The inverse spectral transform for the dispersionless Camassa–Holm equation

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The dispersionless Camassa–Holm equation

$$\omega_t + 2\omega u_x + \omega_x u = 0, \quad u - u_{xx} = \omega, \qquad x,\, t \in \mathbb{R}$$

can be solved by the inverse spectral method: Given a solution of this equation, some particular (time dependent) spectral quantities of the weighted Sturm–Liouville spectral problems (parametrized by time $t \in \mathbb{R}$)

$$-f''(x) + \frac{1}{4} f(x) = z f(x) \omega(x,t), \qquad x \in \mathbb{R},\ z \in \mathbb{C}$$

evolve according to a simple linear flow, which can be solved explicitly. The transformation which takes some ω to the spectral quantities of the corresponding spectral problem (and hence the Camassa–Holm flow to a simple linear flow) is referred to as the inverse spectral transform. We discuss some properties of this transformation and employ them to derive several facts for the dispersionless Camassa–Holm equation.

The talk is based on joint work with G. Teschl.