Spectral multiplicity for the one-dimensional Schrödinger operator on the line

D. J. Gilbert

We consider the one-dimensional Schrödinger operator H on the real line in the case where Weyl's limit point case holds at both of the infinite endpoints. In this situation it is well-known that the spectral multiplicity of H may be one or two, and that it is often convenient to decompose H into a direct sum of two half-line operators, $H_{-\infty}$ and H_{∞}, when investigating the spectrum of H.

We will present necessary and sufficient conditions for H to have simple and/or degenerate spectrum, with particular reference to one or more of the following criteria:

(i) the boundary behaviour on the real axis of the corresponding Titchmarsh-Weyl functions, $m_{-\infty}$ and m_{∞}, which are associated with the Dirichlet half-line operators, $H_{-\infty}$ and H_{∞}, on $(-\infty, 0]$ and $[0, \infty)$, respectively,

(ii) the asymptotic behaviour of solutions of the Schrödinger equation at each of the limit point endpoints, in terms of the theory of subordinacy,

(iii) the rank of the spectral density for H.

The stability of the absolutely continuous and essential spectra under various perturbations will also be discussed, with a view to enabling the multiplicity properties of a family of operators to be inferred from complete knowledge of the location and multiplicity of the absolutely continuous and essential spectra associated with a single operator. This can be achieved provided the location and multiplicity of the absolutely continuous and essential spectra of the original operator are invariant under the chosen perturbation.