On asymptotic stability of kinks for relativistic Ginzburg-Landau equation

E. Kopylova

We consider nonlinear relativistic wave equation in one space dimension

$$\ddot{\psi}(x,t) = \psi''(x,t) + F(\psi(x,t)), \quad x \in \mathbb{R}, \quad F(\psi) = -U'(\psi), \tag{1}$$

where $U(\psi)$ is a potential of Ginzburg-Landau type

$$U(\psi) \sim (\psi^2 - 1)^2/4.$$

The kink is a nonconstant finite energy solution of stationary equation

$$s(x) \sim \tanh x/\sqrt{2}.$$

The corresponding moving kinks or solitary waves

$$s_{q,v}(t) = s(x - vt - q), \quad q, v \in \mathbb{R}, \quad |v| < 1, \quad \gamma = 1/\sqrt{1-v^2}$$

are the solutions to equation (1). Our main results are the following asymptotics

$$(\psi(x,t), \dot{\psi}(x,t)) \sim (s_{q_\pm,v_\pm}(x - v_\pm t - q_\pm), \dot{s}_{q_\pm,v_\pm}(x - v_\pm t - q_\pm)) + W_0(t)\Phi_\pm,$$

$t \to \pm\infty$, for solutions to (1) with initial states close to a solitary wave. Here $W_0(t)$ is the dynamical group of the free Klein-Gordon equation, $\Phi_\pm$ are the corresponding asymptotic states, and the remainder converges to zero as $t^{-1/2}$ in the "global energy norm" of the Sobolev space $H^1(\mathbb{R}) \oplus L^2(\mathbb{R})$.

Crucial role in the proof play our results on dispersion decay for the corresponding linearized Klein-Gordon equations. Moreover, we introduce a new class of piece-wise quadratic potentials which allow an exact description of the spectral properties for the linearized dynamics.

The research was supported partly by the Austrian Science Fund (FWF): M1329-N13.

References

[1] 1. Kopylova E., Komech A., *On asymptotic stability of moving kink for relativistic Ginsburg-Landau equation*, Comm. Math. Phys. 302 (2011), no. 1, 225-252.

[2] 2. Kopylova E., Komech A., *On asymptotic stability of kink for relativistic Ginzburg-Landau equation*, Arch. Rat. Mech. Anal. 202 (2011), no. 2, 213-245.

[3] 3. Komech A., Kopylova E., *Weighted energy decay for 1D Klein-Gordon equation*, Comm. PDE. 35 (2010), no. 2, 353-374.