Technische Universität Graz

5. Workshop on

Fast Boundary Element Methods in Industrial Applications

Söllerhaus, 5.–8.10.2007

U. Langer, O. Steinbach, W. L. Wendland (eds.)

Berichte aus dem Institut für Numerische Mathematik

Book of Abstracts 2007/10

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Program

Friday, 5.10.2007	
15.00–16.20	Coffee
16.20–16.30	Opening
16.30–17.00	W. Kreuzer (Acoustics Research Institute, Wien) A BEM model of a tunnel in an orthotropic layered halfspace
17.00–17.30	S. Engleder (TU Graz) Boundary element methods for the Helmholtz equation
17.30–18.00	D. Copeland (JKU Linz) BEM–based FEM for Helmholtz and Maxwell equations on arbitrary polyhedral meshes
18.00–18.30	C. Fasel (Universität des Saarlandes) Advances in nonlocal electrostatics
18.30	Dinner
Saturday, 6.10.2007	
9.00–9.30	J. O. Watson (UNSW, Sydney) Introduction to enriched boundary elements for fracture mechanics
9.30–10.00	K. Thöni (TU Graz) Efficient calculation of non–linear problems using the boundary element method
10.00–10.30	W. Elleithy (JKU Linz) Adaptive coupling of boundary element and finite element methods for elasto–plastic analysis
10.30–11.00	Coffee
11.00–11.30	D. Pusch (ABB Switzerland) Numerical experiments for 3D eddy–current simulations
11.30–12.00	P. Urthaler (TU Graz) Fast evaluation of Newton potentials in BEM
12.00–12.30	A. Salvadori (Universita di Brescia) Issues on fracture mechanics and boundary integral equations
12.30	Lunch
15.00–15.30	Coffee
15.30–16.00	B. Auchmann (CERN, Genf) A geometrical approach to BEM
16.00–16.30	J. Ostrowski (ABB Switzerland) Robust Maxwell formulations
16.30–17.00	S. Zaglmayr (JKU Linz) High–order finite element methods for computational electromagnetics
17.00–17.30	Break
17.30–18.00	D. Brunner (Universität Stuttgart) Investigation of sound emission from ships by a coupled FE/Fast BE approach
18.00–18.30	O. Steinbach (TU Graz) Coupled FE/BE formulations for the acoustic–structure interaction
18.30	Dinner

Sunday, 7.10.2007	
9.00–9.30	A. Chernov (ETH Zürich) Sparse p version BEM for first kind boundary integral equations with random data
9.30–10.00	L. Illyashenko–Raguin (ETH Zürich) A spectral boundary integral equation method for optical response of plasmon resonant nanoparticles with non–smooth boundaries
10.00–10.30	M. Windisch (TU Graz) Stable boundary element methods for electromagnetic scattering
10.30–11.00	Coffee
11.00–11.30	C. Pechstein (JKU Linz) Hybrid tearing and interconnecting methods in unbounded domains
11.30–12.00	R. Prato (Universität Hannover) FE/BE coupling for time–dependent interface problems in electromagnetics
12.00–12.30	L. Kielhorn (TU Graz) Boundary element methods in time domain by using a symmetric Galerkin method
12.30	Lunch
13.30–18.00	Hiking tour
18.30	Dinner
Monday, 8.10.2007	
9.00–9.30	D. Praetorius (TU Wien) Energy norm based a posteriori error estimation for BEM
9.30–10.00	G. Haase (KFU Graz) A high performance parallel linear algebra toolbox: Building blocks for general parallel solvers
10.00–10.30	Coffee
10.30–11.00	S. Börm (MPI Leipzig) Hierachical compression
11.00–11.30	M. Bebendorf (Universität Leipzig) Recompression techniques for adaptive cross approximation
11.30	Closing

A Geometrical Approach to BEM

B. Auchmann[1], S. Kurz[2], S. Rjasanow[3]
[1]CERN, Genf, [2]ETAS, Stuttgart, [3]Universität des Saarlandes, Saarbrücken

We propose a symmetric BEM which approximates the integral kernel by consecutive application of de Rham map (discretization) and Whitney map (interpolation). The latter concepts belong to the discrete exterior calculus, which constitutes the theoretical framework for discrete electromagnetic theories (Finite Differences, first–order Finite Elements, Finite Integration Technique, Cell Method).
We note that our BEM formulation features so–called pairing matrices, which are known in discrete exterior calculus to represent mappings of degrees of freedom from a primal mesh to a barycentrically dual mesh. As a consequence we can discuss structural similarities between the convential Galerkin BEM and the generalized collocation BEM.
Our method reduces the dimension of the quadrature domains. Furthermore, the method is easy to implement. We thus hope to contribute to a further dissemination of the boundary element method in the engineering community. The 2-D BEM has been coupled to a nonlinear discrete electromagnetic formulation. First numerical experiments have shown encouraging convergence behaviour. The 3-D formulation is under investigation.

Recompression techniques for adaptive cross approximation

M. Bebendorf
Universität Leipzig

The adaptive cross approximation method generates low-rank approximations to suitable $m \times n$ subblocks of discrete integral formulations of elliptic boundary value problems. A characteristic property is that the approximation, which requires $k(m+n)$, $k \sim |\log \varepsilon|^*$, units of storage, is generated in an adaptive and purely algebraic manner using only few of of the matrix entries. In this article we present three recompression techniques which bring the required amount of storage down to kr, where r depends logarithmically on the accuracy of the approximation but is independent of the matrix size.

Hierarchical Compression

S. Börm
MPI Leipzig

Discretizing an integral operator by a standard finite element scheme usually leads to a dense stiffness matrix, and handling this matrix efficiently is a major challenge. Most approaches to this problem fall into two categories: analytical techniques, e.g. the multipole and panel–clustering expansions of the kernel functions or the wavelet representation of the trial and test spaces, and algebraical techniques, e.g., approximative factorizations of matrix blocks.

Both approaches have advantages and disadvantages, but most of them have one thing in common: they are based on low–rank approximations of matrix blocks, therefore they lead to a representation of the dense matrixby a *hierarchical matrix.* Matrices in this form require only a relatively small amount of storage, and they can be used to construct efficient preconditioners for the frequently ill–conditioned integral equations.

This talk focuses on $\mathcal{H}^2$*–matrices*, a combination of hierarchical matrix techniques with ideas of multilevel methods. For large problems, $\mathcal{H}^2$–matrices require far less storage than hierarchical matrices. If we want to take advantage of this property, we need algorithms for constructiong an $\mathcal{H}^2$–matrix approximation with a little additional storage as possible, and we want these algorithms to be flexible enough to use any of the popular low–rank approximation schemes (like multipole expansion, adaptive cross approximation or interpolation).

The standard schemes for the construction of low–rank approximations split the matrix into a number of blocks and treat each block separately. Whiles this approach leads to simple and efficient algorithms, it cannot take advantage of connections between the blocks to reduce the storage requirements even further. In this talk, I present a technique for recovering these connections and using them to reduce storage requirements by representing the dense matrix by an $\mathcal{H}^2$–matrix. The recovery algorithm adds only a small computational overhead, and experiments show that it already provides an advantage for moderate problem dimensions and becomes more efficient as the number of degrees of freedom grows. Since it is based on general low–rank approximations, any of the approximation schemes mentioned above can be used to construct an initial approximation, and the algorithm will turnit into a nearly optimal $\mathcal{H}^2$–matrix.

The basic idea of the algorithm is to convert submatrices into $\mathcal{H}^2$–matrices and then merge these submatrices to form larger submatrices until the entire matrix has been approximated. The merging process is carried out by computing the singular value decomposition of small matrices, and the singular values provide enough information to guarantee that the approximation error can be closely controlled. Since all submatrices are converted into the $\mathcal{H}^2$–matrix representation as soon as possible, the algorithm requires only a small amount of additional storage.

Investigation of Sound Emission from Ships by a Coupled FE/ Fast BE Approach

D. Brunner[1], M. Junge[1], L. Gaul[1], C. Cabos[2]

[1] University of Stuttgart, [2]Germanischer Lloyd, Hamburg

The legal limits of noise level of ships are continuously reduced. Ships should be quiet onboard for people's comfort and the underwater noise pol lution is to be minimized, since the water is the living environment of m arine mammals.

Simulation of the vibro–acoustic behavior of ships needs dealing with fl uid–structure coupled problems, since the surrounding water has a signif icant influence on the vibration behavior. For the structural part, name ly the ship, the finite element method (FEM) is used. The commercial fini te element package ANSYS is applied for setting up the mass and stiffness matrices. The surrounding water is modelled with the fast multipole boun dary element method (FBEM). Different formulations are investigated for the totally submerged and partially immersed case. For the last mentioned case, a special halfspace formulation is applied to incorporate the pres sure boundary condition of the infinite water surface. Iterative precondi tioned solvers are employed and the numerical efficiency is compared. The applicability of the coupling schemes is demonstrated using a real–worl d shipmodel. Finally, a non–conforming coupling scheme is presented, whe re the nodes of the coarse fluid elements are coinciding with some of the nodes of the fine structure mesh. The pressure on the remaining nodes is interpolated linearly.

Sparse p version BEM for first kind boundary integral equations with random data

A. Chernov, C. Schwab
ETH Zürich

We consider the weakly singular boundary integral equation $\mathcal{V}u = g$ on a radomly perturbed smooth closed surface $\Gamma(\omega)$ with deterministic g or on a deterministic closed surface Γ with stochastic $g(\omega)$. The aim is the computation of the moments $\mathcal{M}^k u := \mathbb{E}[\otimes_{i=1}^k u], k \geq 1$, if the corresponding moments of the perturbation are known. The problem on the stochastic surface is reduced to a problem on the nominal deterministic surface Γ with the random perturbation parameter $\kappa(\omega)$. Note, that $u(\omega)$ depends nonlinearly on $\kappa(\omega)$.
Resulting formulation for the kth moment is posed in the tensor product Sobolev spaces and involve the the k–fold tensor product operators $\mathcal{V}^{(k)} = \otimes_{i=1}^k \mathcal{V}$. The standard full tensor product Galerkin BEM requires $\mathcal{O}(N^k)$ unknowns for the kth moment problem, where N is the number of unknowns needed to discretize the normal surface Γ. Based on [3], we develop the p–sparse grid Galerkin BEM to reduce the number of unknowns of $\mathcal{O}(N(\log N)^{k-1})$ (cf. [1], [2] for the wavelet approach).

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BEM–Based FEM for Helmholtz amd Maxwell Equations on Arbitrary Polyhedral Meshes

D. Copeland

Johannes Kepler University Linz

We present new finite element methods for the Helmholtz and Maxwell equations on arbitrary three-dimensional polyhedral meshes, with boundary elements on the surfaces of the polyhedral elements. The methods are based on domain decomposition techniques, treating the polyhedral elements as subdomains. On a triangular mesh of the skeleton, we use the lowest order polynomial spaces and obtain sparse, symmetric linear systems despite the use of boundary elements. Moreover, piecewise constant coefficients are permissible. The resulting approximation on the skeleton mesh can be extended throughout the domain via representation formulas.

Adaptive Coupling of Boundary Element and Finite Element Methods for Elasto-Plastic Analysis [1]

W. Elleithy, U. Langer
Johannes Kepler University Linz

In this talk we present an adaptive finite element-boundary element coupling method for solving problems in elasto-plasticity. In order to obtain a computationally efficient coupling method, considerable attention is devoted to the generation and progressive adaption of the FEM and BEM discretizations.
In an attempt to estimate regions susceptible for FEM discretization, we propose the use of simple, and at the same time fast, post calculations, based on energetic methods which follow a simple hypothetical elastic boundary element computation. The FEM and BEM meshes are automatically generated over the estimated plastic and the remaining linear elastic regions, respectively.
The present adaptive coupling method is practically advantageous as it does not necessitate the predefinition and manual localization of the FEM and BEM subdomains. Moreover, the method is computationally efficient as it substantially decreases the size of FEM meshes, which plainly leads to reduction of required system resources and gain in efficiency. The numerical results confirm the effectiveness of the proposed method.

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Boundary Element Methods for the Helmholtz Equation

S. Engleder, O. Steinbach
TU Graz

Although the exterior boundary value problems for the Helmholtz equation with either Dirichlet or Neumann boundary conditions are unique solvable, related boundary integral equations may not be solvable, or the solutions are not unique. In particular, the boundary integral operators are not injective when the wave number k^2 is an eigenvalue of the interior Dirichlet or Neumann eigenvalue problem, respectively. Considering linear combinations of different boundary integral formulations this results in combined boundary integral equations, which are unique solvable for all wave numbers. The most known formulations are those of Brakhage–Werner and Burton–Miller. However, since the combined boundary integral equation involves boundary integral operators of both first and second kind, the analytical framework offers different settings. The classical combined boundary integral equations are considered in $L_2(\Gamma)$, where the uniqueness results are based on Gårdings inequality and Fredholm's alternative. To ensure the compactness of certain boundary integral operators, sufficient smoothness of the surface Γ is required. Recently, different regularized formulations are discussed, which ensure the unique solvability even for Lipschitz surfaces Γ.
Here we will describe modified regularized boundary integral formulations for the Helmholtz equation with either Dirichlet or Neumann boundary conditions. We analyse the associated boundary element approximation and give numerical examples to illustrate the theoretical results. Moreover we discuss different choices for preconditioners for linear systems, resulting from different boundary integral formulations.

Advances in Nonlocal Electrostatics

C. Fasel[1], S. Rjasanow[1], O. Steinbach[2]
[1]Universität des Saarlandes, [2]TU Graz

Proteins are responsible for nearly all chemical reactions in the human body. They can also be important weapons to fight diseases. To find a fitting reactant to an ailment hundredthousands of possible partners are tested in laboratory.
The number of tests could be significantly reduced, if it was possible to calculate the electric field of the virus and all possible reactants efficiently, thus they cannot react if their fields don't fit. The environment where they come together is structured like water. To describe the electrostatic field of a molecule in water, we have to deal with nonlocal electrostatics because of the hydrogenbond network and this is – in contrast to local electrostatics – very tricky, since the relation between electric field and displacement field is much more complicated than in the local case. The genuine formulation (see for instance [1]) involves one differential equation for the potential inside the molecule and one integro-differential equation on the outside.
We will present an equivalent system of four partial differential equations in each domain. For the spherical symmetric special case of an ion with charge located at the origin, an analytical solution will be given. The problem is an interface problem. The structure of the surface of an biomolecule and its size seem to exclude numerical calculations using finite element methods and therefor we want to use boundary element methods. So we also present a fundamental solution for the operators involved in the PDE-formulation, show their ellipticity and give a boundary integral formulation of the problem. The creation of the single and double layer potential for the operator on the unbounded exterior domain will be discussed.

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A High Performance Parallel Linear Algebra Toolbox: Building Blocks for General Parallel Solvers

G. Haase, M. Liebmann
Karl–Franzens Universität Graz

The Parallel Toolbox, a high performance parallel linear algebra toolbox written in C++is introduced. The toolbox provides building blocks for the construction of advanced parallel solvers for multilevel and domain decomposition aproaches. A parallel algebraic multigrid solver is used as a testbed for problems in fluid mechanics and biomedical engineering. The toolbox automatically generates the necessary parallel communication for unstructured distributed meshes. The communication complexity is handled by the toolbox with only a few simple user accessible routines. Benchmarks on high performance computing clusters are presented to validate the viability of the approach.
The potential of using the toolbox for FEM/BEM–couplings will be discussed as well as new processor developments which have to be taken into account for the algorithm development.

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A Spectral Boundary Integral Equation Method for Optical Response of Plasmon Resonant Nanoparticles with Non–Smooth Boundaries

L. Illyashenko–Raguin

ETH Zürich

In recent years metallic nanoparticles have attracted much theoretical and experimental attention due to their ability to exhibit a large scattering cross–section (SCS) and high near–field amplitude when a surface plasmon resonance (SPR) is excited. Such nanoparticles find variety of applications in today's industrial engineering research. They play a major role in surface–enhanced Raman spectroscopy and may act as localized light sources for scanning near–field optical microscopy and nanolithography [1]. Another interesting application is based on the energy transfer between evanescently coupled SPR particles [2].

It is still requested to enhance the design and optimization of the devices based on SPR applications. Since such resonances are found to exist for silver or gold particles and in the neighborhood of their geometrical singularities the dramatic field enhancement was observed [1], fast and accurate determination of the local fields and SCS for nanoparticles with complex dielectric permittivities and non–smooth boundaries remains a theoretical challenge. Therefore the goal of this work is to develop an efficient numerical calculation scheme for analyzing the optical response of a single SPR particle and their clusters.

The mathematical formulation of such problems results in the direct electromagnetic scattering and transmission problem for Helmholtz equation. The Boundary Integral Equation (BIE) methods are well suited for their treatment [3] due to implicitly fulfilled Sommerfeld radiation condition. Using classical approach based on layer–potential technique we obtain indirect BIE formulation. We use a kind of Spectral–Galerkin method with trigonometric polynomial approximants (spectral harmonics of the unit circle) combined with the singularity subtraction technique, which has been shown to converge exponentially when the boundary of the scatterer is sufficiently smooth [4]. Galerkin's method is further discretized, where a reduction of the complexity of the computational scheme is achieved by using the Fast Fourier Transform (FFT). Mapping and patching permits us to overcome the shape limitations of the classical spectral methods and to extend the applicability to arbitrary complicate geometry. The novel idea is the global parameterization of polygonal boundaries in terms of fast convergent Fourier series which arises from the use of confomal mapping technique providing excellent accuracy, reduced complexity and fast convergence of numerical algorithm is whole.

The performance of the proposed method is demonstrated by calculation of the near– and far–field characteristics including corresponding error anaylsis for several single and coupled SPR particles with various shapes and sizes.

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Boundary Element Methods in Time Domain by Using a Symmetric Galerkin Method

L. Kielhorn, M. Schanz
TU Graz

The Symmetric Galerkin Boundary Element Method (SGBEM) is well established for the treatment of elliptic partial differential equations. In the present work, this method is extended to the hyperbolic case, especially to the field of 3-d elastodynamics.
When dealing with time-dependent problems the underlying boundary integral equations contain convolution integrals with respect to the time variable. These integrations can be performed in several ways. Here, the Convolution Quadrature Method (CQM) proposed by Lubich is used. This time stepping procedure's benefit is the usage of the Laplace transformed fundamental solution which makes it attractive also for problems where time-domain fundamental solutions might not be known.
The symmetric Galerkin formulation requires the usage of the second boundary integral equation involving the hypersingular integral operator. Hence, a regularization for closed surfaces based on Stokes theorem is given resulting in a representation suitable for the numerical treatment. Some numerical studies are presented in order to validate this approach with respect to different spatial and time discretizations. Unfortunately, one of the biggest advantages of boundary element methods in time domain, namely the modelling of wave propagation phenomena within semi-infinite domains, has been lost since the regularization process is based on a closed boundary surface. Therefore, the talk ends with a short discussion of some recently developed ideas to overcome this drawback.

A BEM Model of a tunnel in an orthotropic layered halfspace

W. Kreuzer, W. Waubke
Austrian Academy of Sciences, Acoustics Research Institute, Wien

For the simulation of vibrations in orthotropic and anisotropic layers some work already has been done (i.e. [1]). Nevertheless the combination with the boundary element method turns out to be rather challenging because the lack of an usable analytic form of Green's function. For our model we use a Fourier based numeric approximation for the fundamental function (see for example also [2]).
In a first stage deformations and stresses that are caused by a pointload at the origin are calculated for a layered orthotropic halfspace. All calculations are done in the Fourier domain with respect to the spatial coordinates x and y. As the loadfunction we use the δ-functional which will be transformed into a uniform one load in the Fourier domain.
With the results from the first stage we can construct an approximation for the Green's function. The BEM formulation is still in the Fourier domain with respect to x which enables us to reduce the 3D problem into a series of 2D problems on the grid of the tunnels cross section, because the boundary integral equation is decoupled for every wavenumber k_x.

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Robust Maxwell formulations

R. Hiptmair[1], F. Krämer[2], J. Ostrowski[2]
[1]ETH Zürich, [2]ABB Schweiz

A novel unified method for the stable numerical solution of the time–harmonic Maxwell's equations for any frequency is presented. The method is based on an extended A–φ variational formulation of the full linear Maxwell's equations. This formulation avoids stability problems in the stationary limit, where it reduces to the equations of electrostatics and magnetostatics. Both capacitive and inductive effects are taken into account in a robust fashion for all frequencies.

Hybrid tearing and interconnecting methods in unbounded domains[2]

U. Langer, C. Pechstein
Johannes Kepler Universität Linz

Finite element tearing and interconnecting (FETI) and boundary element tearing and interconnecting (BETI) as well as the closely related dual–primal methods (FETI–DP and BETI–DP) are robust domain decomposition solvers for partial differential equations. These methods have proved to be parallely scalable and the condition number of of the corresponding preconditioned system is rigorously bounded by $C(1 + \log(H/h))^2$ where the constant C is independent of the subdomain diameter H, the mesh size h, and the coefficient if it is piecewise constant on the subdomains.
In this talk, we analyze BETI and BETI–DP methods for two– and three–dimensional potential equations where one of the subdomains is unbounded, i.e. one of the sub–problems is an exterior problem with a radiation condition. Under appropriate assumptions the condition number can be bounded by $C(1 + \log(H_F/h))^2$ where H_F denotes the maximal diameter of the interfaces between adjacent subdomains. In certain situations C can be shown to be robust with respect to the (typically large) diameter of the boundary of the unbounded domain. We sketch the idea of the proof and give some numerical results.

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Energy norm based a posteriori error estimation for BEM

C. Erath[1], S. Ferraz–Leite[2], S. Funken[1], D. Praetorius[2]
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The h–$h/2$–strategy is one very basic and well-known technique for the a posteriori error estimation for Galerkin discretizations of energy minimization problems. Let ϕ denote the exact solution. One then considers

$$\eta_H := \|\phi_{h/2} - \phi_h\|$$

to estimate the error $\|\phi - \phi_h\|$, where ϕ_h is a Galerkin solution with respect to a mesh $\mathcal{T}_h$ and $\phi_{h/2}$ is a Galerkin solution for a mesh $\mathcal{T}_{h/2}$ obtained by a uniform refinement of $\mathcal{T}_h$. We stress that η_H is always efficient – even with known efficiency constant $C_{\rm eff} = 1$, i.e.

$$\eta_H \le \|\phi - \phi_h\|.$$

Reliability of η_H follows immediately from the saturation assumption

$$\|\phi - \phi_{h/2}\| \le q\,\|\phi - \phi_h\|$$

with some uniform constant $q \in (0,1)$. Under this assumption, there holds

$$\eta_H \le \|\phi - \phi_h\| \le \frac{1}{\sqrt{1-q^2}}\,\eta_H.$$

However, for boundary element methods, the energy norm $\|\cdot\|$ is non-local and thus the error estimator η_H does not provide information for a local mesh-refinement. Recent localization techniques from [1] for $\widetilde{H}^{-\alpha}$-norms and [3] for $\widetilde{H}^{\alpha}$-norms allow to replace the energy norm in this case by h-weighted L^2-norms resp. H^1-norms, where h denotes the local mesh-size. In particular, this very basic error estimation strategy is also applicable to steer an h-adaptive mesh-refinement. For instance, for Symm's integral equation, the L^2-norm based estimator

$$\mu_H := \|h^{1/2}(\phi_{h/2} - \phi_h)\|_{L^2(\Gamma)}$$

is equivalent to η_H. We thus may use μ_H to steer the mesh and η_H to estimate the error.

Recently [2], we observed that η_H is equivalent to the averaging error estimator from [1] as well as to the two-level error estimator from [5]. In particular, this generalizes the proof of [5] from the case of uniform mesh-refinement to adaptively generated meshes. Numerical examples for Symm's integral equation conclude the talk.

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FE/BE coupling for time–dependent interface problems in electromagnetics

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We present an h-version of the FE/BE coupling method to solve the eddy current problem for time dependent Maxwell's equations. For the time discretization we use the discontinuous Galerkin method with piecewise linear test and trial functions; for the space discretization we take $\mathbf{H}(\mathbf{curl}, \Omega)$ -conforming vector-valued polynomials to approximate the electric field in the conductor Ω and surface curls of continuous piecewise polynomials on the boundary Γ of Ω to approximate the twisted tangential trace of the magnetic field on Γ. Singular, double singular and hypersingular boundary integral operators appearing in the variational formulation. Finally we present both a priori and a posteriori estimates.

Numerical Experiments for 3D Eddy–Current Simulations

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In many industrial applications the appearing physical phenomenon of eddy currents is playing an important role which has to be considered. They are responsible for inducing force on device units as well as for thermal heat production in the device material.
Starting with the H–φ formulation for modeling eddy currents in 3D we end up with essentially four equations in the space of complex numbers. In order to solve these boundary integral equations we are applying boundary element discretization methods like the collocation or Galerkin method.
An efficient solving procedure is obtained by implementing common compression techniques for dense boundary element matrices. Depending on the underlying electromagnetic model, we are using either the multipole method or the adaptive cross approximation, both of them have advantages in certain cases.
Finally, we are going to show some numerical experiments for various combinations of integration and acceleration techniques.

Issues on fracture mechanics and boundary integral equations

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The present note deals with recent issues on fracture mechanics and boundary integral equations. It aims at addressing the following topics: i. criteria and algorithms of quasi-static crack propagation in brittle and quasi brittle solids: theoretical aspects, formulation via boundary integral equations, implementation strategies; ii. SIFs and T stresses approximation via the boundary element method; iii. potential advantages in the use of iterative solvers.

Coupled FE/BE formulations for the acoustic–structure interaction

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The coupling of finite and boundary element methods is well suited to handle different physical models and phenomena, in particular when including exterior boundary value problems. Here we will focus on a time–harmonic acoustic–structure interaction where the acoustic field is modeled by using boundary integral equations. We discuss different boundary integral formulations which will be stable for all wave numbers, and we will describe different discretization strategies.

Efficient Calculation of non–linear problems using the Boundary Element Method

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The Boundary Element Method (BEM) is a useful tool for solving infinite and semi–finite numerical problems. By using this method, only the boundary of the problem has to be discretized. Thus, the dimension of the problem is reduced by one. regarding mesh generation, dimension of the system of equations, data storage, and post–processing, this is considered a significant advantage.
When dealing with non–linear problems, not only boundary integrals but also domain integrals arise. In the standard 2d or 3d approach, a mesh of area or volume cells is used respectively for the evaluation of the domain integrals (see e.g. [1,2,4]). The drawback of the cell method is that the main advantage of the BEM is lost because cells have to be generated by the user, either in the whole domain or in parts of the domain that are expected to behave non–linearly. Moreover, the evaluation of the internal results and the integration over the cells are rather time consuming. However, the size of the system of equations does not depend on the domain discretization, which means that no additional degrees of freedom will be introduced using internal cells. In addition, internal cells can be generated automatically [3] and the effort in discretization of internal cells can be optimized. It will be shown that non–conforming hybrid cell meshes can be used to reduce computational time. However, the idea is to generate such meshes automatically by using adaptive strategies based on an error estimator. Furthermore, a new engineering approach to computing internal stresses on cells very efficiently will be presented. Some preliminary results will be shown and be up for discussion.

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Fast Evaluation of Newton Potentials in BEM

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This talk is concerned with the evaluation of Newton potentials in the case of the Poisson equation and the system of linear elastostatics. First, a direct evaluation of the Newton potentials is considered. This can be accelerated by the Fast Multipole Method. Alternatively, the Newton potential $N_1 f$ can be computed indirect by solving a boundary integral equation if $N_0 f$ is known. The efficiency of direct and indirect computation is compared in numerical examples.
If the volume function satisfies a certain partial differential equation, applying integration by parts may reduce the Newton potentials to surface integrals only. Thus a meshing of the volume is not needed for the simulation at all. Such an approach is applied to electromechanical coupling.

Introduction to Enriched Boundary Elements for Fracture Mechanics

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Cracks may be modelled in two and three dimensions by coincident isoparametric boundary elements on each face. The primary objective of analysis is the computation of stress intensity factors, from which it is possible to predict whether a crack is stable or will propagate. Stress intensity factors are generally computed from crack opening displacements or the Rice J–integral. It is usually necessary to refine the mesh towards the crack root, and for quadratic elements the midside nodes of elements adjoining the root are shifted to allow the singular component of displacement to be modelled more accurately by the shape functions. There are, however, uncertainties in choice of contour for the J–integral and interpretation of crack opening displacements, and midside node shifting impairs the ability to model other components of the displacement field.
In an alternative approach, the isoparametric shape functions are supplemented by singular functions which exhibit the same singularities as terms of the Williams eigenfunction expansion. In the three–dimensional implementation outlined in this presentation, all three modes are taken into account for the first three eigenvalues of that expansion. There is no needfor local mesh refinement, and stress intensity factors (including T–stresses) are computed simulataneously with nodal displacements and so the uncertainties referred to above are eliminated. Notch singularities can also be modelled, with the dominant eigenvalues only in each mode taken into account. Results are compared with published data, and possible future developments are discussed.

Stable Boundary Element Methods for Electrogmagnetic Scattering [3]

O. Steinbach, M. Windisch
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For exterior electromagnetic scattering problems, the boundary integral equation method is a suitable choice for a numerical approach, because only the boundary has to be discretised, and the Silver-Müller radiation condition is incorporated. However, the unique solvability of the original problem can get lost in particular when eigenfrequencies of the scattering body appear. A first approach to overcome this problem is to use the approach of Brakhage and Werner, who introduced a combined field integral equation for the acoustic scattering problem. But this approach is considered usually in $L_2(\Gamma)$, where uniqueness results are based on Gardings inequality and Fredholm's alternative. However, the compactness of certain boundary integral operators is needed, i.e. the boundary must be assumed to be sufficiently smooth. That's why modified boundary integral equations were introduced which are formulated in the energy function spaces to ensure unique solvability also for Lipschitz polyeders. In this talk a modified boundary integral equation will be presented that in comparison to already existing approaches neither uses a compact operator in the formulation nor uses the Hodge decomposition, further on first steps of developing some numerical estimates are discussed. Finally a numerical example is presented.

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High–order finite element methods for computational electromagnetics

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A key tool in the design of numerical methods for Maxwell's equations and their numerical analysis is the de Rham Complex, which relates function spaces and their natural differential operators, and reads in 3D:

$$\mathbb{R} \xrightarrow{\text{id}} H^1(\Omega) \xrightarrow{\nabla} H(curl,\Omega) \xrightarrow{\text{curl}} H(Div,\Omega) \xrightarrow{\text{div}} L_2(\Omega) \xrightarrow{0} \{0\}.$$

The sequence is exact: the range of an operator in the sequence coincides with the kernel of the next operator. Especially, the kernel of the curl-operator is stated by the gradient fields of H^1.

The de Rham Complex perfectly fits to electromagnetics: in a variational setting $H^1(\Omega)$ is the natural function space for the electrostatic potential, the magnetic and the electric fields lie in $H(curl,\Omega)$ and their fluxes belong to $H(div,\Omega)$. For a proper conforming finite element method, the discrete spaces have to form an exact sequence as well.

The innovation of the presented work is to introduce high-order $H(curl)$-conforming basis functions, where we respect the de Rham Complex already in their construction process. Namely, we explicitely use gradients of the corresponding H^1-conforming basis functions in the FE-basis. This yields an explicite seperation of the set of basis functions into lowest-order Nedelec functions, higher-order gradients and irrotational fields.

In the second part of the talk, we will focus on two practical advantages: The construction of efficient preconditioners for the curl-curl problem and special gauging strategies for magnetostatics and non-conducting domains in eddy current problems.

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