On the multiplicity of generalized poles

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Generalized poles of generalized Nevanlinna functions are by definition eigenvalues, namely eigenvalues of the self-adjoint relation in a minimal realization of the function.

For scalar generalized Nevanlinna functions Q it is well known that the generalized pole (i.e. eigenvalue) α itself as well as the structure of the algebraic eigenspace can be described in terms of the the asymptotic behavior of Q close to the point α. Basically $-\frac{1}{Q}$ has to vanish of sufficiently high order.

In the case of matrix valued functions the situation is more complicated, e.g. a point α can also be both generalized zero and generalized pole. Here one has to take care also of the fact that the function can have different behavior in different directions. This is done in terms of so-called "pole-cancellation functions". However, this method is rather unpractical and gives only a possibility to characterize the dimension of the non-positive subspace of the algebraic eigenspace, not the whole algebraic eigenspace.

In this talk we are describing how even for matrix valued generalized Nevanlinna functions the asymptotic behavior of a certain transformation of the function can be used in order to describe the structure of the algebraic eigenspace completely.