

Scattering and inverse scattering for a left-definite Sturm-Liouville problem

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This talk reports on work that develops a scattering and an inverse scattering theory for the Sturm-Liouville equation $-u'' + qu = \lambda wu$ where w may change sign but $q \geq 0$. Thus the left-hand-side of the equation gives rise to a positive quadratic form and one is led to a left-definite spectral problem. The crucial ingredient of the approach is a generalized transform built on the Jost solutions of the problem and hence termed the Jost transform and the associated Paley-Wiener theorem linking growth properties of transforms with support properties of functions.

One motivation for this investigation comes from the Camassa-Holm equation for which the solution of the Cauchy problem can be achieved by the inverse scattering transform for $-u'' + \frac{1}{4}u = \lambda wu$.

It is based on joint work with Christer Bennewitz (Lund), and Rudi Weikard (Birmingham AL)