Global solutions of the two-component Camassa–Holm system

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The two-component Camassa-Holm (2CH) system

$$u_t - u_{txx} + \kappa u_x + 3uu_x - 2u_x u_{xx} - uu_{xxx} + \eta \rho \rho_x = 0,$$
$$\rho_t + (u\rho)_x = 0,$$

with arbitrary $\kappa \in \mathbb{R}$ and $\eta \in (0, \infty)$, serves as a model for shallow water. Furthermore, it is a generalization of the famous Camassa–Holm (CH) equation which has been studied intensively. Thus naturally the question arises which results derived for the CH equation are also valid for the 2CH system. In this talk we will show how to describe global solutions. This question is of special interest since the 2CH system, like the CH equation, enjoys wave breaking and in general there are two possibilities how to continue solutions thereafter. Namely, either the energy is preserved which yields conservative solutions or if energy vanishes from the system, we obtain dissipative solutions. Additionally, we will admit initial data and hence solutions with nonvanishing asymptotics.

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