Eigenvalue asymptotics for nonsmooth singular Green operators

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Singular Green operators G appear typically as boundary correction terms in resolvents for elliptic boundary value problems on a domain $\Omega \subset \mathbb{R}^n$, and more generally they appear in the calculus of pseudodifferential boundary problems. In particular, the boundary term in a Krein resolvent formula is a singular Green operator. It is well-known in smooth cases, that when G is of negative order $-t$ on a bounded domain, its s-numbers (eigenvalues, if G is selfadjoint nonnegative) have the behavior

$$s_j(G) \sim c j^{-t/(n-1)} \text{ for } j \to \infty, \qquad (*)$$

governed by the boundary dimension $n-1$. In some nonsmooth cases, upper estimates $s_j(G) \leq C j^{-t/(n-1)}$ are known.

We show that $(*)$ holds when G is a general selfadjoint nonnegative singular Green operator with symbol merely Hölder continuous in x. We also show $(*)$ with $t = 2$ for the boundary term in the Krein resolvent formula comparing the Dirichlet and a Neumann-type problem for a strongly elliptic second-order differential operator (not necessarily selfadjoint), with coefficients in $W^1_p(\Omega)$ for some $p > n$.