Estimate of the decay exponent of an operator semigroup associated with a second-order linear differential equation

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In Hilbert space H we consider a second-order linear differential equation

$$u''(t) + Du'(t) + Au(t) = 0$$

and related quadric pensil $L(\lambda) = \lambda^2 I + \lambda D + A$ with self-adjoint positive definite operator A. By H_s denote a collection of Hilbert spaces generated by operator $A^{1/2}$, $\|\cdot\|_s$ is a norm on H_s. We will assume that D is a bounded operator acting from H_1 to H_{-1} and

$$\inf_{x\in H_1, x\neq 0} \frac{\mathrm{Re}(Dx, x)_{-1,1}}{\|x\|_1^2} = \delta > 0.$$

(here $(\cdot,\cdot)_{-1,1}$ is a duality pairing on $H_{-1}\times H_1$). The second-order differrntial equation can be linearized as a system $w'(t) = \mathcal{T}w(t)$ in "energy" space $H \times H_1$, where

$$w(t) = \begin{pmatrix} u'(t) \\ u(t) \end{pmatrix} \quad \mathcal{T} = \begin{pmatrix} -D & -A \\ I & 0 \end{pmatrix}.$$

We estimate an exponential decay rate for a semigroup generated by operator $\mathcal{T}$ in the space $H \times H_1$. We also obtained localization of the spectrum of the pencil $L(\lambda)$:

$$\sigma(L) = \sigma(\mathcal{T}) \subset \{\lambda \in \mathbb{C} \mid \mathrm{Re}\lambda \leq -\omega, |\mathrm{Im}\lambda| \leq \kappa(b)|\mathrm{Re}\lambda| + b\}$$

for some positive ω and for all $b > 0$.

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