Does diffusion determine the domain?

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In the talk we will give a survey on answers to Kac's famous question: Can one hear the shape of a drum? The main part of the talk is devoted to a slightly different inverse problem, though. Given two domains, to say that the corresponding Laplace operators with Dirichlet boundary conditions have the same spectrum means that there exists a unitary operator intertwining the corresponding heat semigroups. Kac's originial question is whether this implies that the two underlying domains are congruent. Instead of an intertwining unitary operator we consider an intertwining order isomorphism (i.e. a bijective linear mapping U between the L^2 spaces such that $Uf \leq 0$ iff $f \leq 0$). This may be interpreted by saying that U maps the positive solutions of the diffusion equation to positive solutions. Formulated in this way, the answer is positive: the domains are indeed congruent if such intertwining order isomorphism exists. One may interprete the result by saying that "diffusion determines the domain". But much more is true: also the boundary conditions are determined. In fact, we consider such an order isomorphism which intertwines the Laplacian with a priori different boundary conditions on both sides. The result is that they have to be the same and the domains have to be congruent. Diverse quite subtle regularity considerations are needed even though the results are valid under very mild regularity conditions on the domains. The result also holds on Riemannian Manifolds (joint work with Markus Biegert and Tom ter Elst). In the general case the related regularitiy problems are delicate. However, if both manifolds are compact so that no boundary conditions are imposed, a direct easy proof is possible (see the joint paper with Tom ter Elst below).

References

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