uses hierarchical matrices in order to approximate the integral operators. One of the challenge here is the employment of two different grids for the discretisation with constant and linear test functions. According to [1], a finer discretisation is required for the constant test functions compared to the linear functions to ensure the stability of the method.
By avoiding the computation of the hypersingular integral operator, we expect an acceleration of the simulation, which we would like to show within this work. Furthermore, in order to decrease the simulation time even more, we would like to replace the Galerkin approach by collocation.

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Regularization and discretization error estimates for Dirichlet control problems

R. Löscher, O. Steinbach
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In recent work, we have developed an abstract framework to analyze regularization as well as finite element error estimates for tracking type optimal control problems subject to partial differential equations. One particular application involves Dirichlet boundary control problems subject to the Laplace equation. In this talk we present a series of regularization error estimates which depend on the regularity of the given target. While we first consider harmonic and smooth target functions, our main interest is to consider discontinuous targets where we have to use appropriate dual norms. These regularization error estimates are then combined with finite element discretization error estimates which results in an optimal choice of the regularization parameter when the finite element mesh size is given. We will present first numerical results, and we discuss our theoretical findings.

Numerical evaluation of retarded potentials

Ernst P. Stephan
Leibniz University Hannover, Germany

We focus on the numerical evaluation of retarded potentials. We present a composite quadrature rule (which converges exponentially fast) together with an optimal grading strategy. Our error analysis is based on regularity results in countably normed spaces for the potential. The computation of the matrix entries in a marching-on-in-time scheme corresponding to the discretization of a space-time variational formulation of retarded potential integral equations involves the numerical approximation of a special class of integrals. These integrals can be understood as a composition of double integrals over triangles intersected with domains of influence determined by the time discretization. We also give numerical results which underline our theoretical results.

First-kind Galerkin BEM for the Hodge-Helmholtz equation

Ralf Hiptmair[1], Carolina Urzúa-Torres[2], Anouk Wisse[2]
[1]Seminar for Applied Mathematics, ETH Zürich, Switzerland
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We consider exterior boundary value problems for the Euclidean Hodge–Helmholtz operator

$$-\Delta_{HH} := \mathbf{curl}\,\mathbf{curl} - \eta\nabla\mathrm{div} - \kappa^2.$$

Although the corresponding first-kind boundary integral equations were derived and analyzed in [2], no numerical results are currently available for $\kappa \neq 0$.
In this talk, we follow the cue from [2] and discuss the low-order Galerkin discretization of the corresponding single layer operator using BEMpp [1]. Then, we validate our implementation by comparing the eigenvalues related to the equivalent saddle point formulation for $\kappa = 0$ with those found in [3]. Finally, we present some preliminary results for $\kappa \neq 0$.

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A coercive boundary element method for the wave equation for flat objects

Martin Costabel[1], Marco Zank[2]
[1]IRMAR, Université de Rennes 1, France
[2]Fakultät für Mathematik, Universität Wien Austria

In this talk, we consider a boundary element method for the wave equation based on the single layer operator. First, we give an overview of boundary integral equations for the wave equation. Second, a new approach is introduced for the case of a flat screen. For this purpose, new space-time Sobolev spaces are derived by Fourier representations, and we present their most important properties. Applying the (classical) Hilbert transformation leads to a coercive and continuous single layer operator in a new space-time Sobolev space. Hence, this results in a new space-time variational formulation for the wave equation in the framework of the Lax–Milgram lemma. Thus, any conforming discretization is unconditionally stable. Based on this, we present a new space-time boundary element method for the wave equation. In the last part of the talk, numerical examples are shown.

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