On the buckling operator of an unbounded operator

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Let S be a densely defined, closed and linear operator in a Hilbert space $\mathcal{H}$, let $\mathcal{G}_S$ denote its graph equipped with the graph norm and let i_S denote the canonical injection $\mathcal{G}_S \to \mathcal{H}$.

The buckling operator of S is given as

$$S_B = i_S^* S i_S : \ \mathcal{G}_S \to \mathcal{G}_S.$$

It follows easily that S_B is bounded and has the same numerical range as S; therefore S is symmetric if and only if S_B is selfadjoint. [M. S. Ashbaugh, F. Gesztesy, M. Mitrea, R. Shterenberg and G. Teschl: The Krein-von Neumann Extension and its connection to an abstract buckling problem. Math. Nachr. 283, No. 2, 165-179 (2010)] proved : If S is positive definite then S_B is (for an approbriate scalar product on $\mathcal{G}_S$) unitarily equivalent to the inverse of the reduced Krein-von Neumann extension of S.
We extend this result to the von Neumann extension S_N where

$$\mathrm{Dom}\,(S_N) = \mathrm{Dom}\,(S) + \mathrm{Ker}\,(S^*) \qquad S_N(u+v) = Su.$$

for a symmetric operator S bounded from below.

This is a joint work with N. Lubrich.