Conference on Rings and Polynomials

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Conference information

Date

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Venue

Graz University of Technology
Department of Analysis and
Number Theory
Steyrergasse 30
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Scientific Comittee

Karin Baur, Jean-Luc Chabert, Marco Fontana, Alfred Geroldinger, Sarah Glaz, Irena Swanson

Local Organizers

Sophie Frisch, Roswitha Rissner, Carmelo A. Finocchiaro

Idempotent and regular in semirings associated to transformation semigroups with restricted range

Ananya Anantayasethi

The transformation semigroup on a set X, denoted by $T(X)$, is a semigroup with base set is all mapping from X to X and usual coposition. For $Y \subseteq X$, the set $T(X,Y) = \{\alpha \in T(X) \mid x\alpha \subseteq Y\}$ is a transformation subsemigroup with restricted range. In [3], gave a characterization of all regular elements in $T(X,Y)$. In [4], J. Sanwong and W. Sommanee determined a largest regular subsemigroup of $T(X,Y)$. For any semigroup S, it is associated to a semiring $\mathcal{P}(S)$ of the power set of S under the usual set union and the complex product induced from binary operation of S. In this work, we give a characterization of all idempotent and regular elements of a semiring $\mathcal{P}(T(X,Y))$.

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Unique factorization in torsion-free modules

Gerhard Angermüller

A generalization of unique factorization in integral domains to torsion-free modules ("factorial modules") has been proposed by A.-M. Nicholas in the 1970s and subsequently refined by D. L. Costa, C.-P. Lu and D. D. Anderson - S. Valdes-Leon. The aim of this lecture is to present new results on this theory. In particular, it is shown that locally projective modules, flat Mittag-Leffler modules and torsionfree content modules are factorial modules. Moreover, inert extensions of factorial domains are characterized with help of factorial modules.

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Cox rings and unique factorization

Ivan Arzhantsev

In this talk we introduce and discuss a total coordinate ring, or a Cox ring, of an algebraic variety with emphasis on unique factorization properties. Our aim is to give an elementary introduction to the theory of Cox rings, to discuss recent algebraic characterizations of Cox rings and connections to divisor theories for semigroups, and to formulate several open problems.

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The valuative capacity of the set of sums of d-th powers

Marie-Andrée B.Langlois

If E is a subset of the integers then the n-th characteristic ideal of E is the fractional ideal of $\mathbb{Z}$ consisting of 0 and the leading coefficients of the polynomials in $\mathbb{Q}[x]$ of degree no more than n which are integer valued on E. For, p, a prime the characteristic sequence of $Int(E, \mathbb{Z})$ is the sequence $\alpha_E(n)$ of negatives of the p-adic valuations of these ideals. The asymptotic limit $\lim_{n\to\infty} \frac{\alpha_{E,p}(n)}{n}$ of this sequence, called the valuative capacity of E, gives information about the geometry of E. We compute these valuative capacity for the sets E of sums of $\ell \geq 2$ integers to the power of d, by observing the p-adic closure of these sets.

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On weakly 2-absorbing δ-primary ideals of commutative rings

Ayman Badawi

Let R be a commutative ring with $1 \neq 0$. We recall that a proper ideal I of R is called a weakly 2-absorbing primary ideal of R if whenever $a, b, c \in R$ and $0 \neq abc \in I$, then $ab \in I$ or $ac \in \sqrt{I}$ or $bc \in \sqrt{I}$. In this paper, we introduce a new class of ideals that is closely related to the class of weakly 2-absorbing primary ideals. Let $I(R)$ be the set of all ideals of R and let $\delta : I(R) \to I(R)$ be a function. Then δ is called an expansion function of ideals of R if whenever L, I, J are ideals of R with $J \subseteq I$, then $L \subseteq \delta(L)$ and $\delta(J) \subseteq \delta(I)$. Let δ be an expansion function of ideals of R. Then a proper ideal I of R (i.e., $I \neq R$) is called a *w*eakly 2-absorbing δ-primary ideal if $0 \neq abc \in I$ implies $ab \in I$ or $ac \in \delta(I)$ or $bc \in \delta(I)$. For example, let $\delta : I(R) \to I(R)$ such that $\delta(I) = \sqrt{I}$. Then δ is an expansion function of ideals of R and hence a proper ideal I of R is a weakly 2-absorbing primary ideal of R if and only if I is a weakly 2-absorbing δ-primary ideal of R. A number of results concerning weakly 2-absorbing δ-primary ideals and examples of weakly 2-absorbing δ-primary ideals are given.

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Factorizations of Block Triangular Matrices

Nicholas Baeth

Transfer homomorphisms have long been used to study factorizations in commutative semigroups, allowing one to compute certain invariants in simpler commutative semigroups. In [1], weak transfer homomorphisms were introduced, allowing the transfer of information about factorization in a commutative semigroup to a noncommutative semigroup. In this talk we generalize this concept even further, constructing weak transfer homomorphisms from semigroups of block triangular matrices over not necessarily commutative rings to easier-to-understand semigroups.

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Torsion pairs generated by a module

Silvana Bazzoni

The notion of torsion pair in an abelian category was introduced formally by Dickson in the sixties and treated extensively in the books of Stenström (1975) and Golan (1986.) Since then the use of torsion pairs became an indispensable tool for the localization theory of rings and abelian categories.
Torsion pair have been generalized in the context of triangulated categories and used to study localizations and recollements of triangulated categories.
I will consider a particular class of torsion pairs in the module category, namely the torsion pairs generated by a module M. The torsion free class is easily described: it consists of the modules F such that there are no nonzero morphisms from M to F. The problem in general is to describe the torsion class.
Important examples are the tilting and silting torsion pairs which recently have been fully characterized in the case of commutative rings. Another example is the torsion pair generated by the quotient field of a commutative domain whose description is easy in case the torsion class coincides with the divisible module, and challenging in other cases.

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On n-trivial Extensions of Rings

Driss Bennis

The notion of trivial extension of a ring by a module has been extensively studied and used in ring theory as well as in various other areas of research like cohomology theory, representation theory, category theory and homological algebra. In this paper we extend this classical ring construction by associating a ring to a ring R and a family $M = (M_i)_{i=1}^n$ of n R-modules for a given integer $n \geq 1$. We call this new ring construction an n-trivial extension of R by M. In particular, the classical trivial extension will be just the 1-trivial extension. Thus we generalize several known results on the classical trivial extension to the setting of n-trivial extensions and we give some new ones. Various ring-theoretic constructions and properties of n-trivial extensions are studied and a detailed investigation of the graded aspect of n-trivial extensions is also given. We end the paper with an investigation of various divisibility properties of n-trivial extensions. In this context several open questions arise.

This is a joint work with D.D. Anderson, Brahim Fahid and Abdulaziz Shaiea (arXiv:1604.01486).

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Pure semisimple rings and direct products

Simion Breaz

We present some characterizations for pure-semisimple rings which use direct products of modules.
One of them depends on the (non-)existence of some large cardinals: Let R be a ring and let W be the direct sum of all finitely presented right R-modules. Under the set theoretic hypothesis ($V = L$), the ring R is right pure semisimple if and only if there exists a cardinal λ such that $\mathrm{Add}(W) \subseteq \mathrm{Prod}(W^{(\lambda)})$. On the other hand, there is a set theoretic model such that for every ring R there exists a cardinal λ such that $\mathrm{Add}(W) \subseteq \mathrm{Prod}(W^{(\lambda)})$.
Secondly, we will see that a left pure-semisimple ring R is of finite representation type (i.e. it is right pure-semisimple) if and only if for every finitely presented left R-module M the right R-module $\mathrm{Hom}_{\mathbb{Z}}(M, \mathbb{Q}/\mathbb{Z})$ is Mittag-Leffler.

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n-universal subsets and Newton sequences

Paul-Jean Cahen (with Jean-Luc Chabert)

We let E be a subset of an integral domain D with quotient field K. A subset S of E is said to be an n-universal subset of E if every integer-valued polynomial $f(X) \in K[X]$ on S (that is, such that $f(S) \subseteq D$), with degree at most n, is integer-valued on E (that is, $f(E) \subseteq D$). A sequence $a_0, \ldots, a_n$ of elements of E is said to be a Newton sequence of E of length n if, for each $k \leq n$, the subset $\{a_0, \ldots, a_k\}$ is a k-universal subset of E. Our main results concern the case where D is a Dedekind domain, where both notions are strongly linked to p-orderings, as introduced by Bhargava. We extend and strengthen previous studies by Volkov, Petrov, Byszewski, Frączyk, and Szumowicz that concerned only the case where $E = D$. In this case, but also if E is an ideal of D, or if E is the set of prime numbers $> n + 1$ (in $D = \mathbb{Z}$), we prove the existence of sequences in E of which $n + 2$ consecutive terms always form an n-universal subset of E.

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When is Int(E, V) a Prüfer domain?

Jean-Luc Chabert

Let K be a valued field, V be the corresponding valuation domain, and E be any subset of V. Under which hypotheses on E is the ring $\mathrm{Int}(E,V) = \{f \in K[x] \mid f(E) \subseteq V\}$ a Prüfer domain? It is known that the precompactness of E is sufficient [1] but, following a recent work of Loper and Werner, it is not necessary in general. To what extend is it necessary? It is when the valuation is discrete [1], or when the subset E is an additive subgroup of V [2]. In this talk, we generalize both previous results: the precompactness of E is necessary when the completion of K is maximally complete, or when the subset E is regular.

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Characterizations and representations of Moore-Penrose inverses,

Jianlong Chen

Moore-Penrose inverses and group inverses are very important generalized inverses. Core inverse of a complex matrix was first introduced by Baksalary and Trenkler in 2010. Rakic, Dincic and Djordjevic generalized this notion to the case of a ring in 2014. In this talk, we will give some new characterizations and representations of Moore-Penrose inverses, group inverses and core inverses, by using equations and units in a ring.

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Radical braces and the Yang-Baxter equation

Ilaria Colazzo

The Yang-Baxter equation is a basic equation of the statistical mechanics that arose from Yang's work in 1967 and Baxter's one in 1972. In 1992, Drinfeld *[in Quantum Groups (Leningrad, 1990), Lecture Notes in Math.Springer, Berlin, 1992, 1–8, 1510]* posed the question of finding all set-theoretic solutions of the Yang-Baxter equation. Later, in the seminal paper, Etingof, Schedler and Soloviev *[Duke Math. J., 1999, 169–209, 100]* laid the groundwork for the study of a particular class of these solutions, the non-degenerate involutive ones.

From a radical ring we may construct a non-degenerate involutive solution of the Yang-Baxter equation. In 2007, Rump *[J. Algebra, 2007, 153–170, 307]* found a strict link between a generalization of radical rings, called *radical braces*, and the non-degenerate involutive solutions.

Later in this talk, we introduce more general algebraic structures, the *skew braces* and the *semi-braces* that further generalize radical rings and have a link with solutions of the Yang-Baxter equation that are not necessary involutive (see *[L. Guarnieri, L. Vendramin, Skew braces and the Yang-Baxter equation, Accepted for publication in Math. Comp., 2015]* and *[F. Catino, I. Colazzo, P. Stefanelli, Semi-braces and the Yang-Baxter equation, in preparation]*).

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Finitely generated powers of prime ideals

François Couchot

Let P be a prime ideal of a commutative ring R. Assume there exists an integer $n > 1$ such that P^n is finitely generated. Is P finitely generated too? This question was first studied by Gilmer, Heinzer and Roitman in [1]. They proved that any reduced ring for which each prime ideal has a finitely generated power is Noetherian. They also got some positive answers to this question when R is an integral domain, but some negative responses too. In [2] Roitman showed that this question has always a positive answer when R is a coherent domain. In my talk I shall presents some new results about this question. The answer is positive for any reduced ring R which is either coherent or arithmetical. When R is not reduced the response remains positive except for minimal prime ideals which are not maximal. If R is a polynomial ring over either a ring of global dimension ≤ 2 or a reduced arithmetical ring then we shall see that the answer is positive too. I shall end my talk by giving some examples. In particular there exists examples of rings R for which the answer to this question is positive but negative for R_M for each maximal ideal M.

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Artinianness of Certain Graded Generalized Local Cohomology Modules

Fatemeh Dehghani-Zadeh

Let $R = \oplus_{n\geq 0} R_n$ be a homogeneous Noetherian ring with irrelevant ideal $R_+ = \oplus_{n>0} R_n$. Let M, N be two finitely generated graded R-modules and $\mathfrak{a} = \mathfrak{a}_0 + R_+$ an ideal of R. We get properties about the artinianness, tameness and asymptotic of the graded R-modules $H^i_{\mathfrak{b}_0}(H^j_{\mathfrak{a}}(M,N))$ for some i's and j's with a specified property, where $\mathfrak{b}_0$ is an ideal of R_0 such that $\dim(\frac{R_0}{\mathfrak{a}_0+\mathfrak{b}_0}) = 0$.

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A model structure approach to the Tate-Vogel cohomology

Nanqing Ding

We study Tate-Vogel cohomology of complexes by applying the model structure induced by a complete hereditary cotorsion pair $(\mathcal{A}, \mathcal{B})$ of modules. Vanishing of Tate-Vogel cohomology characterizes the finiteness of $\mathcal{A}$ dimension and $\mathcal{A}$ dimension of complexes defined in "X. Y. Yang and N. Q. Ding, On a question of Gillespie, Forum Math. 27 (6) (2015), 3205-3231". Applications go in three directions. The first is to characterize when a left and right Noetherian ring is Gorenstein. The second is to obtain some criteria for the validity of the Finitistic Dimension Conjecture. The third is to investigate the relationships between flat dimension and Gorenstein flat dimension for complexes. This talk is a report on joint work with J. S. Hu.

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Separating polynomial invariants of abelian groups

Mátyás Domokos

Let G be a group of linear transformations of a finite dimensional vector space V. A set S of G-invariant polynomial functions on V is called a *s*eparating system if whenever $x, y \in V$ can be separated by a polynomial invariant (i.e. there exists a G-invariant polynomial function f on V with $f(x) \neq f(y)$), then x, y are separated by an appropriate element of S (i.e. there exists an $h \in S$ with $h(x) \neq h(y)$). The study of separating systems has become an active area of invariant theory in the past fifteen years. Obviously a set of generators of the algebra of polynomial invariants on G is a separating system, so the notion of separating systems is a weakening of the notion of generating systems. In the talk the following question will be addressed: is this weakening reflected when we compare degree bounds for separating systems with degree bounds for generating systems? The case of finite abelian groups (in non-modular characteristic) will be discussed. In this case the question about degree bounds can be tied up with the theory of zero-sum sequences over the group.

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Zero-dimensionality and Artinian Subrings

Karim Driss

The study of zero-dimensionality in commutative rings has been widely treated in the literature (see [1, 2, 3, 4, 5]). In particular, many recent papers investigate zero-dimensional overrings, zero-dimensional subrings and Artinian subrings of a commutative ring. In this talk we are interested in the Artinian overring of pair of rings, that means, we are looking for intermediate Artinian rings between R and T, where R is a subring of a ring T.

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On perinormal domains

Tiberiu Dumitrescu

In their recent paper (J. Algebra 451 (2016)) N. Epstein and J. Shapiro introduced and studied the perinormal domains: those domains A whose overrings satisfying going down over A are flat A-modules. We show that every Prüfer v-multiplication domain is perinormal and has no proper lying over overrings. Conversely, a w-treed perinormal domain is a Prüfer v-multiplication domain. (Joint work with Anam Rani.)

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Some new constructions of spectral spaces and a topological version of Hilbert's Nullstellensatz

Marco Fontana

I will present some recent results on spectral spaces constructions obtained in collaboration with Carmelo Finocchiaro and Dario Spirito and, in particular, I will show that the space of radical ideals of a ring R, endowed with the hull-kernel topology, is a spectral space (after Hochster), and that it is canonically homeomorphic to the space of the non-empty Zariski closed subspaces of the prime spectrum of R, endowed with a Zariski-like topology.

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$\star$-Reductions of Ideals and Prüfer v-Multiplication Domains

Evan Houston

Recall that an ideal is said to be *basic* if it has no proper reductions. J. Hays (1973, 1975) characterized Prüfer domains as domains in which every finitely generated ideal is basic and one-dimensional Prüfer domains as domains in which every ideal is basic. We extend this to Prüfer v-multiplication domains. The extension is somewhat surprising, and we produce examples to explain why.

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On redundancy of ideal-projectivity for superfluous ideals in the finitely generated setting

Peter Kálnai

Let R be an associative ring with unity and let I be a two-sided ideal of R contained in the Jacobson radical $\mathcal{J}(R)$ of R. For a right module M over the ring R we call the right module M/MI ($M/M\mathcal{J}(R)$) *the ideal* (*the radical*) *factor* of M.

We (re)introduce four ideal related notions of *an ideal supplement*, *an ideal superfluity*, *an ideal projectivity* and *a projective ideal-cover*. The main result then states an equivalence of redundancy of these properties and their relation to the following conditions: the projectivity of a finitely generated flat module with the projective ideal factor and the finite generation of a (countably generated) projective module with the finitely generated ideal factor. The result slightly extends known facts for the boundary case of the Jacobson radical in this finitely generated setting. We also examine how idempotent-lifting modulo ideals in matrix rings of finite sizes is related to the conditions. As a consequence, superfluous ideals satisfying some form of nilpotency are examples for which the ideal projectivity gives (or might give under particular circumstances) no new property.

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On the generalized Clifford algebra

Jung-Miao Kuo

The Clifford algebra of a homogeneous polynomial was introduced by Roby as a generalization of the classical notion of the Clifford algebra of a quadratic form. It was proved by Haile that in characteristic not 2 or 3, among other things, the Clifford algebra of a nondegenerate binary cubic form is Azumaya of rank 9. In 2000, Pappacena generalized the notion of the Clifford algebra to the algebra associated to a monic (with respect to the first variable) homogeneous polynomial, called the generalized Clifford algebra. In this talk, I will present some results on the structure of this generalized Clifford algebra, generalizing those obtained by Haile.

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Geometric theories for constructive algebra

Henri Lombardi

In [1], a general method for getting constructive versions of Nullstellensatz-like theorems by deciphering classical nonconstructive proofs based on model theory was developed. There, the link between dynamical methods, coherent theories and coherent topoi was emphasized. Since [1], by using similar methods, a lot of abstract proofs in classical algebra have been transformed into algorithms. These constructive proofs give also new versions that are fully acceptable in classical mathematics, following the Bishop style. See, *e.g.*, the book [2]. *A posteriori*, it seems that almost all these contributions fall into the field of a constructive version of geometric theories (an infinitary generalization of coherent first order theories). We will explain some classical examples and show how their constructive versions can be understood in the framework of constructive geometric theories.

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Additively Regular Rings, Weakly Additively Regular Rings and Marot Rings

Tom Lucas

An ideal of a commutative ring is regular if it contains a regular element (an element that is not a zero divisor). A ring R is additively regular if for each pair of elements $a, b \in R$ with b regular, there is an element $r \in R$ such that $a + br$ is a regular element of R. A related notion introduced in this talk is that of a weakly additively regular ring. A ring R is weakly additively regular if for each pair of elements $a, b \in R$ with b regular, there is a pair of elements $p, q \in R$ such that $ap + bq$ is regular and $pR + bR = R$. If R is weakly additively regular, then each regular ideal can be generated by a set of regular elements (in other words, R is a Marot ring). However, there are weakly additively regular rings that are not additively regular. Like additively regular rings [1], if R is a weakly additively regular ring with only finitely many regular maximal ideals, then each invertible ideal is principal. The same conclusion need not hold if R is simply a Marot ring with only finitely many regular maximal ideals [1]. Examples will be given of weakly additively regular rings with only finitely many regular maximal ideals, but with infinitely many maximal ideals that are not regular.

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Precovers and preenvelopes by phantom and Ext-phantom morphisms

Lixin Mao

In this talk, we will discuss precovers and preenvelopes by phantom and Ext-phantom morphisms. A morphism $f : M \to N$ of left R-modules is called a phantom morphism if the induced morphism $\mathrm{Tor}_1^R(A, f) = 0$ for every (finitely presented) right R-module A. Similarly, a morphism $g : M \to N$ of left R-modules is said to be an Ext-phantom morphism if the induced morphism $\mathrm{Ext}_R^1(B, g) = 0$ for every finitely presented left R-module B. We prove that every left R-module has a phantom preenvelope if R is a right coherent ring and every left R-module has an Ext-phantom cover if R is a left coherent ring. In addition, we investigate the properties of precovers and preenvelopes by phantom and Ext-phantom morphisms under change of rings.

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Exponent of local ring extensions of Galois rings and digraphs of the kth power mapping

Yotsanan Meemark

In this talk, we consider a local extension R of the Galois ring of the form $GR(p^n, d)[x]/(f(x)^a)$, where n, d and a are positive integers and p is a prime and $f(x)$ is a monic polynomial in $GR(p^n, d)[x]$ of degree r such that the reduction $\overline{f}(x)$ in $\mathbb{F}_{p^d}[x]$ is irreducible. We establish the exponent of R without completely determination of its unit group structure. We obtain better analysis of the iteration graphs $G^{(k)}(R)$ induced from the k-th power mapping including the conditions on symmetric digraphs. In addition, we work on the digraph over a finite chain ring R.

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Nil-Clean Companion Matices

George Ciprian Modoi

Recall that an element of a ring is called nil-clean provided that it decomposes as a sum between an idempotent and a nilpotent. It turns out that decompositions like this are important in the study of some properties of rings, see [2]. We will work in the ring of all square matrices over a field. Here we solve a matrix completion problem, more precisely we write any companion matrix as sum between an idempotent matrix and one with prescribed characteristic polynomial. Using this we describe nil-clean companion matrices. Some examples are also provided.
This presentation is based on a joint work with Simion Breaz, [1].

[1] S. Breaz, G. C. Modoi, Nil-clean companion matrices, Linear Alg. Appl., 489(2016), 50–60.

[2] W.K. Nicholson, Lifting idempotents and exchange rings, Trans. Amer. Math. Soc., 229(1977), 269–278.

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Lorenzen's free semilattices over integral domains

Stefan Neuwirth

In his habilitation (published in 1949), Paul Lorenzen proposed an analysis of the concept of ideal in terms of the preorder given by divisibility. He wrote: "An ideal system of a preordered set is nothing but an embedding into a semilattice". In 1951, he showed that such an embedding is characterised by a single-statement entailment relation; an embedding into a distributive lattice is characterised by an entailment relation. In 1953, he used this to show how to embed a noncommutative preordered group into a lattice group.
In our work, done in collaboration with Thierry Coquand and Henri Lombardi, we describe Lorenzen's lattice theoretic approach to multiplicative ideal theory in its last form (1952–1953). We also provide a complete and constructive description of embeddings of commutative partially ordered groups into distributive lattice groups in terms of "unbounded" entailment relations. In particular, we obtain a constructive proof of the so-called Lorenzen-Dieudonné theorem.

[1] Important Author, Title of Important Article or Book, Important Journal or Publisher, Year of Publication, Pages, Volume, Edition

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Anomalous primes of the elliptic curve $E_D : y^2 = x^3 + D$

Hourong QIN

It is open whether there exists a polynomial in one variable of degree > 1 that represents infinitely many primes. The Hardy-Littlewood conjecture gives an asymptotic formula for the number of primes of the form $ax^2 + bx + c$. We establish a relationship between the Hardy-Littlewood conjecture and the Mazur conjecture.
Let $D \in \mathbb{Z}$ be an integer which is neither a square nor a cube in $\mathbb{Q}(\sqrt{-3})$, and let E_D be the elliptic curve defined by $y^2 = x^3 + D$. Mazur conjectured that the number of anomalous primes less then N should be given asymptotically by $c\sqrt{N}/\log N$(c is a positive constant), and in particular there should be infinitely many anomalous primes for E_D. We show that the Hardy-Littlewood conjecture implies the Mazur conjecture, except for $D = 80d^6$, where $0 \neq d \in \mathbb{Z}[\frac{1+\sqrt{-3}}{2}]$ with $d^6 \in \mathbb{Z}$. Conversely, if the Mazur conjecture holds for some D, then the polynomial $12x^2 + 18x + 7$ represents infinitely many primes. All anomalous primes belong to the quadratic progression $q(h) = \frac{1}{4}(1 + 3h^2)$. Assuming the Hardy-Littlewood conjecture, we obtain the density of the anomalous primes in the primes in $q(h)$ for any D. The density is $1/6$ in some cases, as Mazur had conjectured, but it fails to be true for all D. Our results are more general.

The main results of my talk have appeared in Proc. London Math. Soc. (3) 112 (2016) 415-453.
See also http://plms.oxfordjournals.org/content/112/2/415.full.pdf+html

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Divisor-class groups of monadic submonoids of $\mathbf{Int}(R)$

Andreas Reinhart

Let R be a factorial domain. In this talk we investigate the structure of special divisor-closed submonoids of $\mathrm{Int}(R)$ (the so called monadic submonoids). This is motivated by the fact that every monadic submonoid of $\mathrm{Int}(R)$ is a Krull monoid. It is well-known that the arithmetic of a Krull monoid can be described by using its divisor-class group, and thus it is natural to study these objects in detail. If $f \in \mathrm{Int}(R) \setminus \{0\}$, then $[\![f]\!] = \{g \in \mathrm{Int}(R) \setminus \{0\} \mid g \mid_{\mathrm{Int}(R)} f^k \text{ for some } k \in \mathbb{N}_0\}$ is called the monadic submonoid of $\mathrm{Int}(R)$ generated by f. We present a complete description of the divisor-class group of $[\![f]\!]$ if $f \in R[X] \setminus \{0\}$. Furthermore, we show that every finitely generated, torsion-free abelian group is isomorphic to the divisor-class group of $[\![f]\!]$ for some nonzero $f \in \mathrm{Int}(\mathbb{Z})$. We elaborate a few connections between the arithmetic of $\mathrm{Int}(R)$ and the structure of the divisor-class group of its monadic submonoids. In particular, we explain the infinitude of the elasticity and the tame degree of $\mathrm{Int}(R)$ for a non-trivial class of factorial domains R.

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Sets of Lengths of Powers of a Variable

Mark Rogers

A positive integer k is a *length* of a polynomial if that polynomial factors into a product of k irreducible polynomials. We find the sets of lengths of polynomials of the form x^n in $R[x]$, where $(R, \mathfrak{m})$ is an Artinian local ring with $\mathfrak{m}^2 = 0$.

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How to use ultraproducts in Commutative Algebra

Hans SCHOUTENS

I will give a brief survey of the use of ultraproducts in commutative algebra (although originally a concept from logic, ultraproducts of rings have an entirely algebraic construction; there are also two related, algebraic constructions, called proto-products and cata-products). I will discuss two applications: uniform bounds and transfer. The first is originally due to Schmidt-van den Dries, and can be best understood via proto-products; the second uses good algebraic properties of cata-products. As an application of the latter, I will show how characteristic p tight closure theory can be lifted to characteristic zero using ultraproducts. We may paraphrase this as: there exists a Frobenius endomorphism over $\mathbb{C}$!

[1] Hans Schoutens, *The use of ultraproducts in commutative algebra*, Lecture Notes in Mathematics, vol. 1999, Springer-Verlag, 2010.

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Graded local cohomology modules over polynomial rings are generalized Eulerian

Jyoti Singh (with Tony J. Puthenpurakal)

Let $R = K[X_1, \ldots, X_n]$, where K is a field of characteristic zero and R is standard graded. Let $\mathfrak{m} = (X_1, \ldots, X_n)$ and let E be the *injective hull of $R/\mathfrak{m}$. Let $A_n(K)$ be the n^{th} Weyl algebra over K. If $\mathcal{T}$ is graded Lyubeznik functor on $^*Mod(R)$, then we show that $\mathcal{T}(R)$ is generalized Eulerian $A_n(K)$-module. As an application, we show that $H^i_{\mathfrak{m}}\mathcal{T}(R) \cong E(n)^{a_i}$ for some $a_i \geq 0$. (This is a characteristic zero version of a result due to Ma and Zhang in characteristic $p > 0$).

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A transfer homomorphism for factorizations in bounded hereditary Noetherian prime rings

Daniel SMERTNIG

Let R be a (noncommutative) hereditary Noetherian prime (HNP) ring. Every element a of the monoid of non-zero-divisors $R^\bullet$ can be written as a product of atoms (irreducible elements), say $a = u_1 \cdots u_k$. The set of all possible factorization lengths k of a fixed element a, denoted by $\mathsf{L}(a)$, is a finite non-empty set. The system of sets of lengths $\mathcal{L}(R^\bullet) = \{ \mathsf{L}(a) \mid a \in R^\bullet \}$ is a basic arithmetical invariant of $R^\bullet$.
If R is bounded and every stably free right R-ideal is free, then there exists a transfer homomorphism from $R^\bullet$ to the monoid B of zero-sum sequences over a subset $G_{\max}(R)$ of the ideal class group $G(R)$. In a number-theoretic setting, the condition on the right R-ideals is not only sufficient but also necessary. The existence of this transfer homomorphism implies that the systems of sets of lengths, together with further arithmetical invariants, of the monoids $R^\bullet$ and B coincide. The arithmetic of monoids of the latter type has been the subject of much research.
The proof is based on the structure theory of finitely generated projective modules over HNP rings, as established in the recent monograph by Levy and Robson. We complement our results by giving an example of a non-bounded HNP ring in which every stably free right R-ideal is free but which does not allow a transfer homomorphism to a monoid of zero-sum sequences over any subset of $G(R)$.

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Module analogues of coincidence of nilpotent elements of a ring and its prime radical

David Ssevviiri

It is a well-known fact that in a commutative ring, the set of all nilpotent elements coincides with the intersection of all prime ideals. McCasland and Moore generalised this notion to modules by defining modules that satisfy the radical formula. A not necessarily commutative ring is 2-primal if its nilpotent elements coincide with the intersection of all its prime ideals. Groenewald and Ssevviiri generalised 2-primal rings to 2-primal modules. In this talk, I compare modules that satisfy the radical formula and 2-primal modules.

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Eggert's conjecture

Cora Stack

Eggert's conjecture (N. H. Eggert 1971 Pacific J Maths) states that for a finite dimensional commutative nilpotent algebra R over a perfect field F of characteristic p that $\dim(R) \geq p \dim(R^{(p)})$ where $R^{(p)} = \{x^p \in R\}$. We will discuss the significance and some of the history of this conjecture and prove some special cases.

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Radical braces over a field and the asymmetric product

Paola Stefanelli

Commutative associative radical algebras partially answer to the problem of finding all regular subgroups of an affine group (see *[Caranti, et al., Publ. Math. Debrecen, 2006, 297–308, 69]*), that is still an open problem formalized by Liebeck, Praeger and Saxl (see *[Memoirs of the AMS 203, 2010, no. 952]*.
A systematic way to obtain all regular subgroups was found in *[Bull. Aust. Math. Soc., 2009, 103–107, 79]* by Catino and Rizzo through a generalization of radical associative algebras, *radical braces over a field*. These structures were introduced independently by Rump in *[J. Algebra, 2007, 153–170, 307]* for their connection with the involutive non-degenerate solutions of the Yang-Baxter equation.
In this talk, we introduce the *asymmetric product* of radical braces over a field (see *[F. Catino, I. Colazzo, P. Stefanelli, J. Algebra, 2016, 164–182, 455]*), a construction that generalizes the semidirect product and allows us to systematically obtain regular subgroups of the affine group. In this way, we may put in a more general context the subgroups constructed by Hegedűs in *[J. Algebra, 2000, 740–742, 225 (2)]*.

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On locally invertibility and finite character for general ring extensions

Francesca TARTARONE

It is well-known that in an integral domain the invertible ideals are exactly the finitely generated ideals that are also locally principal. In many cases, like for Noetherian domains, the hypothesis "finitely generated" can be omitted. For these domains (also called LPI domains) the invertible ideals are exactly the locally principal.
In the context of integral domains the invertible ideals are exactly the projective ideals (viewed as modules on the domain) and the locally principal ideals provide a characterization of the faithfully flat ideals. Moreover, always in the integral case, invertible ideals correspond to the class of flat and finitely generated ideals.
Many generalizations of these facts involving the t-have been investigated over the recent years.
The study of this interplay among the concepts of invertible, faithfully flat and flat ideal can be extended to unitary rings A with respect to any their ring extension B. This is the content of a work of mine in collaboration with C.A. Finocchiaro [1].

[1] C.A. Finocchiaro, F. Tartarone, Invertibility of ideals in Prüfer extensions, submitted for publication

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Ring constructions on spectral spaces

Christopher Tedd

In his celebrated 1969 thesis, Mel Hochster provides, from a spectral space X, a construction of a ring R_X such that $\operatorname{Spec} R_X$ is homeomorphic to X, thus demonstrating that the spectral spaces exactly characterise the topological spaces that arise as the prime spectrum of a ring. However, the rings so constructed are generally regarded as rather inaccessible due to what Hochster himself refers to as the intricacy of the construction. In the case of a finite spectral space, Lewis[1] provides a more accessible construction, particularly in light of the observation of Fontana[2] that the technique employed utilises the amalgamated sum of spaces, and the corresponding fibre product of rings, thereby setting it within a topological/categorical framework. In this talk I will give an overview of this construction, including explicit descriptions of the rings obtained, show how it can be extended to represent morphisms of finite spectral spaces, and indicate how it may be used to address questions such as that of which spectral spaces arise as the spectrum of a Noetherian ring. For example it can be shown that every 1-dimensional Noetherian spectral space is the spectrum of a Noetherian ring.

[1] W. J. Lewis, The spectrum of a ring as a partially ordered set, J. Algebra, 1973, 419–434, 25

[2] M. Fontana, Topologically defined classes of commutative rings, Ann. Mat. Pura Appl., 1980, 331–355, 123

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Right-left symmetry of nonsingularity and CS condition in Utumi rings

Do Van Thuat

In associative ring theory, right-left symmetry of extending properties is extensively investigated under the assumption of primeness by authors [1, 2, 3]. In this work, we aim to find right-left symmetry of the CS and max-min CS conditions on some classes of rings without primeness. It is proved that if R is a right nonsingular right CS and left Utumi if and only if it is left nonsingular left CS right Utumi. A nonsingular Utumi ring is right max (resp. right min, right max-min) CS if and only if it is left max (resp. left min, left max-min) CS. We also study CS property on right and left ideals generated by an idempotent. Then, the results are generalized to nonsingular Utumi modules and their endomorphism rings.

[1] D. V. Huynh et al., On the symmetry of the Goldie and CS conditions for prime rings, Proceedings of the American Math. Soc., 2000, 3153-3157, 128:11.

[2] D. V. Huynh, The symmetry of the CS condition on one-sided ideals in a prime ring, J. Pure and Applied Algebra, 2008, 9-13, 212.

[3] S. K. Jain et al., Husain S. Al-Hazmi, and Adel N. Alahmadi, Right-Left Symmetry of Right Nonsingular Right Max-Min CS Prime Rings, Communications in Algebra, 2006, 3883-3889, 34.

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Krull dimension and unique factorization in Hurwitz polynomial rings

Phan Thanh Toan

Let R be a commutative ring with identity and let $R[x]$ be the collection of polynomials with coefficients in R. We observe that there are a lot of multiplications in $R[x]$ such that together with the usual addition, $R[x]$ becomes a ring that contains R as a subring. These multiplications are from a class of functions λ from $\mathbb{N}_0$ to $\mathbb{N}$. The trivial case when $\lambda(i) = 1$ for all i gives the usual polynomial ring. Among nontrivial cases, there is an important one, namely, the case when $\lambda(i) = i!$ for all i. For this case, it gives the well-known Hurwitz polynomial ring $R_H[x]$. In this paper, we study Krull dimension and unique factorization in $R_H[x]$. We show in general that $\dim R \leq \dim R_H[x] \leq 2\dim R + 1$. When the ring R is Noetherian, we prove that $\dim R \leq \dim R_H[x] \leq \dim R + 1$. A condition for the ring R is also given in order to determine whether $\dim R_H[x] = \dim R$ or $\dim R_H[x] = \dim R + 1$ in this case. We show that $R_H[x]$ is a unique factorization domain (resp., a Krull domain) if and only if R is a unique factorization domain (resp., a Krull domain) containing all the rational numbers.

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Rings generated by companion matrices

Stefan Veldsman

Full matrix rings, structural matrix rings and Morita rings have been studied extensively and there is a well-developed radical theory for all three. On the other hand, matrix rings generated by companion matrices of polynomials over rings provide the basis of many well-known rings (complex numbers, Gaussian integers, quadratic extension rings, circulant matrix rings, ...), but there is not yet a systematic study of these matrix rings. Here we initiate the radical theory for this class of matrix rings.

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Integer-valued Polynomials on Algebras: New Results and New Questions

Nicholas Werner

Let D be an integral domain with field of fractions K. Let A be a torsion-free D-algebra such that $A \cap K = D$, and let $B = K \otimes_D A$ be the extension of A to a K-algebra. We define the set of integer-valued polynomials on A to be $\mathrm{Int}(A) := \{f \in B[x] \mid f(A) \subseteq A\}$, and we denote the intersection of $\mathrm{Int}(A)$ with $K[x]$ by $\mathrm{Int}_K(A) := \{f \in K[x] \mid f(A) \subseteq A\}$, which is a subring of $K[x]$. Both $\mathrm{Int}(A)$ and $\mathrm{Int}_K(A)$ are generalizations of the traditional ring of integer-valued polynomials $\mathrm{Int}(D) = \{f \in K[x] \mid f(D) \subseteq D\}$.
After giving a brief overview of this subject, we will discuss some problems concerning $\mathrm{Int}(A)$ and $\mathrm{Int}_K(A)$ that have been investigated over the past few years. Questions that may be addressed include:

- When is $\mathrm{Int}_K(A)$ integrally closed? If $\mathrm{Int}_K(A)$ is not integrally closed, how can we describe its integral closure?
- We always have $D[x] \subseteq \mathrm{Int}_K(A) \subseteq \mathrm{Int}(D)$. When is $D[x] = \mathrm{Int}_K(A)$? When is $\mathrm{Int}_K(A) = \mathrm{Int}(D)$?
- $\mathrm{Int}(A)$ always contains A and $\mathrm{Int}_K(A)$ as subrings. When can $\mathrm{Int}(A)$ be generated (as a subring of $B[x]$) by A and $\mathrm{Int}_K(A)$?

We will also suggest questions and topics for further research, including:

- When is $\mathrm{Int}_K(A)$ a Prüfer domain?
- Studying integer-valued polynomials on subsets of algebras.
- Studying integer-valued rational functions on algebras.

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Polynomials Inducing the Zero Function on Local Rings

Cameron Wickham

For a Noetherian local ring $(R, \mathfrak{m})$ having a finite residue field of cardinality q, let $\mathcal{Z}(S)$ denote the ideal of $R[x]$ consisting of polynomials that vanish on the subset S of R. In this talk, the connections between the ideal $\mathcal{Z}(R)$ and the ideal $\mathcal{Z}(\mathfrak{m})$ will be discussed. In particular, using what we call *π-polynomials* (polynomials of the form $\pi(x) = \prod_{i=1}^{q}(x - c_i)$, where $c_1, \dots, c_q$ is a set of representatives of the residue classes of $\mathfrak{m}$) we show that, when R is Henselian, a generating set for $\mathcal{Z}(R)$ may be obtained from a generating set for $\mathcal{Z}(\mathfrak{m})$ by composing with $\pi(x)$. We also determine when $\mathcal{Z}(R)$ is nonzero, regular, or principal, respectively, and do the same for $\mathcal{Z}(\mathfrak{m})$. This is joint work with Mark Rogers.

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The semigroup of Betti diagrams over a short Gorenstein algebra

Roger Wiegand

Let k be a field and R a short, standard-graded, Gorenstein k-algebra. ("Short" means that $R = k \oplus R_1 \oplus R_2$.) When the embedding dimension $e := \dim_k R_1$ is three or more, R has wild representation type, but nonetheless one can classify the Betti tables of modules and describe the semigroup consisting of all Betti tables. This semigroup is atomic but is very far from being factorial. I will give an explicit description of the atoms of this semigroup and show that most of them are not *strong* atoms. In fact, for a typical atom α, one can find two other atoms β and γ, together with a large positive integer n such that $n\alpha = \beta + \gamma$.

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Prime ideal spectra in rings of power series

Sylvia Wiegand

We consider examples of prime spectra that arise rings constructed using power series. The examples are based on two projects, one with Ela Celikbas, Christina Eubanks-Turner, and the other with William Heinzer, and Christel Rotthaus.

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Algorithms for computing syzygies over $\mathbf{V}[X_1, \ldots, X_k]$, where $\mathbf{V}$ is a valuation ring

Ihsen Yengui

The following new result [1] will be presented:

A valuation domain $\mathbf{V}$ *has Krull dimension at most* 1 *if and only if for every finitely generated ideal* I *of* $\mathbf{V}[X_1, \ldots, X_k]$*, fixing the lexicographic order as monomial order, the ideal generated by the leading terms of the elements of* I *is also finitely generated.*

This proves the Gröbner Ring Conjecture. The same result is valid for Prüfer domains. As a “scoop", contrary to the common idea that Gröbner bases can be computed exclusively on Noetherian ground, we prove that computing Gröbner bases over $\mathbf{R}[X_1, \ldots, X_k]$, where $\mathbf{R}$ is a Prüfer domain, has nothing to do with Noetherianity; it is only related to the fact that the Krull dimension of $\mathbf{R}$ is at most 1, thus, opening the doors to a wider class of rings over which Gröbner bases can be computed (the class of Prüfer domains of Krull dimension at most one instead of that of Dedekind domains).

[1] Yengui I. Constructive Commutative Algebra - Projective modules over polynomial rings and dynamical Gröbner bases. Lecture Notes in Mathematics, no 2138, Springer 2015.

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On some cancellation algorithms

Maciej Zakarczemny

Let $g : \mathbb{N} \to \mathbb{N}$, where $\mathbb{N}$ is the set of positive integers, be fixed. Then the number

$$b(n) := \min\{m \in \mathbb{N} : \ g(1), g(2), \ldots, g(n) \,\text{are distinct modulo}\, m\}$$

is generally known as the *discriminant*. In the talk we will show how $b(n)$ can be computed for some special functions g by elementary number theory methods. This methods can be generalized to yield certain modifications of the Eratosthenes sieve.
Let $f : \mathbb{N}^k \to \mathbb{N}$, for some k. In the sieve we cancel all divisors of values of f at the points for which the sum of coordinates is not greater than n, and next we select the smallest number left. This leads to new problems for polynomial functions or quadratic and cubic forms that can be investigated by methods initiated in [1], and its companion papers.

[1] J. Browkin, H-Q. Cao, *Modifications of the Eratosthenes sieve*, Colloq. Math., 2014, vol. 135.

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The lattice of primary ideals of orders in quadratic number fields

Paolo Zanardo

This is a joint work with Giulio Peruginelli.
Let O be an order in a quadratic number field K with ring of integers D, such that the conductor $F = fD$ is a prime ideal of O. When P is a prime ideal of O different from F, it is easily see that the P-primary ideals form a chain. We give a complete description of the F-primary ideals of O. They form a lattice with a particular structure by layers; the first layer, which is the core of the lattice, consists of those F-primary ideals not contained in F^2. We get three different cases, according to whether the prime number f is split, inert or ramified in D. We also provide generating sets for each F-primary ideal, to make the relations of containment of these ideals evident.

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Polynomial constants of cyclic factorizable derivations

Janusz ZIELIŃSKI

We describe all polynomial constants of generic cyclic factorizable derivations over an arbitrary field of characteristic zero. Factorizable derivations are important in derivation theory. Namely, we may associate the factorizable derivation with any given derivation of a polynomial ring and that construction helps to determine constants of arbitrary derivations. Moreover, such systems play a significant part in population biology, laser physics and plasma physics.

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Index