Right-left symmetry of nonsingularity and CS condition in Utumi rings

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Right-left symmetry of extending properties under the assumption of primeness

- **D. V. Huynh** et al., On the symmetry of the Goldie and CS conditions for prime rings, Proceedings of the American Math. Soc., 2000, 3153-3157, 128:11.
- **D. V. Huynh**, The symmetry of the CS condition on one-sided ideals in a prime ring, J. Pure and Applied Algebra, 2008, 9-13, 212.
- **S. K. Jain** et al., Husain S. Al-Hazmi, and Adel N. Alahmadi, RightLeft Symmetry of Right Nonsingular Right Max-Min CS Prime Rings, Communications in Algebra, 2006, 3883-3889, 34.

OUR AIMS

Find right-left symmetry of

- the CS condition
- the max-min CS condition

on some classes of rings without primeness.

Key words and Related concepts

- CS modules (rings)
- Max CS modules (rings)
- Min CS modules (rings)
- Max-min CS modules (rings)
- Uniform modules
- Nonsingularity and cononsingularity
- Utumi and co-Utumi
- Self-generator
- Nondegenerate modules

Lemmas

Let M be a right R−module with the endomorphism ring S.

- If M is a CS module, then M is Utumi. In particular, a right CS ring is right Utumi.
- If M is a nonsingular CS module, then M is cononsingular. In particular, a right nonsingular right CS ring is left nonsingular.

Main results

The first theorem

The following statements are equivalent for a ring R:

1. R is a right nonsingular, right CS and left Utumi ring,
2. R is a left nonsingular, left CS and right Utumi ring.

Corollary

A right nonsingular, right CS and left Utumi ring is directly finite.

Main results

The second theorem

The following statements hold for every right nonsingular left Utumi ring R.

1. R is right min CS if and only if R is left max CS.
2. R is right max CS if and only if R is left min CS.
3. R is right max-min CS if and only if R is left max-min CS.

Main results

The third theorem

Let M be a right R–module with endomorphism S. If M is either a finitely generated, quasi-projective self-generator or a nondegenerate self-generator,

then

M is a nonsingular, co-Utumi, CS module if and only if S is a nonsingular, Utumi, right and left CS ring.

Main results

The fourth theorem

Let M be a right R−module with endomorphism S. If M is either a finitely generated, quasi-projective self-generator or a nondegenerate self-generator. Then the followings hold.

1. M is min CS if and only if S is right min CS and left max CS.
2. M is max CS if and only if S is right max CS and left min CS.
3. M is max-min CS if and only if S is right and left max-min CS.

Conclusion

Our question: If primeness is omitted, can we find a class of rings in which CS, max CS, min CS and max-min CS properties are right-left symmetric?

Our results:

1. On the class of nonsingular and Utumi rings, the conditions of CS, max CS, min CS and max-min CS are right-left symmetric?
2. Generalization on module is investigated, in particular, for the classes of finitely generated, quasi-projective self-generators and nondegenerate self-generators.

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